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Solving Simultaneous Equations by Using A New Transformation

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Abstract: *sometimes you will encounter two or more unknown quantities, and two or more equations relate them. These are called simultaneous equations and when you are asked to solve them you must find the unknown values that satisfy all the given equations at the same time. In this work, we explain one way in which this can be done. It is the use of the Shaban transformation method. The purpose of demonstrating the applicability of the Shaban transformation for analysis of simultaneous differential equations*

Keyword: simultaneous equations, Shaban transformation

1- introduction:

There are several transformations like Laplace transforms , Elzaki , sumudu , Abood , … Available for solving linear differential equations [1-4] and they also comes out to be very effective tools to analyze the simultaneous differential equations which appear mostly in electric circuit analysis[5,6]

, in 2021, a new transformation was found called the Shaban transform [7] to solve the differential equations whose coefficients are trigonometric functions and which the rest of the transformations could not solve, such as the Laplace Transform, Elzaki, Sumudu,…,etc . . In this work , we explain way to solve simultaneous equations . It is the use of the Shaban transformation method. The purpose of demonstrating the applicability of the Shaban transformation for analysis of simultaneous differential equations

2. BASIC DEFINITIONS

Definition (2.1) [7]:-

Shaban transformation for the function $y(x)$, is define by the following integral

$$Sh[y(x)] = \int_0^{\frac{\pi}{2}} sinx\,(cosx)^p y(x)dx$$

Such that this integral is convergent , p is constant , The inverse of Shaban transform is given by

Linearity property (2.2) [7] :

Let $y(x), \beta(x)$ are functions for $x$and δ, k are constants , then

$$Sh\big(k\,y(x) \pm \delta\,\beta(x)\big) = kSh[y(x)] \pm \delta\,Sh[\beta(x)]$$

2.3 Shaban transform of some special functions [7]

n	$f(x)$	$G(\rho)$
1	$\boldsymbol{\delta}$	$\dfrac{\boldsymbol{\delta}}{\boldsymbol{\rho+1}}$
2	$(\boldsymbol{cosx})^{\boldsymbol{n}}$	$\dfrac{\boldsymbol{1}}{\boldsymbol{\rho+(n+1)}}$
3	$\boldsymbol{sin^2x}$	$\dfrac{\boldsymbol{2}}{\boldsymbol{(\rho+2)^2-1}}$
4	$\boldsymbol{cos(\delta lncosx)}$	$\dfrac{\boldsymbol{\rho+1}}{\boldsymbol{(\rho+1)^2+\delta^2}}$

5	$\boldsymbol{sin(\delta lncosx)}$	$\boldsymbol{\frac{-\delta}{(\rho+1)^2+\delta^2}}$
7	$\boldsymbol{sinh(\delta lncosx)}$	$\boldsymbol{\frac{-\delta}{(\rho+1)^2-\delta^2}}$
8	$\boldsymbol{cos\,h(\delta lncosx)}$	$\boldsymbol{\frac{\rho+1}{(\rho+1)^2-\delta^2}}$
9	$\boldsymbol{(secx)^n}$	$\boldsymbol{\frac{1}{p+(1-n)}}$
10	$U(x-x_0)$	$\boldsymbol{\frac{(cos\,x_0)^{\rho+1}}{\rho+1}}$

2.4. Shaban Transform of Some Derivatives

1- $Sh[cosx \quad f'(cosx)] = \vartheta(1) - (\rho+1)\ Sh\ \{f(cosx)\}$

2- $Sh[cos^2x \quad f''(cosx)] = f'(1) - (\rho+2)f\ (1) + (\rho+2)(\rho+1)Sh\,\{f\ (cosx)\}$

3- $Sh\ \left(cos^n x\, f^{(n)}(cosx)\right) = \ f^{(n-1)}(1) - (\rho+n)f^{(n-2)}(1) + \cdots$

$+(\rho+n)\big(\rho+(n-1)\big)\big(\rho+(n-2)\big)\ldots(\rho+2)f(1)$

$+(-1)^n\ (\rho+n)\big(\rho+(n-1)\big)\big(\rho+(n-2)\big)+\cdots$

$+(\rho+2)\ (\rho+1)\ \ Sh\,\{\ f\ (cosx)\}$

3 .SIMULTANEOUS EQUATIONS

Example (3.1): Solve simultaneous equations for each of the following

1-

$cos\,t\ \ x'(cos\,t) + 4y(cos\,t) = 0$

$cos\,t\ \ y'(cos\,t) - 9x(cos\,t) = 0 \quad , x(1) = 2 \quad , \ y(1) = 1$

Solution:

$cos\,t\ \ x'(cos\,t) + 4y(cos\,t) = 0 \qquad \ldots\ (1)$

$cos\,t\ \ y'(cos\,t) - 9x(cos\,t) = 0 \qquad \ldots(2)$

We take Shaaban transform for both sides equation (1)

$Sh[\cos t\ \ x'(\cos t) + 4y(\cos t)] = Sh[0]$

Using the linear equation , we get

$Sh[\cos t\ \ x'(\cos t)] + Sh[4y(\cos t)] = Sh[0]$

$x(1) - (p+1)Sh[x(\cos t)] + 4Sh[y(\cos t)] = 0$

$2 - (p+1)Sh[x(\cos t)] + 4Sh[y(\cos t)] = 0$ ... (3)

We take Shaaban transform for both sides equation (2)

