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Derivation of Composition Formula for Evaluating Triple Integrals Numerically BY Using Trapezoidal rule and Midpoint and when the Singularity at both of End Integrals.

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Abstract- *The main aim of this paper to find values of the triple integration numerically which is improper (singular) of the partial derivatives or improper at both of of the integration .Also in this paper, we find general formula of the errors according behaviour of the integrands using new approach is different from the previous approached they are taken by most of the researchers . The $RMTM$ method is a composition method of using trapezoidal on the dimension of y and midpoint rule on the two dimensions interior x and exterior z with applying Romberg acceleration method when the number of subintervals of interval of interior integral are equal to the number of subintervals of exterior integral $\left(h = \bar{h} = \bar{\bar{h}}\right)$ such that $\bar{h}$ is the distances between coordinates of x and h is the distances between coordinates of y and $\bar{\bar{h}}$ is the distances between coordinates of z such that we can depend on it to calculate the triple integrations , and given higher accuracy in the results by few subintervals and time less than the request time for the researchers in the same subject .*

Keywords-:Region;integral;dimensions; subintervals;Romberg.

1. Introducation

The subject of numerical analysis is characterized by devising various methods to find approximate solutions to specific mathematical problems in an effective manner. The efficiency of these methods depends on both the accuracy and ease with which they can be implemented. Modern numerical analysis is the numerical interface to the broad field of applied analysis. Since the triple integrals are important in finding the volumes, the mean centers, the moment of inertia of the volumes, and finding the masses of variable density, for example the volume inside $x^2 + y^2 = 4x$ over $z = 0$,under $x^2 + y^2 = 4z$ calculate the average center of the volume falling within $x^2 + y^2 = 9$ And above $z = 0$ the plane and $x + z = 4$ below the plane, as well as finding masses of variable density, such as a piece of thin wire or a thin sheet of metal. Frank Ayers [1] therefore, a number of researchers have worked in the field of triple integrals.

In 2014, Salman [4] presented a numerical method to calculate the disrupted three integrals at one end of the integration by using the RMMM method resulting from Romberg acceleration with the midpoint rule applied to the dimensions x, y and z when the number of partial partial periods on the three dimensions is equal and he obtained good results in terms of accuracy and quickly Good and very short time.

In this paper, I present a theorem with the proof to derive a new rule for calculating approximate values of the triple integrals whose integrals are partial derivatives in both terms of the integration with the error formula for it and this rule is a result of applying Romperk's acceleration method to the values resulting from the use of the two midpoint rules on the internal x and external z dimensions x and the rule Trapezoid on y the middle dimension when $m = n = n_1$ (n the number of sub-periods into which the internal dimension period is fragmented $\left[x_0, x_n\right], n_1$ the number of sub-periods into which the period of the middle dimension is fragmented $\left[y_0, y_{n1}\right], m$ the number of partial periods that the external dimension period is divided into $\left[z_0, z_m\right]$, and we'll symbolize this method $RMTM$ In R terms of the Rombrk acceleration method and MTM the derivative base, we obtained good results in terms of accuracy, approach velocity, relatively few partial intervals, and very short time.

1.2. Calculate triple integrals with impaired and disruptive integrals and partial derivatives in both terms numerically

Theorem:

Let $f\ (x\ , y\ )$ the function be continuous and derivative at every point in the region $[x_0, x_n]\times[y_0, y_n]\times[z_0, z_n]$ Except at the points $(x_n, y_n, z_n), (x_0, y_0, z_0)$ The value of the triple integral $I = \int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f(x, y, z)dxdydz$ It can be calculated according to the following rule

