International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 5 Issue 7, July - 2021, Pages: 40-47

Characterization Of Fuzzy Soft Tri-*semi*-open set In Fuzzy Soft Tri-topological Spaces

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Abstract— *In this paper, we define a fuzzy soft tri-semi-open set (fuzzy soft $\tau_1\tau_2\tau_3$-semi-open set) in a Fuzzy Soft Tri-topological spaces and investigate it is properties, a detailed study is carried out on properties of Fuzzy Soft Tri-semi-interior, Fuzzy Soft Tri-semi-closure of Fuzzy Soft sets and Fuzzy Soft Tri-semi-neighborhood of a Fuzzy Soft point which are fundamental for further research on the theory of Fuzzy Soft Tri-topological spaces (w.r.t. Fuzzy Soft Tri-semi-open set).*

2010 AMS Classification: 54A05, 54A10, 54E55, 54E99.

Keywords— Fuzzy Soft Tri-topological space; Fuzzy Soft $\tau_1\tau_2\tau_3$-semi-open set (Fuzzy Soft Tri-semi-open set); Fuzzy Soft $\tau_1\tau_2\tau_3$-semi-interior; Fuzzy Soft $\tau_1\tau_2\tau_3$-semi-closure, Fuzzy Soft $\tau_1\tau_2\tau_3$-semi-neighborhood.

1. INTRODUCTION (*Heading 1*)

In 1965, Fuzzy set was introduced by Zadeh in [1] as a mathematical way to represent and deal with vagueness in everyday life. And the applications of Fuzzy set theory can be found in many branches of sciences, see [2], [3].
In 1999, Molodtsov [4] pioneered soft set theory by describing it as a new mathematical method, presenting many fundamental findings, and successfully applying it to a number of mathematical areas such as smoothness of functions, theory of probability, Riemann-integration, operations analysis, Perron integration, and so on A soft set is a list of approximations to an object's definition. He also showed that Soft set theory is immune to the parameterization inadequacy problem that plagues Fuzzy, Rough, Probability, and Game theory. Soft sets have significant applications in decision-making problems and information systems [5], [6].
In 2001, Maji et al. [7] By embedding the ideas of Fuzzy sets, the definition of Fuzzy Soft sets was introduced. Many important applications of Soft set theory have been extended by researchers using this concept of Fuzzy Soft sets. Roy and Maji [8] presented some Fuzzy Soft set applications.. Aktas and Cagman [9] compared Soft sets with Fuzzy sets and rough sets are similar definitions. Yang et al. [10] described the operations on Fuzzy Soft sets as negation, triangular norm, and triangular conorm, which are based on three Fuzzy logic operators. The combination of interval-valued Fuzzy set and Soft set was proposed by Xiao et al. [11]. In 1963, Kelly By embedding the ideas of Fuzzy sets, the definition of Fuzzy Soft sets was introduced. Many important applications of Soft set theory have been extended by researchers using this concept of Fuzzy Soft sets. Roy and Maji [8] presented some Fuzzy Soft set applications..
Ittanagi [15] introduced the concept of Soft Bitopological spaces in 2014, which he described as spaces defined over an initial universal set with a fixed set of parameters, as well as some forms of Soft separation axioms in Soft Bitopological spaces. Fuzzy Soft Bitopological spaces are a generalization of Fuzzy Soft topological spaces.
Mukherjee and Park [16] first proposed the concept of Fuzzy Soft Bitopological Space in 2015, and investigated some of its basic properties, see [17], [18].
In 2000, By modifying - regularity for spaces with three topologies, Kovar [19] introduced the definition of Tri-topological spaces, which they describe as spaces with three topologies., i.e. triple of topologies on the same set, Palaniammal [20] studied Tri-topological spaces and introduced semi-open and pre-open sets in Tri-topological spaces and he also introduced Fuzzy Tri-topological space.
In 2004, Asmhan was introduced the definition of δ^*-open set in Tri-topological spaces [21]. And in [22] she defined the δ^*-connectedness in Tri-topological spaces, also Asmhan et al. [23] defined the δ^*-base in Tri-topological spaces. In [24], [25] the reader can find a relationships among separation axioms, and a relationships among some types of continuous and open functions in topological, Bitopological and Tri-topological spaces, and in 2017, Asmhan introduced the new definitions of countability and separability in Tri-topological spaces namely δ^*-countability and δ^*-separability [26].
In 2017, Asmhan F.H. presented the concept of the Soft Tri-topological spaces [27], and by the same author the concept of Fuzzy Soft topological spaces have been generalized to initiate the study of Fuzzy Soft Tri-topological spaces in [28].
In the present paper, we introduce and characterize the Fuzzy Soft open set in a Fuzzy Soft Tri-topological spaces namely Fuzzy Soft $\tau_1\tau_2\tau_3$- semi -open set (Fuzzy Soft Tri- semi -open set). Also, look at certain fundamental properties..

In section 2, we present some preliminary concepts that will play a key role in our work. The third and fourth parts of the manuscript include the characterization of the Fuzzy Soft $\tau_1\tau_2\tau_3$-semi-open set, as well as examples and theorems. Finally, in section 5, the conclusions are presented, as well as some suggestions for future research.

2. Preliminaries :

In this section, we present the basic definitions of Fuzzy Soft set theory, Soft set theory and Fuzzy set theory that are useful for subsequent discussions and which will be a central role in our work.

