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Solving Gas Diffuses Phenomenon Using Variational Technique

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Abstract--- *This scientific study is concerned with an integral equation, especially with Volterra integral equation and used new variational formulation which corresponds on the linear operator* ***Lu = f*** *. The gas diffuses phenomenon has been solved using this variational formulation.*

keyword:- integral equation **,** variational formulation **,**gas diffuses phenomenon

1.Introduction

Integral equations appeared and used in various applied areas as important as differential equations but the integral formulations are more elegant and compact than the differential formulations due to the accuraey of solutions that obtained from the integral equations **[1].** The present study works in solving Volterra integral equation directly using variational approach (Magri approach) **[3,10].** Moreover, this new approach will used for solving gas diffuses phenomenon, as an application to Magri approach. **[7].** Diffusion refers to the process of particles moving from an area of high concentration to one of low concentration. The rate of this movement is a function of temperature, viscosity of the medium, and the size (mass) of the particles. Diffusion results in the gradual mixing of materials, and eventually, it forms a homogeneous mixture **[9].**

2. Integral Equations

An integral equation is an equation in which the unknown function $f(s)$ to be determined appears under the integral sign. A typical form of an integral equation in *f(s)* is of the form

$$f(s) = ɣ(s) + \lambda \int_{v(s)}^{u(s)} k(s,t) f(t) dt \qquad ...(1)$$

where *k(s,t)* is called the kernel of the integral equation , the function ɣ *(s)* is called the driving term , $u(s)$ and $v(s)$ are the limits of integration and may be both variables ,constants or mixed . It is easily observed that the unknown function *f(s)* appears under the integral sign as stated above, and out of the integral sign in most other cases. Integral equations arise naturally in physics, chemistry, biology and engineering applications modelled by initial value problems for a finite interval *[a, b]*. More details about the sources and origins of integral equations and as well how the initial value problem be conveted to the from of integral equations can be found in **[4,12].**

Our goal is to classify integral equations , and solving the gas diffuses problem using Magri approach.

3. Types of Integral Equations

According to the limets of integrations, the driving terms and the kernel, Arfken **[5]** gave a simple classification for integral equations , as follows:

1. If the limits of integration are fixed, the equation is called Fredholm integral equation, whereas if one of these limits is variable, then the equation will be Volterra integral equation.

2. If the unknown function appears only under the integral sing, we shall label it, first kind. While if it appears both inside and outside the integral sing, it will be labeled as, second kind. Symbolically , we write the Fredholm integral equation of the first kind and second kind respectively as follows :

$ɣ(s) = \lambda \int_a^b k(s,t\,(f(t)dt \quad , f(s) = ɣ(s) + \lambda \int_a^b k(s,t) f(t) dt$

whereas the Volterra integral equation of the first, second kind will be written respectively as:

$ɣ(s) = \int_a^s k(s,t)\, f(t) dt \quad , f(s) = ɣ(s) + \lambda \int_a^s k(s,t)\, f(t) dt$

For more types , see **[2, 1,5].**

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4. Integro - Differential Equations

Many scientific applications can represent mathematically in the form of an integro – differential equation, especially when we convert initial value problems or boundary value problems to integral equation. It contains both integral and derivatives of the unknown function $\Psi(s)$; therefore, a general Fredholm integro – differential equation is given by the form **[13]:**

$$\Psi^{(k)}(s) = \gamma(s) + \lambda \int_a^b k(s,t)\, \Psi(t)\, dt \,,\ \Psi^{(k)} = \frac{d^k \Psi}{ds^k} \qquad \ldots (2)$$

and a general Volterra integro – differential equation is given by :

$$\Psi^{(k)}(s) = \gamma(s) + \lambda \int_a^s k(s,t)\, \Psi(t)\, dt \,,\quad \Psi^{(k)} = \frac{d^k \Psi}{ds^k} \qquad \ldots (3)$$

Whereas a general Fredholm integro-differential equation is

$$\Psi^{(n)}(s) = \gamma(s) + \lambda_1 \int_a^s k_1(s,t)\, \Psi(t) dt + \lambda_2 \int_a^b k_2(s,t)\, \Psi(t) dt \qquad \ldots (4)$$

For more detiels see **[13]**

5. The Connection Between Differential and Integral Equations: [6]

There is a close connection between these kinds of equations many initial and boundary value problems can formulated as integral equations and vice versa. In general, the initial problems , dynamical systems and boundary value problem can be formulated as Volterra and Fredholm integral equations respectively.

Consider the differential problem (initial value problem):

$$\dot{y}(s) = f\,(s, y), y\,(s_0) = y_0 \qquad \ldots (5)$$

then $\int_s^{s_0} y'(t)dt = \int_{s_0}^s f(t, y(t))d\,t$ which means that:

$$y\,(s) = y_0 + \int_{s_0}^s f\big(t, y(t)\big)d\,t \qquad \ldots (6)$$

On the other hand, if (6) holds, we get $y\,(s_0) = y_0$ and $\dot{y}(s = f\,(s, y)$ which implies that (5) holds. Thus (5) and (6) are equivalent.

6. The Variational Formulation

Definition 1: (***Linear Operator)*** **[1]**

A functional is an operator whose range lies on the real line R or in the complex $\mathbb{C}$. A functional β on a linear space U is called **linear** if for any $u_1, u_2 \in$ U and any scalar α we have :

1. $\beta\,(u_1 + u_2) = \beta\,(u_1) + \beta\,(u_2)$.

2. $\beta\,(\alpha u_1) = \alpha\beta\,(u_1)$.