$$
\begin{aligned}
MTM &= \int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f(x, y, z)dxdydz \\
&= \frac{h^3}{2}\sum_{k=0}^{n-1}\sum_{j=0}^{n-1}[\, f(x_i + .5h, y_0, z_k + .5h) + f(x_j + .5h, y_n, z_k + .5h) + 2\sum_{i=1}^{n-1} f(x_j + .5h, y_i, z_k + .5h)] + \\
&+ h^5\{[\frac{1}{24}D_x{}^2 - \frac{1}{12}D_y{}^2 + \frac{1}{24}D_z{}^2] \\
&+ h^6[(\frac{-1}{48}D_x{}^3 + \frac{1}{24}D_y{}^3 - \frac{1}{48}D_z{}^3) + \frac{1}{24}(D_y{}^2D_z + D_y{}^2D_x) - \\
&\frac{1}{48}(D_x{}^2D_z + D_z{}^2D_x + D_z{}^2D_y + D_x{}^2D_y)] + h^7[\cdots]\} f(x_1, y_1, z_1) + \\
&\{-h^5[\frac{1}{24}D_x{}^2 - \frac{1}{12}D_y{}^2 + \frac{1}{24}D_z{}^2] + h^6[(\frac{1}{48}D_x{}^3 - \frac{1}{24}D_y{}^3 + \frac{1}{48}D_z{}^3) + \\
&\frac{1}{48}(D_x{}^2D_z + D_z{}^2D_x) - \frac{1}{24}(D_y{}^2D_x + D_y{}^2D_z + D_x{}^2D_y + D_z{}^2D_y)] \\
&+ h^7[\cdots] + \cdots\} f(x_{n-1}, y_{n-1}, z_{n-1})
\end{aligned}
$$

And the formula for the error is $E_{MTM}(h) = I - MTM(h) = A_{MTM}h^2 + B_{MTM}h^4 + C_{MTM}h^6 + \cdots$ Where $A_{MTM}, B_{MTM}, C_{MTM}, \ldots$ Constants that depend on the partial derivatives of the function f .

Proof:

Suppose we have an integral

$$I=\int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f(x,y,z)dxdydz \tag{1}$$

Suppose that the function f (x, y, z) is continuous in the region of integration $[x_0,x_n]\times[y_0\times y_n]\times[z_0\times z_n]$ except at points (x_n,y_n,z_n) and (x_0,y_0,z_0) the above integration can be written as follows

$$I=\int_{z_1}^{z_k}\int_{y_0}^{y_m}\int_{x_0}^{x_n} f(x,y,z)dxdydz = MTM(h)+E(h) \qquad ...(2)$$

As $MTM(h)$ the integral value is numerically represented using the formula MTM and that is the correction limits chain correction terms can be added to the $MTM(h)$ values,and $h=\frac{(b-a)}{n}=\frac{(d-c)}{n_1}=\frac{(g-e)}{m}$ the error formula for single integrals with continuous integrals using the trapezoid rule is:

$$E_T(h)=-\frac{1}{12}h^2(f_n^{(1)}-f_0^{(1)})+\frac{1}{720}h^4(f_n^{(3)}-f_0^{(3)})-\cdots \qquad \cdots(3)$$

And the error formula for single integrals with continuous integrals using the midpoint rule is

$$E_M(h)=\frac{1}{6}h^2(f_{n_1}^{(1)}-f_0^{(1)})-\frac{6}{360}h^4(f_{n_1}^{(3)}-f_0^{(3)})-\cdots \qquad \cdots(4)$$

Fox [2]

Integration can be written $I=\int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f(x,y,z)dxdydz$ As follows, for the purpose of(isolating improper) the morbidity of the binomial

$$I=\int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f(x,y,z)dxdydz=$$

$$\int_{z_0}^{z_1}\int_{y_0}^{y_1}\int_{x_0}^{x_1} f(x,y,z)dxdydz+\int_{z_{n-1}}^{z_n}\int_{y_{n-1}}^{y_n}\int_{x_{n-1}}^{x_n} f(x,y,z)dxdydz+\int_{z_1}^{z_n}\int_{y_1}^{y_n}\int_{x_0}^{x_1} f(x,y,z)dxdydz+\int_{z_0}^{z_{n-1}}\int_{y_0}^{y_{n-1}}\int_{x_{n-1}}^{x_n} f(x,y,z)dxdydz+$$
$$\int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_1}^{x_{n-1}} f(x,y,z)dxdydz+\int_{z_0}^{z_1}\int_{y_1}^{y_n}\int_{x_1}^{x_n} f(x,y,z)dxdydz+\int_{z_{n-1}}^{z_n}\int_{y_0}^{y_{n-1}}\int_{x_0}^{x_{n-1}} f(x,y,z)dxdydz+\int_{z_1}^{z_{n-1}}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f(x,y,z)dxdydz$$