Definition 2.1. [1] Let $\mathcal{U}$ be an universe. A Fuzzy set X over $\mathcal{U}$ is a set defined by a function μ_X representing a mapping $\mu_X: \mathcal{U} \to [0,1]$, μ_X is called the membership function of X and the value $\mu_X(u)$ is called the grade of membership of $u \in \mathcal{U}$. The value represents the degree of u belonging to the Fuzzy set X .Thus , a Fuzzy set X over $\mathcal{U}$ can be represented as follows :$X = \{(\mu_X(u)/u): u \in \mathcal{U}, \mu_X \in [0,1]\}$
Definition 2.2. [4] Let the set $\mathcal{U}$ be an initial universe and E be a set of parameters. Let $\mathcal{P}(\mathcal{U})$ denotes the power set of $\mathcal{U}$ and $\mathbb{A}$ be a non-empty subset of E. A pair $(\mathrm{F}, \mathbb{A})$ is said to be a $soft$ −set over $\mathcal{U}$ where F is a mapping given by $\mathrm{F}: \mathbb{A} \to \mathcal{P}(\mathcal{U})$.
In other words, a $soft$ −set over $\mathcal{U}$ is a parametrized family of subsets of the universe $\mathcal{U}$. For e $\in \mathbb{A}$, F(e) may be considered as the set of e−approximate elements of the $soft$ −set $(\mathrm{F}, \mathbb{A})$. Clear that, a $soft$ −set is not a set.
Definition 2.3. [29] A Fuzzy Soft set S_A over $\mathcal{U}$ is a set defined by a function ξ_A representing amapping $\xi_A: E \to F(\mathcal{U})$ such that $\xi_A(x) = \emptyset$ if $x \notin A$. Here ξ_A is called Fuzzy approximate function of the Fuzzy Soft set S_A and the value $\xi_A(x)$ is a set called $x-$ element of the Fuzzy Soft set for all $x \in E$. Thus, an Fuzzy Soft set S_A over $\mathcal{U}$ can be represented by the set of ordered pair $S_A = \{(x, \xi_A(x): x \in E, \xi_A(x) \in F(\mathcal{U})\}$.
Note that the set of all Fuzzy Soft sets over $\mathcal{U}$ will be denoted by F.S.$(\mathcal{U})$ or F.S.$(\mathcal{U}, E)$.
Definition 2.4.[7] Let $S_A \in \mathrm{F.S.}(\mathcal{U}, E)$.If $\xi_A(x) = \emptyset$ for all$x \in E$, then S_A is called an empty Fuzzy Soft set, denoted by $S_\emptyset$ or(0_E) .
Definition 2.5.[7] Let $S_A \in \mathrm{F.S.}(\mathcal{U}, E)$.If $\xi_A(x) = \mathcal{U}$ for all $x \in A$, then S_A is called A- universal Fuzzy Soft set, denoted by S_E or (1_E) .
Definition 2.6. [7] Let $S_A, S_B \in \mathrm{F.S.}(\mathcal{U}, E)$. Then S_A is called a Fuzzy Soft subset of S_B, denoted by $S_A \sqsubseteq S_B$ If $\xi_A(x) \sqsubseteq \xi_B(x)$ for all $x \in E$.
Definition 2.7. [30] $S_A \sqsubseteq S_B$ does not mean as in the classical subset. (i.e. does not imply that every element of S_A is an element of S_B) .
Definition 2.8. [7] Let $S_A, S_B \in \mathrm{F.S.}(\mathcal{U}, E)$. Then the two Fuzzy Soft sets S_A and S_B are equal, written as $S_A = S_B$ If and only if $\xi_A(x) = \xi_B(x)$ for All $x \in E$.
Definition 2.9. [7] Let $\mathcal{S}_A \in \mathrm{F.S.}(\mathcal{U}, E)$. Then the complement $\mathcal{S}_A^c$ of $\mathcal{S}_A$ is a Fuzzy Soft set such that $\xi_{A^c}(x) = \xi_A^c(x)$ for all $x \in E$, where $\xi_A^c(x)$ is complement of all set $\xi_A(x)$. Clear that $(\mathcal{S}_A^c)^c = \mathcal{S}_A$, $\mathcal{S}_\emptyset^c = \mathcal{S}_E$ and $\mathcal{S}_E^c = \mathcal{S}_\emptyset$.
Definition 2.10.[7] Let $S_A, S_B \in \mathrm{F.S.}(\mathcal{U}, E)$. Then the union of S_A and S_B, denoted by $S_A \sqcup S_B$, is defined by its Fuzzy approximate function $\xi_{A \sqcup B} = \xi_A(x) \sqcup \xi_B(x)$ for all $x \in E$.
Definition 2.11.[7] Let S_A, $S_B \in \mathrm{F.S.}(\mathcal{U}, E)$. Then the intersection of S_A and S_B , denoted by $\mathcal{S}_A \sqcap \mathcal{S}_B$, is defined by its Fuzzy approximate function $\xi_{A \sqcap B} = \xi_A(x) \sqcap \xi_B(x)$ for all $x \in E$.
Definition 2.12. [31] Let τ be the collection or sub family of Fuzzy Soft set over $\mathcal{U}$ (i.e. $\tau \sqsubseteq \mathrm{F.S.}(\mathcal{U}, E)$. Then τ is said to be a Fuzzy Soft topology on the universal set $\mathcal{U}$ if satisfying the following properties:
(i) $\mathcal{S}_\emptyset$, $\mathcal{S}_E \in \tau$
(ii) If $\mathcal{S}_A, \mathcal{S}_B \in \tau$, then $\mathcal{S}_A \sqcap \mathcal{S}_B \in \tau$
(iii) If $\mathcal{S}_{Aj} \in \tau$, $\forall$ j $\in \Lambda$, where Λ is some index set, then $\sqcup_{j \in \Lambda} \mathcal{S}_{Aj} \in \tau$.
Then the triple $(\mathcal{U}, E, \tau)$ is called a Fuzzy Soft topological space over $\mathcal{U}$. And each member of τ is called Fuzzy Soft open set in $(\mathcal{U}, E, \tau)$. Also Fuzzy Soft set is called Fuzzy Soft closed if and only if its complement is Fuzzy Soft open.
Definition 2.13. [16] Let $(\mathcal{U}, E, \tau_1)$ and $(\mathcal{U}, E, \tau_2)$be the two Fuzzy Soft topological spaces over $\mathcal{U}$. Then $(\mathcal{U}, E, \tau_1, \tau_2)$ is called a Fuzzy Soft bitopological space .
Definition 2.14. [28] Let $(\mathcal{U}, E, \tau_1)$, $(\mathcal{U}, E, \tau_2)$ and $(\mathcal{U}, E, \tau_3)$ be the three Fuzzy Soft topological spaces on $\mathcal{U}$. Then a space equipped with three Fuzzy Soft topologies, i.e. triple of Fuzzy Soft topologies on the same set is called a Fuzzy Soft Tri-topological space and denoted by $(\mathcal{U}, \mathrm{E}, \tau_1, \tau_2, \tau_3)$. Where the three Fuzzy Soft topological space are independently satisfy the axioms of Fuzzy Soft topological space.
Definition 2.15. [28] Let $(\mathcal{U}, \mathrm{E}, \tau_1, \tau_2, \tau_3)$ be a Fuzzy Soft Tri-topological space and let $\mathcal{S}_E$ be a Fuzzy Soft set over $(\mathcal{U}, E)$. Then :
i. The $\tau_j(\mathrm{j} = 1,2,3)$ −Fuzzy Soft closure of $\mathcal{S}_E$ denoted by $\tau_j(\mathrm{j} = 1,2,3)cl(\mathcal{S}_E)$, is the intersection of all $\tau_j(\mathrm{j} = 1,2,3)$ Fuzzy Soft closed supersets.