Definition 2 (***Bilinear Form***) **[3]:**

A functional (u, v) depending on $u, v \in U$ is said to be **bilinear** form, if it satisfies the following properties for $w \in U$ and $\alpha \in R$:

$(u+w,v)= \beta(u, v)+ \beta(w, v)$, $\beta(\alpha u, v) = \alpha\beta(u, v)$ and

$\beta(u,v+w)=\beta\,(u, v)+ \beta(u, v)$, $\beta(u,\alpha v)=\alpha\beta(u, v)$.

Definition 3 (Non-Degenerate Bilinear Form) **[3,11]:**

The functional $(u, v)$is denoted by $<u,v>$ which called **non- degenerate** on the two linear space U and V if for every $u \in U$ and $v \in V$, the following conditions are hold:

1.$< u, \bar{v} > 0$, $then\ \bar{v} = 0$.

2. $< \bar{u}, v > 0$, $hen\ \overline{u} = 0$.

Theorem : [3]

For a given symmetric linear operator L with respect to the chosen bilinear form $<u, v>$, the solutions of the equation Lu= f are the critical points of the functional:

$$F[f] = < Lu, L\,u > - < f, u >$$

proof : see **[1]** .

7. Gas Diffuses Phenomenon

In many fields, including physics, chemistry and life sciences, the concept of diffusion appears widely, so in general we mean diffusion is the net movement of anything (atom, ions, and molecules) from a region of higher concentration to a region of lower concentration. In this example, we will assume that the diffusion of gas into the liquid is in a long and narrow tube and that this process will take a long period of time as the concentration of gas in the tube depends only on the distance s from some initial point 0 (and is independent of time) **[8].**

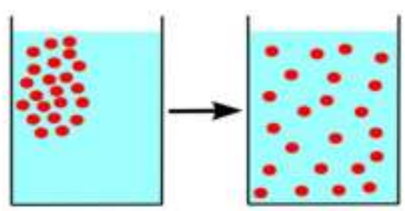

Fig.1: *gas diffuses into the liquid*

So we will write the equation as follows:

$$y''(s) + k\,y(s) = 0 \quad y(0) = s\,, \;\; y(1) = 0 \qquad ...(7)$$

In the beginning, we convert the differential equation in to an integral equation as follows:

Let $y''(s) = \Psi(s)$ ...(8)

Integral both sides from 0 to s to obtain:

$$\int_0^s y''(t)\,dt = \int_0^s \Psi(t)\,dt,$$

then we have:

$$y'(s) = y'(0) + \int_0^s \Psi(t)\,dt$$

Integrating again the last equation from 0 to s and substitute the values for the boundary conditions $(y(0) = \alpha)$ to obtain:

$$y(s) = y(0) + \int_0^s \int_0^s \Psi(t)\,dt$$

$$y(s) = s + \int_0^s (s-t)\cdot\Psi(t)\,dt \qquad \ldots(9)$$

Since $\Psi(s) = A\,y$

$\therefore \Psi(s) = A\,\{\, s + \int_0^s (s-t)\cdot\Psi(t)\,dt\,\}$

Thus

$\Psi(s) = As + A\,\int_0^s (s-t)\,\Psi(t)\,dt$

Equation (3.20) is a homogeneous 2nd order differential equation , as :

$\Psi(s) = D + A\,\int_0^s (s-t)\,\Psi(t)\,dt\;;\;(where\ D\ is\ constant\,).$ …(10)

which is a second kind of Volterra integral equation.

To solve this Volterra equation (10) using variational formulation Defined the linear operator as :

$L = I - \int_0^s (s-t\,)\Psi(t)dt$ with the bilinear form :

$< m,n > = \int_s m\,(s)n(t)ds$, the functional $F[\Psi]$ will be :

$F\,[\Psi] = \frac{1}{2}\,(Lm,m) - (f,L\,m)$

$F\,[\Psi]\ = \frac{1}{2}\int_s^0 (I\ -\ IN)\,\Psi(s)\ \ (I - IN)\,\Psi(s) - \gamma\,(s)(I - IN)\,\Psi\,(s)ds$

$\mathrm{F}\,[\Psi] = \frac{1}{2}\int_0^s\ \left[\Psi\,(s) + \int_0^s k(s,t\,)\Psi(t)dt\right]^2 ds - - \int_0^s \gamma\,(s)\,\left[\Psi\,(s) + \int_0^s k(s,t\,)\Psi(t)dt\right] ds$

... (11)

Suppose that the function $\Psi\,(t)$ as a linear combination of some basis as: $\Psi(t) = a_1 + a_1 s + a_1 s^2$ As with the critical points of the functional $F[\,\Psi]$ in (10) are found by equating the derivatives of $F[\Psi]$ with respect to the coefficient of the solution (a_1, a_2 and a_3) to zero, then solve the obtained system of linear algebraic equations:

$\frac{43}{60}a_1 + \frac{25}{72}a_2 + \frac{187}{840}a_3 = 3$

$\frac{25}{72}a_1 + \frac{341}{1260}a_2 + \frac{121}{576}a_3 = \frac{24}{10}$

$\frac{187}{840}a_1 + \frac{121}{576}a_2 + \frac{4327}{9072}a_3 = \frac{17}{9}$

which will give the values of a_1, a_2 and a_3:

a_1= -0,5504 , a_2 = 10,0602 , a_3= -0,5887

This result shortens a lot of efforts and time for programming rather than solving the required problem, since using the linear system is easier for programming , as differential equation where the solution is difficult . Moreover, using the linear system is easier for solving in order to solve it's physics equation gas diffuses phenomenon.

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