With regard to the six integrals, except for the first and the last, the integral is continuous derivatives in the period of their integration, and the values of the integrals can be calculated according to the following theorem $f(x,y,z)$ continuous and all its derivatives are present at every point in the region $[a,b]\times[c,d]\times[e,g]$ the approximate value of the triple integral $I=\int_e^g\int_c^d\int_a^b f(x,y,z)dxdydz$ it can be calculated from the following base:

$$MTM = \int_e^g \int_c^d \int_a^b f(x,y,z)dxdydz =$$

$$\frac{h^3}{2}\sum_{i=0}^{n-1}\sum_{k=0}^{n-1}\left(f(x_i+\frac{h}{2},c,z_k+\tfrac{h}{2})+f(x_i+\frac{h}{2},d,z_k+\tfrac{h}{2})+2\sum_{j=1}^{n-1} f(x_i+\frac{h}{2},y_j,z_k+\tfrac{h}{2})\right)$$

Hilal [3]

The calculation of integrals is as follows:

$$1)\int_{z_1}^{z_n}\int_{y_1}^{y_n}\int_{x_0}^{x_1} f(x,y,z)dxdydz = \sum_{k=1}^{n-1}\int_{z_k}^{z_{k+1}}\sum_{i=1}^{n-1}\int_{y_i}^{y_{i+1}}\int_{x_0}^{x_1} f(x,y,z)dxdydz =$$

$$\frac{h^3}{2}\sum_{k=1}^{n-1}\sum_{i=1}^{n-1}[f(x_0+\frac{h}{2},y_i,z_k+\frac{h}{2})+f(x_0+\frac{h}{2},y_{i+1},z_k+\frac{h}{2})+A_1h^2+B_1h^4+\cdots \qquad (5)$$

$$2)\sum_{k=0}^{n-2}\int_{z_k}^{z_{k+1}}\sum_{i=0}^{n-2}\int_{y_i}^{y_{i+1}}\int_{x_{n-1}}^{x_n} f(x,y,z)dxdydz = \frac{h^3}{2}\sum_{k=0}^{n-2}\sum_{i=0}^{n-2}[f(x_{n-1}+\frac{h}{2},y_i,z_k+\frac{h}{2})+f(x_{n-1}+\frac{h}{2},y_{i+1},z_k+\frac{h}{2})]$$

$$+A_2h^2+B_2h^4+\cdots \qquad \cdots(6)$$

$$3)\int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_1}^{x_{n-1}} f(x,y,z)dxdydz =$$

$$\sum_{k=0}^{n-1}\int_{z_k}^{z_{k+1}}\sum_{i=0}^{n-1}\int_{y_i}^{y_{i+1}}\sum_{j=1}^{n-2}\int_{x_1}^{x_n} f(x,y,z)dxdydz = \frac{h^3}{2}\sum_{k=0}^{n-1}\sum_{i=1}^{n-1}\sum_{j=1}^{n-2}[f(x_j+.5h,y_i,z_k+.5h)+f(x_j+.5h,y_{i+1},z_k+.5h)]$$

$$+A_2h^2+B_2h^4+\cdots \qquad \cdots(7)$$

$$4)\int_{z_0}^{z_1}\int_{y_1}^{y_n}\int_{x_1}^{x_n} f(x,y,z)dxdydz =$$

$$\int_{z_0}^{z_1}\sum_{i=1}^{n-1}\int_{y_i}^{y_{i+1}}\sum_{j=1}^{n-1}\int_{x_j}^{x_{j+1}} f(x,y,z)dxdydz = \frac{h^3}{2}\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}[f(x_j+\frac{h}{2},y_i,z_0+\frac{h}{2})+f(x_j+\frac{h}{2},y_{i+1},z_0+\frac{h}{2})]$$