ii. The τ_j(j = 1,2,3) −Fuzzy Soft interior of $\mathcal{S}_E$ denoted by $\tau_j(\mathrm{j}=1,2,3)int(\mathcal{S}_E)$, union of all τ_j(j = 1,2,3) −Fuzzy Soft open sets contained in $\mathcal{S}_E$.

Definition 2.16. [28] Let $(\mathcal{U}, \mathrm{E}, \tau_1, \tau_2, \tau_3)$ be a fuzzy soft tritopological space and Γ_E is a fuzzy soft set in $\mathcal{U}$, then:

(i) Γ_E is called a fuzzy soft $\tau_1\tau_2\tau_3$-open set if $\Gamma_E = f_E \sqcup g_E \sqcup h_E$, where $f_E \in \tau_1$, $g_E \in \tau_2$ and $h_E \in \tau_3$. The complement of fuzzy soft $\tau_1\tau_2\tau_3$-open set is called fuzzy soft $\tau_1\tau_2\tau_3$-closed. The family of all fuzzy soft $\tau_1\tau_2\tau_3$-open sets is denoted by $\mathrm{FS}.\tau_1\tau_2\tau_3.\mathrm{O}(\mathcal{U})$. And the family of all fuzzy soft $\tau_1\tau_2\tau_3$-closed sets is denoted by $\mathrm{FS}.\tau_1\tau_2\tau_3.\mathrm{C}(\mathcal{U})$.

Definition 2.17. [28] Let $(\mathcal{U}, E, \tau_1, \tau_2, \tau_3)$ be a fuzzy soft tritopological space, and Γ_E is a fuzzy soft set in $\mathcal{U}$, then:

i. The fuzzy soft $\tau_1\tau_2\tau_3$-closure of Γ_E denoted by $\mathrm{FS}.\tau_1\tau_2\tau_3cl(\Gamma_E)$ is defined by:
$\mathrm{FS}.\tau_1\tau_2\tau_3cl(\Gamma_E) = \sqcap\{g_E\colon \Gamma_E \sqsubseteq g_E,\ and\ g_E$ is fuzzy soft $\tau_1\tau_2\tau_3$-closed$\}$

ii. The fuzzy soft $\tau_1\tau_2\tau_3$-interior of Γ_E, denoted by $\mathrm{FS}.\tau_1\tau_2\tau_3int(\Gamma_E)$ is defined by:
$\mathrm{FS}.\tau_1\tau_2\tau_3int(\Gamma_E) = \sqcup\{h_E\colon h_E \sqsubseteq \Gamma_E,\ and\ h_E$ is fuzzy soft $\tau_1\tau_2\tau_3$-open$\}$

3. FUZZY SOFT $\tau_1\tau_2\tau_3$-semi-OPEN SET (FUZZY SOFT TRI-semi-OPEN SET)

In this section a study of fuzzy soft tri-semi-open set is initiated.

Definition 3.1. [28] Let $(\mathcal{U}, \mathrm{E}, \tau_1, \tau_2, \tau_3)$ be a Fuzzy Soft Tri-topological space and let F_E be a Fuzzy Soft set over $(\mathcal{U}, E)$ then F_E is called a Fuzzy Soft $\tau_1\tau_2\tau_3$-semi-open set (or Fuzzy Soft Tri-semi-open set) if $F_E \sqsubseteq (\mathrm{F.S.}\tau_1\tau_2\tau_3\mathrm{cl}(\mathrm{F.S.}\tau_1\tau_2\tau_3\mathrm{int}(F_E))$. The complement of Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*-open set is defined to be Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*-closed.

Example 3.2. Let $\mathcal{U} = \{u_1, u_2\}$, $E = \{x_1, x_2\}, \tau_1 = \{0_E, 1_E, \psi_{1E}, \psi_{2E}\}, \tau_2 = \{0_E, 1_E, \gamma_{1E}, \gamma_{2E}\}$ and $\tau_3 = \{0_E, 1_E, \beta_E\}$. Where $\psi_{1E}, \psi_{2E}, , \gamma_{1E}, \gamma_{2E}$ and β_E are Fuzzy Soft sets over $(\mathcal{U}, E)$ defined as follows ;

$\psi_{1E} = \{(x_1, \{0.3/u_1, 0.4/u_2\}), (x_2, \{0.1/u_1, 0.2/u_2\})\}$
$\psi_{2E} = \{(x_1, \{0.4/u_1, 0.6/u_2\}), (x_2, \{0.2/u_1, 0.5/u_2\})\}$
$\gamma_{1E} = \{(x_1, \{0.3/u_1, 0.4/u_2\}), (x_2, \{0.3/u_1, 0.8/u_2\})\}$
$\gamma_{2E} = \{(x_1, \{0.2/u_1, 0.4/u_2\}), (x_2, \{0.2/u_1, 0.5/u_2\})\}$
And $\beta_E = \{(x_1, \{0.3/u_1, 0.0/u_2\}), (x_2, \{0.0/u_1, 0.2/u_2\})\}$