$$+A_4h^2+B_4h^4+\cdots \qquad (8)$$

$$5)\int_{z_{n-1}}^{z_n}\int_{y_0}^{y_{n-1}}\int_{x_0}^{x_{n-1}} f(x,y,z)dxdydz =$$

$$\int_{z_{n-1}}^{z_n}\sum_{i=0}^{n-2}\int_{y_i}^{y_{i+1}}\sum_{j=0}^{n-2}\int_{x_j}^{x_{j+1}} f(x,y,z)dxdydz = \frac{h^3}{2}\sum_{i=0}^{n-2}\sum_{j=0}^{n-2}[f(x_j+\frac{h}{2},y_i,z_{n-1}+\frac{h}{2})+f(x_j+\frac{h}{2},y_{i+1},z_{n-1}+\frac{h}{2})]$$

$$+A_4h^2+B_4h^4+\cdots \qquad (9)$$

$$6)\int_{z_1}^{z_{n-1}}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f(x,y,z)dxdydz =$$

$$\sum_{k=1}^{n-2}\int_{z_1}^{z_{n-1}}\sum_{i=0}^{n-1}\int_{y_i}^{y_{i+1}}\sum_{j=0}^{n-1}\int_{x_j}^{x_{j+1}} f(x,y,z)dxdydz = \frac{h^3}{2}\sum_{k=1}^{n-2}\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}[f(x_j+\frac{h}{2},y_i,z_1+\frac{h}{2})+f(x_j+\frac{h}{2},y_{i+1},z_1+\frac{h}{2})]$$

$$+A_4h^2+B_4h^4+\cdots \qquad (10)$$

whereas $h=\dfrac{x_n-x_0}{n}$ and $j=1,2,\cdots n-1 \quad x_j=a+jh \qquad i=1,2,\cdots n-1 \quad y_i=c+ih$ $,k=1,2,3,\cdots,n+1$

whereas $A_i, B_i, C_i,\cdots$ and $i=1,2,3$ constants that depend on the values of the partial derivatives with respect to the two variables x,y it does not depend on h. The first and last integrations are calculated according to the following theorem:

Let $f(x,y,z)$ the function be continuous and derivative at every point in the region $[x_0,x_n]\times[y_0,y_n]\times[z_0,z_n]$ except for one of its derivatives at least not capable of being derivative at the point $(x,y,z)=(x_n,y_n,z_n)$, so the approximate value of the triple integral can be calculated from the following rule:

$$MTM=\int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f(x,y,z)dxdydz=\frac{h^3}{2}\sum_{k=0}^{n-2}\sum_{i=0}^{n-2}[f(x_i+.5h,y_0,z_k+.5h)+f(x_i+.5h,y_n,z_k+.5h)$$

$$+2\sum_{j=1}^{n-1} f(x_i+.5h,y_j,z_k+.5h)]\cdots$$

Hilal [3]

This is when the integral is undefined at the upper bound and its computation is as follows:

$$1)\int_{z_{n-1}}^{z_n}\int_{y_{n-1}}^{y_n}\int_{x_{n-1}}^{x_n} f(x,y,z)dxdydz=\frac{h^3}{2}[f(x_{n-1}+\frac{h}{2},y_{n-1},z_{n-1}+\frac{h}{2})+f(x_{n-1}+\frac{h}{2},y_n,z_{n-1}+\frac{h}{2})]$$

$$-\{h^5[\frac{1}{24}D_x^{\ 2}-\frac{1}{12}D_y^{\ 2}+\frac{1}{24}D_z^{\ 2}]+h^6[(\frac{-1}{48}D_x^{\ 3}+\frac{1}{24}D_y^{\ 3}-\frac{1}{48}D_z^{\ 3})+$$

$$\frac{1}{24}(D_y^{\ 2}D_z+D_y^{\ 2}D_x)-\frac{1}{48}(D_x^{\ 2}D_z+D_z^{\ 2}D_x+D_z^{\ 2}D_y+D_x^{\ 2}D_y)]+h^7[\cdots]]+\cdots\}f(x_1,y_1,z_1)+$$

$$+A_{MTM}h^2+B_{MTM}h^4\cdots \qquad \cdots(11)$$

And when the impairment is also at the lower end, according to the theorem derived from it Hilal [3], it is as follows:

$$2)\int_{z_0}^{z_1}\int_{y_0}^{y_1}\int_{x_0}^{x_1} f(x,y,z)dxdydz = \frac{h^3}{2}[f(x_0+\frac{h}{2},y_0,z_0+\frac{h}{2})+f(x_0+\frac{h}{2},y_1,z_0+\frac{h}{2})]$$
$$+\{h^5[\frac{1}{24}D_x{}^2-\frac{1}{12}D_y{}^2+\frac{1}{24}D_z{}^2]+h^6[(\frac{-1}{48}D_x{}^3+\frac{1}{24}D_y{}^3-\frac{1}{48}D_z{}^3)+$$
$$\frac{1}{24}(D_y{}^2D_z+D_y{}^2D_x)-\frac{1}{48}(D_x{}^2D_z+D_z{}^2D_x+D_z{}^2D_y+D_x{}^2D_y)]+h^7[\cdots]]+\cdots\}f(x_1,y_1,z_1)+$$
$$A_{MTM}h^2+B_{MTM}h^4\cdots f(x_1,y_1) \qquad \cdots(12)$$

and by adding the equations from… (6), (5) to (12), we get

$$MTM=\int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f(x,y,z)dxdydz$$
$$=\frac{h^3}{2}\sum_{k=0}^{n-1}\sum_{j=0}^{n-1}[f(x_i+.5h,y_0,z_k+.5h)+f(x_j+.5h,y_n,z_k+.5h)+2\sum_{i=1}^{n-1}f(x_j+.5h,y_i,z_k+.5h)]+$$
$$+h^5\{[\frac{1}{24}D_x{}^2-\frac{1}{12}D_y{}^2+\frac{1}{24}D_z{}^2]$$
$$+h^6[(\frac{-1}{48}D_x{}^3+\frac{1}{24}D_y{}^3-\frac{1}{48}D_z{}^3)+\frac{1}{24}(D_y{}^2D_z+D_y{}^2D_x)-$$
$$\frac{1}{48}(D_x{}^2D_z+D_z{}^2D_x+D_z{}^2D_y+D_x{}^2D_y)]+h^7[\cdots]\}f(x_1,y_1,z_1)$$

$$+\{-h^5[\frac{1}{24}D_x{}^2-\frac{1}{12}D_y{}^2+\frac{1}{24}D_z{}^2]+$$
$$h^6[(\frac{1}{48}D_x{}^3-\frac{1}{24}D_y{}^3+\frac{1}{48}D_z{}^3)+\frac{1}{48}(D_x{}^2D_z+D_z{}^2D_x)-$$
$$\frac{1}{24}(D_y{}^2D_x+D_y{}^2D_z+D_x{}^2D_y+D_z{}^2D_y)]$$
$$+h^7[\cdots]+\cdots\}f(x_{n-1},y_{n-1},z_{n-1})+A_{MTM}h^2+B_{MTM}h^4+C_{MTM}\cdots$$

whereas $A_{MTM}, B_{MTM}, C_{MTM},\cdots$

They are constants that depend on the value of partial derivatives with respect to the variables x y, Z, and do not depend on h. As the above formula includes the use of the mid- point rule on the inner and outer dimensions and the trapezoid rule on the middle dimension Y including the terms of correction plus the error due to the impairment in the derivative in the two points (x_n,y_n,z_n) ، (x_0,y_0,z_0) and about the two points $(x_{n-1},y_{n-1},z_{n-1})$, (x_1,y_1,z_1).