Then τ_1, τ_2 and τ_3 are three Fuzzy Soft topologies over $(\mathcal{U}$,E). Therefore $(\mathcal{U}, E, \tau_1, \tau_2, \tau_3)$ is a Fuzzy Soft Tri-topological space. It is clear that the family of all Fuzzy Soft $\tau_1\tau_2\tau_3$- open sets are: $F.S.\tau_1\tau_2\tau_2 .\mathrm{O}(\mathcal{U}) = \{0_E, 1_E, \psi_{1E}, \psi_{2E}, \gamma_{1E}, \gamma_{2E}, \beta_E, \lambda_E, \delta_E\}$
$= \tau_1 \sqcup \tau_2 \sqcup \tau_3 \sqcup \{\lambda_E, \delta_E\}$. Where $\psi_{2E} \sqcup \gamma_{2E} \sqcup \beta_E = \psi_{2E}$, $\psi_{1E} \sqcup \gamma_{1E} \sqcup \beta_E = \gamma_{1E}$
$\psi_{2E} \sqcup \gamma_{1E} \sqcup \beta_E = \lambda_E = \{(x_1, \{0.4/u_1, 0.6/u_2\}), (x_2, \{0.3/u_1, 0.8/u_2\})\}$
$\psi_{1E} \sqcup \gamma_{2E} \sqcup \beta_E = \delta_E = \{(x_1, \{0.3/u_1, 0.4/u_2\}), (x_2, \{0.2/u_1, 0.5/u_2\})\}$

Now, we find the Fuzzy Soft $\tau_1\tau_2\tau_3$ closed sets :
$F.s.\tau_1\tau_2\tau_3 .\mathrm{C}(\mathcal{U}) = \{1_E, 0_E, \psi^c_{1E}, \psi^c_{2E}, \gamma^c_{1E}, \gamma^c_{2E}, \beta^c_E, \lambda^c_E, \delta^c_E\}$, where defined as follows; $\psi^c_{1E} = \{(x_1, \{0.7/u_1, 0.6/u_2\}), (x_2, \{0.9/u_1, 0.8/u_2\})\}$
$\psi^c_{2E} = \{(x_1, \{0.6/u_1, 0.4/u_2\}), (x_2, \{0.8/u_1, 0.5/u_2\})\}$
$\gamma^c_{1E} = \{(x_1, \{0.7/u_1, 0.6/u_2\}), (x_2, \{0.7/u_1, 0.2/u_2\})\}$
$\gamma^c_{2E} = \{(x_1, \{0.8/u_1, 0.6/u_2\}), (x_2, \{0.8/u_1, 0.5/u_2\})\}$
$\beta^c_E = \{(x_1, \{0.7/u_1, 1.0/u_2\}), (x_2, \{1.0/u_1, 0.8/u_2\})\}$
$\lambda^c_E = \{(x_1, \{0.6/u_1, 0.4/u_2\}), (x_2, \{0.7/u_1, 0.2/u_2\})\}$
$\delta^c_E = \{(x_1, \{0.7/u_1, 0.6/u_2\}), (x_2, \{0.8/u_1, 0.5/u_2\})\}$

It is clear, the Fuzzy Soft set ψ_{1E} is a Fuzzy Soft $\tau_1\tau_2\tau_3$-semi-open set since
$F.S.\tau_1\tau_2\tau_3int(\psi_{1E}) = (\psi_{1E})$, then
$F.S.\tau_1\tau_2\tau_3cl(\psi_{1E}) = \{\psi^c_{1E} \sqcap \psi^c_{2E} \sqcap \gamma^c_{1E} \sqcap \gamma^c_{2E} \sqcap \lambda^c_E \sqcap \delta^c_E \sqcap 1_E\} = \lambda^c_E$
, thus
$F.S.\tau_1\tau_2\tau_3cl\big(F.S.\tau_1\tau_2\tau_3int(\psi_{1E})\big) = \lambda^c_E$, hence $\psi_{1E} \sqsubseteq \lambda^c_E$.

The Fuzzy Soft set ψ_{2E} is a Fuzzy Soft $\tau_1\tau_2\tau_3$-semi-open set since
$F.S.\tau_1\tau_2\tau_3int(\psi_{2E}) = \psi_{2E}$, $F.S.\tau_1\tau_2\tau_3cl(\psi_{2E}) = \{\psi^c_{1E} \sqcap \gamma^c_{2E} \sqcap \delta^c_E \sqcap 1_E\} = \delta^c_E$
, thus
$F.S.\tau_1\tau_2\tau_3cl\big(F.S.\tau_1\tau_2\tau_3int(\psi_{2E})\big) = \delta^c_E$, hence $\psi_{2E} \sqsubseteq \delta^c_E$,

And, the all Fuzzy Soft $\tau_1\tau_2\tau_3$-open sets $\{0_E, 1_E, \gamma_{1E}, \gamma_{2E}, \beta_E, \lambda_E, \delta_E\}$ are Fuzzy Soft $\tau_1\tau_2\tau_3$-semi-open sets.

Also, if we take the Fuzzy Soft set D_E which defined as; $\{(x_1, \{0.3/u_1, 0.1/u_2\}),\ (x_2, \{0.1/u_1, 0.2/u_2\}$, then D_E is a Fuzzy Soft $\tau_1\tau_2\tau_3$-semi-open set since $F.S.\tau_1\tau_2\tau_3 int(D_E) = \{\beta_E \sqcup 0_E\} = \beta_E$
$F.S.\tau_1\tau_2\tau_3 cl(\beta_E) = \lambda_E^c$, thus
$F.S.\tau_1\tau_2\tau_3 cl(F.S.\tau_1\tau_2\tau_3 int(D_E)) = \lambda_E^c$ hence $D_E \sqsubseteq \lambda_E^c$
And, if we take the Fuzzy Soft set G_E which defined as; d $G_E = \{(x_1, \{0.3/u_1, 0.3/u_2\}),\ (x_2, \{0.2/u_1, 0.2/u_2\})\}$, then G_E is a Fuzzy Soft $\tau_1\tau_2\tau_3$-semi-open set since $F.S.\tau_1\tau_2\tau_3 int(G_E) = \{\beta_E \sqcup 0_E\} = \beta_E$
$F.S.\tau_1\tau_2\tau_3 cl(\beta_E) = \lambda_E^c$, thus
$F.S.\tau_1\tau_2\tau_3 cl(F.S.\tau_1\tau_2\tau_3 int(G_E)) = \lambda_E^c$ hence $G_E \sqsubseteq \lambda_E^c$