3. Examples

1- $\int_0^1\int_0^1\int_0^1 \sqrt{(x+y+z)/3-xyz}\,dxdydz$ analytical value unknown

2- $\int_0^1\int_0^1\int_0^1 (1/\sqrt{1-xyz}+1/\sqrt{x+y+z})dxdydz$ analytical value unknown

3- $\int_{-1}^0\int_0^1\int_{-1}^0 (x.y.z(x^2+y^2-z\wedge 2-z)^{1/2})dxdydz$ whose value is analytical 0.132598984165

1- $\int_0^1\int_0^1\int_0^1 \sqrt{(x+y+z)/3-xyz}\,dxdydz$ the integral here is continuous in the region of integration, but the partial derivatives are impaired at points(1,1,١),((0,0,0) and the type of impairment, so the radical correction limits according to the Theorem above are as follows

$$E_{MTM}(h)=A_{MTM}h^2+b_1h^{2.5}+b_2h^{3.5}+B_{MTM}h^4+\cdots$$

whear $i=1,2,3,\cdots \quad b_i, \quad A_{MTM}, B_{MTM},\cdots$ is constant

And by using the RMTM method, we obtained the results recorded in table (1) although the integral is unknown the analytical value, but we see through the table of the RMTM base that the same value is fixed horizontally (for five columns when n=256) say that the value is true to at least twelve decimal places) (0.607511819343)

2- $\int_0^1\int_0^1\int_0^1 (1/\sqrt{1-xyz}+1/\sqrt{x+y+z})dxdydz$ the integral here continues in the integration region, but the modulus of partial derivatives at points(1, 1,1) (0,0,0) and the type of radical and relative morbidity is therefore

$$E_{MTM}(h)=a_1h^{1.5}+A_{MTM}h^2+b_1h^{2.5}+b_2h^{3.5}+B_{MTM}h^4+b_ih^{4.5}+\cdots$$

using the RMTM method, we obtained the results listed in table (2) although the integral is unknown the analytical value, but we see through the table of the RMTM rule that the value itself is fixed horizontally (for three columns when n = 256) so we can say that the value is true at least to eleven decimal places 1.94964355464)) and here lies The benefit of this method is to find approximate values for these types of integrals .

3- $\int_{-1}^0\int_0^1\int_{-1}^0 (x.y.z(x^2+y^2-z\wedge 2-z)^{1/2})dxdydz$ It has a continuous integral, but it is impaired of the partial derivatives in the two points $(x,y,z)=(0,0,-1)$, $(x,y,z)=(0,0,0)$ and the type of radical morbidity and the analytical value of this integral is 1325989841650 rounded to twelve decimal places.

$$E_{MTM}(h)=a_1h^2+A_{MTM}h^4+b_1h^{5.5}+b_2h^6+B_{MTM}h^{6.5}+b_ih^{7.5}+\cdots$$

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from applying the MTM rule, we obtained an integer value for five decimal places after the comma when m = n = n1 = 256 compared with the analytical value of the integral, but after using Romprk acceleration, i.e. when applying the RMTM method based on these correction limits, the result improved as we obtained a correct value for one Ten decimal places after the comma, as shown in Table (3), noting that the time taken by the Matlab program to calculate is 7.50 seconds despite the presence of impairment in the two points.