But, if we take the Fuzzy Soft set A_E defined as $\{(x_1, \{0.3/u_1, 0.3/u_2\}), (x_2, \{0.3/u_1, 0.4/u_2\})\}$, then A_E is not a Fuzzy Soft $\tau_1\tau_2\tau_3$-semi-open set since
$F.S.\tau_1\tau_2\tau_3 int(A_E) = \{\beta_E \sqcup 0_E\} = \beta_E$, then $F.S.\tau_1\tau_2\tau_3 cl(\beta_E) = \lambda_E^c$
Thus $F.S.\tau_1\tau_2\tau_3 cl(F.S.\tau_1\tau_2\tau_3 int(\psi_{2E})) = \lambda_E^c$, hence $A_E \not\sqsubseteq \lambda_E^c$,
Also, the Fuzzy Soft sets $B_E = \{(x_1, \{0.4/u_1, 0.4/u_2\}), (x_2, \{0.3/u_1, 0.4/u_2\})\}$, $C_E = \{(x_1, \{0.5/u_1, 0.5/u_2\}), (x_2, \{0.3/u_1, 0.4/u_2\})\}$, …..etc. , are not Fuzzy Soft $\tau_1\tau_2\tau_3$-semi-open set.

Definition 3.3. Let $(\mathcal{U}, E, \tau_1, \tau_2, \tau_3)$ be a Fuzzy Soft Tri-topological space over $(\mathcal{U}, E)$ and F_E be a Fuzzy Soft set over $(\mathcal{U}, E)$. Then F_E is said to be a Fuzzy Soft $\tau_1\tau_2\tau_3$ -*semi*-closed if $\mathrm{F.S.}\tau_1\tau_2\tau_3 int(\mathrm{F.S.}\tau_1\tau_2\tau_3 cl(F_E)) \sqsubseteq F_E$.

Definition 3.4. let$(\mathcal{U}, \mathrm{E}, \tau_1, \tau_2, \tau_3)$ be a Fuzzy Soft Tri-topological space and $F_E \in \mathrm{F.S.}(\mathcal{U}, \mathrm{E})$ then the Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*-closure of F_E is defined as;
$\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-cl$(F_E) = Inf\{g_E \mid F_E \sqsubseteq g_E$, where g_E is $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-closed set$\}$
i.e., $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*- cl(F_E) is the smallest Fuzzy Soft $\tau_1\tau_2\tau_3$- *semi*-closed set containing F_E .

Theorem 3.5. Let $(\mathcal{U}, E, \tau_1, \tau_2, \tau_3)$ be a Fuzzy Soft Tri – topological space and $F_E, G_E \in \mathrm{F.S.}(\mathcal{U}, \mathrm{E})$. Then:
(1) $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*- $cl(0_E) = 0_E$ *and* $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*- $cl(1_E) = 1_E$
(2) $F_E \sqsubseteq \mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-$cl(F_E)$
(3) F_E is a Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*-closed if and only if $F_E = \mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*- $cl(F_E)$
(4) $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-$cl(\mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*- $cl(F_E))$ = $\mathrm{F.S.}\tau_1\tau_{02}\tau_3$ -*semi*- cl(F_E)
(5) $F_E \sqsubseteq G_E \Rightarrow \mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*- $cl(F_E) \sqsubseteq \mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*- $cl(G_E)$
(6) $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*- $cl(F_E \sqcup G_E) = \mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*- $cl(F_E) \sqcup \mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*- $cl(G_E)$
(7) $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*- $cl(F_E \sqcap G_E) \sqsubseteq \mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*- $cl(F_E) \sqcap \mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*- $cl(G_E)$
Proof. (1) Since 0_E is Fuzzy Soft $\tau_1\tau_2\tau_3$-semi-closed we have by (3) from this theorem $\mathrm{F.S.}\tau_1\tau_2\tau_3\text{-semi- cl}(0_E) = 0_E$ and $\mathrm{F.S.}\tau_1\tau_2\tau_3 \text{ -semi- cl}(1_E) = 1_E$
(2) By Definition 3.7 $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -semi- cl(F_E) is the intersection of any Fuzzy Soft $\mathrm{F.S.}\tau_1\tau_2\tau_3$- semi-closed supersets of F_E. Hence $F_E \sqsubseteq \mathrm{F.S.}\tau_1\tau_2\tau_3$-semi- cl$(F_E)$
(3) Let F_E be a Fuzzy Soft $\tau_1\tau_2\tau_3$-semi-closed set. By (2), we have $F_E \sqsubseteq \mathrm{F.S.}\tau_1\tau_2\tau_3$ -semi- cl(F_E). Since $\mathrm{F.S.}\tau_1\tau_2\tau_3$-semi- cl$(F_E)$ is the smallest Fuzzy Soft $\tau_1\tau_2\tau_3$ -semi-closed set over $(\mathcal{U}, \mathrm{E})$ which contains F_E , $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -semi-cl$(F_E) \sqsubseteq F_E$. Hence $F_E = \mathrm{F.S.}\tau_1\tau_2\tau_3$-semi-cl$(F_E)$
Conversely, let $F_E = \mathrm{F.S.}\tau_1\tau_2\tau_3$-semi-cl$(F_E)$ since $\mathrm{F.S.}\tau_1\tau_2\tau_3$-semi-cl$(F_E)$ is a Fuzzy Soft $\tau_1\tau_2\tau_3$ -semi-closed set, F_E is a Fuzzy Soft $\tau_1\tau_2\tau_3$ -semi-closed set.
(4) Since $\mathrm{F.S.}\tau_1\tau_2\tau_3$-semi-cl$(F_E)$ is Fuzzy Soft $\tau_1\tau_2\tau_3$-semi-closed set therefore, by (3) we have $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -semi-cl ($\mathrm{F.S.}\tau_1\tau_2\tau_3$-semi- cl$(F_E)$) = $\mathrm{F.S.}\tau_1\tau_2\tau_3$-semi- cl$(F_E)$
(5) Let $F_E \sqsubseteq G_E$. Then every Fuzzy Soft $\tau_1\tau_2\tau_3$-semi- closed supper set of G_E will also contain F_E . That is, every Fuzzy Soft $\tau_1\tau_2\tau_3$-semi- closed supper set of G_E is also a Fuzzy Soft $\tau_1\tau_2\tau_3$-semi- closed supper set of F_E . Hence the intersection of Fuzzy Soft $\tau_1\tau_2\tau_3$-semi- closed supper set of F_E is contained in the intersection of Fuzzy Soft $\tau_1\tau_2\tau_3$ -semi- closed supper set of G_E . Thus $\tau_1\tau_2\tau_3$-semi- cl$(F_E) \sqsubseteq \tau_1\tau_2\tau_3$-semi-cl$(G_E)$
(6) Since $F_E \sqsubseteq F_E \sqcup G_E$ and $G_E \sqsubseteq G_E \sqcup F_E$, by (5) $\mathrm{F.S.}\tau_1\tau_2\tau_3$-semi-cl$(F_E) \sqsubseteq \mathrm{F.S.}\tau_1\tau_2\tau_3$-semi- cl$(F_E \sqcup G_E)$ and $\mathrm{F.S.}\tau_1\tau_2\tau_3$-semi-cl$(G_E) \sqsubseteq \mathrm{F.S.}\tau_1\tau_2\tau_3$ -semi-cl$(F_E \sqcup G_E)$. Thus $\mathrm{F.S.}\tau_1\tau_2\tau_3$-semi- cl$(F_E) \sqcup \mathrm{F.S.}\tau_1\tau_2\tau_3$-semi-cl$(G_E) \sqsubseteq \mathrm{F.S.}\tau_1\tau_2\tau_3$-semi-cl$(F_E \sqcup G_E)$. Again, $F_E \sqsubseteq \mathrm{F.S.}\tau_1\tau_2\tau_3$-semi- cl$(F_E)$ and $G_E \sqsubseteq \mathrm{F.S.}\tau_1\tau_2\tau_3$-semi- cl$(G_E)$. So $F_E \sqcup G_E \sqsubseteq \mathrm{F.S.}\tau_1\tau_2\tau_3$-semi-cl$(F_E) \sqcup \mathrm{F.S.}\tau_1\tau_2\tau_3$ -semi-cl(G_E). By(5), $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -semi-cl$(F_E) \sqcup \mathrm{F.S.}\tau_1\tau_2\tau_3$ -semi-cl(G_E) is a Fuzzy Soft $\tau_1\tau_2\tau_3$-semi- closed set over $(\mathcal{U}, \mathrm{E})$ being the union of two Fuzzy Soft $\tau_1\tau_2\tau_3$-semi- closed sets .
Then $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -semi-cl$(F_E \sqcup G_E) \sqsubseteq \mathrm{F.S.}\tau_1\tau_2\tau_3$ -semi-cl$(F_E) \sqcup \mathrm{F.S.}\tau_1\tau_2\tau_3$.-semi-cl$(G_E)$. Thus

$\mathrm{F.S.}\tau_1\tau_2\tau_3$ -semi-cl($F_E \sqcup G_E$) = $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -semi-cl(F_E)$\sqcup$ $\mathrm{F.S.}\tau_1\tau_2\tau_3$-semi-cl($G_E$).
(7) Since $F_E \sqcap G_E \sqsubseteq F_E$ and $F_E \sqcap G_E \sqsubseteq G_E$, by (5) , $\mathrm{F.S.}\tau_1\tau_2\tau_3$-semi-cl($F_E \sqcap G_E$) $\sqsubseteq$ $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -semi-cl(F_E) and $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -semi-cl($F_E \sqcap G_E$) $\sqsubseteq$ $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -semi-cl(G_E).
Thus $\mathrm{F.S.}\tau_1\tau_2\tau_3$-semi-cl($F_E \sqcap G_E$) $\sqsubseteq$ $\mathrm{F.S.}\tau_1\tau_2\tau_3$-semi-cl($F_E$)$\sqcap$ $\mathrm{F.S.}\tau_1\tau_2\tau_3$-semi-cl($G_E$) .

Definition 3.6. Let $(\mathcal{U}, E, \tau_1, \tau_2, \tau_3)$ be a Fuzzy Soft Tri-topological space and $F_E \in \mathrm{F.S.}(\mathcal{U}, \mathrm{E})$. Then the Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*-interior of F_E is defined as;
$\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E) = Sup\{g_E | g_E \sqsubseteq F_E$, where g_E is $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-open set}
i.e., $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E)$ is the largest Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi* -open set contained in F_E.