N	MTM	2	2.5	3.5	4
1	0.611423746779				
2	0.607921116893	0.607168971840			
4	0.607551500689	0.607472130328	0.607501524070		
8	0.607515520691	0.607507794446	0.607511252379	0.607511702193	
16	0.607512157559	0.607511435369	0.607511788387	0.607511813171	0.607511815679
32	0.607511849890	0.607511783822	0.607511817607	0.607511818958	0.607511819089
64	0.607511822082	0.607511816111	0.607511819242	0.607511819317	0.607511819325
128	0.607511819588	0.607511819052	0.607511819337	0.607511819341	0.607511819342
256	0.607511819365	0.607511819317	0.607511819342	0.607511819343	0.607511819343

n	5.5	6	6.5
64	0.607511819329		
128	0.607511819342	0.607511819342	
256	0.607511819343	0.607511819343	0.607511819343

Tuble(1)

Calculate triple integration

$$\int_0^1\int_0^1\int_0^1 \sqrt{(x+y+z)/3-xyz}\,dxdydz$$

N	MTM	K=1.5	K=2	K=2.5	K=3.5
1	1.93090365978				
2	1.94565057016	1.95371592327			
4	1.94888363912	1.95065186324	1.94963050990		
8	1.94950529730	1.94984529344	1.94957643684	1.94956482534	
16	1.94961884093	1.94968094000	1.94962615552	1.94963683198	1.94964381362
32	1.94963916708	1.94965028381	1.94964006508	1.94964305198	1.94964365507
64	1.94964277773	1.94964475247	1.94964290869	1.94964351931	1.94964356463
128	1.94964341721	1.94964376695	1.94964343845	1.94964355221	1.94964355540
256	1.94964353034	1.94964359221	1.94964353396	1.94964355447	1.94964355469

K=4	K=4.5	K=5.5	K=6
1.94964364450			
1.94964355860	1.94964355462		
1.94964355478	1.94964355461	1.94964355461	
1.94964355465	1.94964355464	1.94964355464	1.94964355464

Tuble(2)

Calculate triple integration

$$\int_0^1\int_0^1\int_0^1 (1/\sqrt{1-xyz}+1/\sqrt{x+y+z})\,dxdydz$$

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N	MTM	k=2	k=4	k=5.5	k=6
1	0.153093108924				
2	0.137402620883	0.132172458203			
4	0.133790489537	0.132586445755	0.132614044925		
8	0.132896713997	0.132598788817	0.132599611688	0.132599285549	
16	0.132673431384	0.132599003846	0.132599018181	0.132599004770	0.132599000313
32	0.132617598574	0.132598987638	0.132598986557	0.132598985843	0.132598985542
64	0.132603638063	0.132598984559	0.132598984353	0.132598984304	0.132598984279
128	0.132600147668	0.132598984204	0.132598984180	0.132598984176	0.132598984174
256	0.132599275043	0.132598984168	0.132598984166	0.132598984165	0.132598984165

k=6.5	k=7.5	k=8	k=8.5
0.132598985377			
0.132598984265	0.132598984259		
0.132598984173	0.132598984172	0.132598984172	
0.132598984165	0.132598984165	0.132598984165	0.132598984165

Tuble(3)

Calculate triple integration

$$\int_{-1}^{0}\int_{0}^{1}\int_{-1}^{0}(x.y.z(x^2+y^2-z\wedge 2-z)^{1/2})dxdydz$$

4.Discussion and conclusion

In this paper, we dealt with calculating the approximate values of three integrals with continuous integrals but with poor partial or disruptive derivatives in both terms of the integration using the two rules of the middle point on the internal and external dimensions x, z and the base of the trapezoid on the middle dimension y when the partial periods are equal on all dimensions, which are denoted by MTM,and when using precipitate, Romprk stands for RMTM.we conclude from the results and tables of this research that this method gives good results with high accuracy as this rule gave correct results for five decimal places compared to the analytical value, and after using Romperk acceleration, I got a value close to eleven decimal places as in example (3) and in a short time as well when Their use in the first and second examples gave correct values (to several decimal places) despite the fact that the true value of the integral is not known. In the first integration, we obtained an approximate value close to twelve decimal places (0.607511819343) through the congruence of five columns. when $n = 256$ also the second integral, we obtained an approximate value of eleven decimal places 1.94964355464) by congruence of three columns and here lies the benefit of this method for finding approximate values for this type of integrals. And from observing the results of the aforementioned examples, we conclude that the RMTM method has high accuracy, excellent speed, short time and can be relied upon in calculating the triple integrals.

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