Theorem 3.7. Let $(\mathcal{U}, \mathrm{E}, \tau_1, \tau_2, \tau_3)$ be a Fuzzy Soft Tri-topological space and
$F_E, G_E \in \mathrm{F.S.}(\mathcal{U}, \mathrm{E})$. Then:
(1) $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*-int$(0_E) = 0_E$ and $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(1_E) = 1_E$
(2) $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*-int$(F_E) \sqsubseteq F_E$
(3) F_E is Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*- open if and only if F_E= $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*-int(F_E)
(4) $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*-int($\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E)$) = $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E)$
(5) $F_E \sqsubseteq G_E \Rightarrow$ $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E) \sqsubseteq$ $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*-int(G_E)
(6) $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*-int$(F_E \sqcap G_E)$ = $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E)\sqcap$ $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(G_E)$
(7) $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*-int$(F_E)\sqcup$ $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(G_E) \sqsubseteq$ $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E \sqcup G_E)$
Proof . (1) Since 0_E and 1_E are Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*- open sets, we have by (3) of this theorem, $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(0_E) = 0_E$ and $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(1_E) = 1_E$
(2) By Definition 3.8, since $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int($F_E$) is The union of all Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*-open sets contained F_E . Hence $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E) \sqsubseteq F_E$
(3) Let F_E be Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*-open set since $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E)$ is the Biggest Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*- open set contained in F_E, $F_E = \mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E)$.
Conversely, suppose that $F_E = \mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E)$, since $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E)$ is a Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*-open, then F_E is a Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*-open set.
(4) Let $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E) = G_E$, since G_E is a Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*-open set, $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*- int$(G_E) = G_E$, so $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*-int($\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E)$) = $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*-int(F_E)
(5) Let $F_E \sqsubseteq G_E$, $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*-int$(F_E) \sqsubseteq F_E$ and hence $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(G_E)$ is the Biggest Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*-open set contained in G_E and $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E) \sqsubseteq$ $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(G_E)$.
(6) $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E) \sqsubseteq F_E$ and $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(G_E) \sqsubseteq G_E$, Hence
$\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E)\sqcap$ $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(G_E) \sqsubseteq F_E \sqcap G_E$, since the Biggest Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*-open set contained in $F_E \sqcap G_E$ is $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E \sqcap G_E)$, $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E)\sqcap$ $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(G_E) \sqsubseteq$ $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E \sqcap G_E)$
Conversely, $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E \sqcap G_E) \sqsubseteq$ $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E)$ and $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E \sqcap G_E) \sqsubseteq$ $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(G_E)$. Hence $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E \sqcap G_E) \sqsubseteq$ $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E)\sqcap$ $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(G_E)$.
(7) $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E) \sqsubseteq F_E$ and $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(G_E) \sqsubseteq G_E$, then
$\mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*-int$(F_E)\sqcup$$\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(G_E) \sqsubseteq F_E \sqcup G_E$. The Biggest Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*-open set contained in $F_E \sqcup G_E$ is $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E \sqcup G_E)$ and so $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E)\sqcup$ $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(G_E) \sqsubseteq$ $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E \sqcup G_E)$.

Theorem 3.8. Let $(\mathcal{U}$,E,$\tau_1, \tau_2, \tau_3)$ be a Fuzzy Soft Tri-topological space and let $F_E \in \mathrm{F.S.}(\mathcal{U}, \mathrm{E})$. Then:
(1) $(\mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*-cl$(F_E))^C$ = $\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-int$(F_E^c)$
(2) $(\mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*-int$(F_E))^C$ = $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*-cl(F_E^c)
Proof .
(1) $(\mathrm{F.S.}\tau_1\tau_2\tau_3$-*semi*-cl$(f_E))^C$ = $(\sqcap\{G_E / G_E$ is fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*-closed set and $F_E \sqsubseteq G_E\})^C$
= $\sqcup\{G_E^c / G_E^c$ is a fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*-open set and $G_E^c \sqsubseteq F_E^c\}$
= $\tau_1\tau_2\tau_3$-*semi*-int(F_E^c)
(2) $(\tau_1\tau_2\tau_3$-*semi*- int$(F_E))^C$ =$(\sqcup\{G_E | G_E$ is Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*-open set, $G_E \sqsubseteq F_E\})^C$
= $\sqcap\{G_E^c | G_E^c$ is Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*-closed set and $F_E^c \sqsubseteq G_E^c\}$
= $\mathrm{F.S.}\tau_1\tau_2\tau_3$ -*semi*-cl(F_E^c)

Theorem 3.9. Let $(\mathcal{U}$, E, $\tau_1, \tau_2, \tau_3)$ be a Fuzzy Soft Tri-topological space and F_E be a Fuzzy Soft set over $(\mathcal{U}, E)$. Then F_E is Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*-open $\Leftrightarrow$ F_E^c is Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*-closed.
Proof. $\Longrightarrow$ Let F_E be a Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*-open, by definition 3.1 to proof F_E^c a Fuzzy Soft $\tau_1\tau_2\tau_3$-*semi*-closed $[F_E \sqsubseteq (\tau_1\tau_2\tau_3 cl(\tau_1\tau_2\tau_3 int(F_E)))]^C$ by complement

$F_E^c \sqsupseteq [(\tau_1\tau_2\tau_3 cl(\tau_1\tau_2\tau_3 int(F_E))]^C$,
$[((\tau_1\tau_2\tau_3 cl(\tau_1\tau_2\tau_3 int(F_E))]^C = [((\tau_1\tau_2\tau_3 int(\tau_1\tau_2\tau_3 cl({F_E}^c))$
$F_E^c \sqsupseteq [((\tau_1\tau_2\tau_3 int(\tau_1\tau_2\tau_3 cl({F_E}^c))$, hence F_E^c is Fuzzy Soft $\tau_1\tau_2\tau_3$ - $semi$- closed .
$\Leftarrow$ Let F_E^c be a Fuzzy Soft $\tau_1\tau_2\tau_3$ - $semi$-closed to prove F_E is a Fuzzy Soft $\tau_1\tau_2\tau_3$ - $semi$- open
$[(\tau_1\tau_2\tau_3 int(\tau_1\tau_2\tau_3 cl(F_E))] \sqsubseteq {F_E}^c$
$[[((\tau_1\tau_2\tau_3 int(\tau_1\tau_2\tau_3 cl(F_E))] \sqsubseteq {F_E}^c]^c$ by complement
$[((\tau_1\tau_2\tau_3 int(\tau_1\tau_2\tau_3 cl(F_E))]^c \sqsupseteq ({F_E}^c)^c$ by theorem 3.8
$[[((\tau_1\tau_2\tau_3 cl(\tau_1\tau_2\tau_3 int({F_E}^c))] \sqsupseteq ({F_E}^c)^c$
$[[((\tau_1\tau_2\tau_3 cl(\tau_1\tau_2\tau_3 int({F_E}^c))] \sqsupseteq F_E$
Thus F_E is a Fuzzy Soft $\tau_1\tau_2\tau_3$ -$semi$-open .

4. Fuzzy Soft $\tau_1\tau_2\tau_3$-$semi$- Neighborhood at Fuzzy Soft point in Fuzzy Soft Tri-topological Space

In this section, we study and explore Fuzzy Soft neighborhood at Fuzzy Soft point in new definition.

Definition 4.1. [32] Fuzzy Soft point is the composite of Fuzzy point P_α^A with the Soft point F_x^u if $A = \{u\}$ we have the Fuzzy Soft point P_α^x, and if $u \notin A$, and then the composite is the null Fuzzy Soft set.

Definition 4.2. Let F_E be Fuzzy Soft set in Fuzzy Soft Tri-topological space $(\mathcal{U}, E, \tau_1, \tau_2, \tau_3)$ over $(\mathcal{U}, E)$ and P_α^x be Fuzzy Soft point. If there exists a Fuzzy Soft
$\tau_1\tau_2\tau_3$-$semi$-open set $G_E$with $P_\alpha^x \in G_E \sqsubseteq F_E$, then F_E is called Fuzzy Soft $\tau_1\tau_2\tau_3$-$semi$- neighborhood (F.S. $\tau_1\tau_2\tau_3$-$semi$-nbd) of a Fuzzy Soft point P_α^x .
The collection of all F.S.$\tau_1\tau_2\tau_3$-$semi$- nbds of Fuzzy Soft point P_α^x is denoted as $N(P_\alpha^x)$ and is known as Fuzzy Soft $\tau_1\tau_2\tau_3$-$semi$-nbd system of Fuzzy Soft point P_α^x .

Example 4.3. In Example 3.2 $(\mathcal{U}, E, \tau_1, \tau_2, \tau_3)$ is a Fuzzy Soft Tri-topological space, and consider the Fuzzy Soft point $P_\alpha^{x_1} = (x_1, \{0.3/u_1, o.4/u_2)$
Then δ_E is Fuzzy Soft $\tau_1\tau_2\tau_3$ -$semi$- nbd of Fuzzy Soft point $P_\alpha^{x_1}$.
Because there exists a Fuzzy Soft $\tau_1\tau_2\tau_3$ -$semi$- open set ψ_{1E} such that $P_\alpha^{x_1} \in \psi_{1E} \sqsubseteq \delta_E$.
In the following theorem, we discuss some important properties of Fuzzy Soft $\tau_1\tau_2\tau_3$ - $semi$- nbd system.

Theorem 4.4. Let F_E, G_E and H_E be Fuzzy Soft sets in Fuzzy Soft Tri-topolpgical space $(\mathcal{U}, E, \tau_1, \tau_2, \tau_3)$ over $(\mathcal{U}, E)$ and P_α^x be Fuzzy Soft point. Then Fuzzy Soft $\tau_1\tau_2\tau_3$- $semi$ - nbd system N(P_α^x) of Fuzzy Soft point P_α^x has the following properties :
(1) If $G_E \in N(P_\alpha^x)$, then $P_\alpha^x \in G_E$.
(2) If $G_E, H_E \in N(P_\alpha^x)$, then $G_E \sqcap H_E \in N(P_\alpha^x)$.
(3) If $G_E \in N(P_\alpha^x)$, and $G_E \sqsubseteq H_E$, then $H_E \in N(P_\alpha^x)$
(4) G_E is Fuzzy Soft $\tau_1\tau_2\tau_3$- $semi$ -open if and only if it contains a Fuzzy Soft $\tau_1\tau_2\tau_3$- $semi$ - nbd of each of its points .
Proof .
(1) Is obvious, since G_E is a Fuzzy Soft $\tau_1\tau_2\tau_3$- $semi$- open nbd of Fuzzy Soft point P_α^x . Therefore, G_E is a Fuzzy Soft $\tau_1\tau_2\tau_3$-$semi$- open set with $P_\alpha^x \in G_E$.
(2) If $G_E, H_E \in N(P_\alpha^x)$, then there exist Fuzzy Soft $\tau_1\tau_2\tau_3$-$semi$- open sets M_E and L_E such that $P_\alpha^x \in M_E \sqsubseteq G_E$ and $P_\alpha^x \in L_E \sqsubseteq H_E$. Therefore $P_\alpha^x \in M_E \sqcap L_E \sqsubseteq G_E \sqcap H_E$ and hence $G_E \sqcap H_E \in N(P_\alpha^x)$.
(3) Since $G_E \in N(P_\alpha^x)$, then there exists a Fuzzy Soft $\tau_1\tau_2\tau_3$- $semi$ - open set M_E such that
$P_\alpha^x \in M_E \sqsubseteq G_E$.Therefore,$P_\alpha^x \in M_E \sqsubseteq G_E \sqsubseteq H_E$ or $P_\alpha^x \in M_E \sqsubseteq H_E$. Hence $H_E \in N(P_\alpha^x)$.
(4)(a) Suppose G_E is a Fuzzy Soft $\tau_1\tau_2\tau_3$- $semi$ -open in$(\mathcal{U}, E, \tau_1, \tau_2, \tau_3)$ then $P_\alpha^x \in G_E \sqsubseteq G_E$ implies G_E is a Fuzzy Soft $\tau_1\tau_2\tau_3$-$semi$- nbd of each $P_\alpha^x \in G_E$.
(b) If each $P_\alpha^x \in G_E$ has a Fuzzy Soft $\tau_1\tau_2\tau_3$-$semi$- nbd $H_E \sqsubseteq G_E$, then
$G_E = \{P_\alpha^x | P_\alpha^x \in G_E\} \sqsubseteq \sqcup_{P_\alpha^x \in G_E} H_E \sqsubseteq G_E$ or $G_E = \sqcup_{P_\alpha^x \in G_E} H_E$. This gives G_E is Fuzzy Soft $\tau_1\tau_2\tau_3$-$semi$- open in $(\mathcal{U}, E, \tau_1, \tau_2, \tau_3,)$. This completes the proof.

5. CONCLUSIONS

Fuzzy soft tritopology is a new and promising domain which can lead to the development of new mathematical models that will significantly contribute to the applications in natural sciences such as and decision making problems, biomathematics and information systems .The topological structures of fuzzy soft $\tau_1\tau_2\tau_3$-$semi$- open (closed) set are initiated in this paper. Some basic notions of generalized concepts have been studied. the purpose of this paper is just to initiate the concept, and there is a lot of scope for the researchers to make their investigations in this field, i.e. this is a beginning of some new generalized structure and the concept like separation axioms and a new kinds of continuous functions and another basic concepts can be studied.

6. REFERENCES

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