$Sh[\cos t\ \ y'(\cos t) - 9x(\cos t)] = Sh[0]$

Using the linear equation , we get

$Sh[\cos t\ \ y'(\cos t)] - Sh[9x(\cos t)] = Sh[0]$

$y(1) - (p+1)Sh[y(\cos t)] - 9Sh[x(t)] = 0$

$1 - (p+1)Sh[y(\cos x)] - 9Sh[x(\cos x)] = 0$ ... (4)

Multiply equation (3) by $(p+1)$ and equation (4) by 4 ,

$$2(p+1) - (p+1)^2 Sh[x(\cos t)] + 4(p+1)Sh[y(\cos t)] = 0 \quad \text{... (5)}$$

$4 - 4(p+1)sh[y(\cos x)] - 36sh[x(\cos x)] = 0$ ... (6)

Sum the two equations (5) ,(6) , we get

$2(p+1) - (p+1)^2 Sh[x(\cos t)] + 4 - 36Sh[x(\cos t)] = 0$

$-[(p+1)^2 + 36]Sh[x(\cos t)] = -2(p+1) - 4$

$$Sh[x(\cos t)] = \frac{2(p+1)}{[(p+1)^2 + 36]} + \frac{4}{[(p+1)^2 + 36]}$$

We take Shaaban inverse transformation $Sh^{-1}[sh[x(\cos t)]] = Sh^{-1}\left[\frac{2(p+1)}{[(p+1)^2+36]}\right] + Sh^{-1}\left[\frac{4}{[(p+1)^2+36]}\right]$.

$$x(\cos t) = 2\cos(6\ln\cos t) - \frac{2}{3}\sin(6\ln\cos t)$$

And

Multiply equation (3) by 9 and equation (4) by $(p+1)$,

$$18 - 9(p+1)Sh[x(\cos t)] + 36Sh[y(\cos t)] = 0 \quad \text{... (7)}$$

$(p+1) - (p+1)^2 Sh[y(\cos x)] - 9(p+1)Sh[x(\cos x)] = 0$ ... (8)

By subtracting the two equations (7) , (8) , we obtain

$18 + 36\,Sh[y(\cos t)] - (p+1) + (p+1)^2 Sh[y(\cos t)] = 0$

$(36 + (p+1)^2)Sh[y(\cos t)] = (p+1) - 18$

$$Sh[y(\cos t)] = \frac{(p+1)}{[(p+1)^2 + 36} - \frac{18}{[(p+1)^2 + 36}$$

We take Shaban inverse transformation

$$Sh^{-1}[Sh[y(\cos t)]] = Sh^{-1}[\frac{(p+1)}{[(p+1)^2 + 36}] - Sh^{-1}[\frac{18}{[(p+1)^2 + 36}]$$

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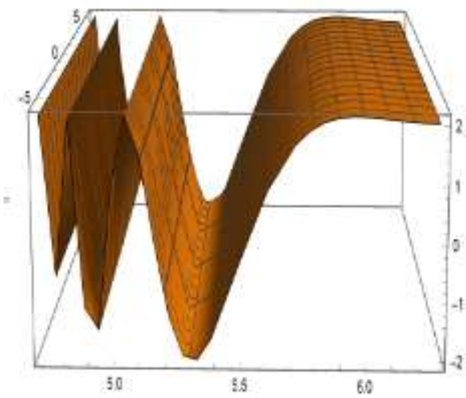
$$Sh^{-1}[Sh[y(\cos t)]] = Sh^{-1}\left[\frac{(p+1)}{[(p+1)^2+36]}\right] + 3Sh^{-1}[\frac{-6}{[(p+1)^2+36]}]$$

$$y(\cos t) = \cos(6\ln\cos t) + 3\sin(6\ln\cos t)$$



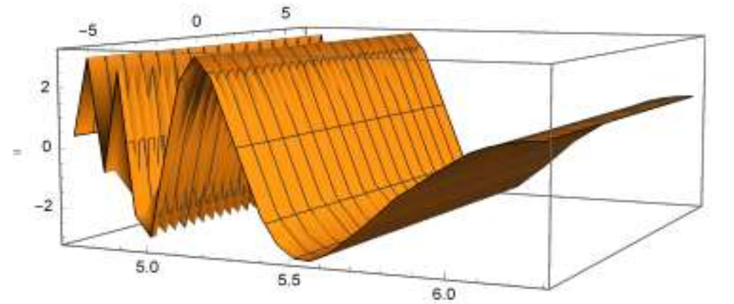


Figure (1) : **surfaces of the analytical solution of $x(cost), y(cost)$**

3-

$$cos\,t\ \ x'(cos\,t) + y(cos\,t) = \sin(\ln\cos t)$$

$$cos\,t\ \ y'(cos\,t) + x(cos\,t) = \cos(\ln\cos t) \quad ,x(1) = 2 \quad , \ y(1) = 0$$

Solution :

$$cos\,t\ \ x'(cos\,t) + y(cos\,t) = \sin(\ln\cos t) \qquad ...(9)$$

$$cos\,t\ \ y'(cos\,t) + x(cos\,t) = \cos(\ln\cos t) \qquad ...(10)$$

We take Shaaban transform for both sides equation (9)

$$Sh[\cos t\ \ x'(\cos t) + y(\cos t)] = Sh[\sin(\ln\cos t)]$$

Using the linear equation , we get

$$Sh[\cos t\ \ x'(\cos t)] + sh[y(\cos t)] = Sh[\sin(\ln\cos t)]$$

$$x(1) - (p+1)Sh[x(\cos t)] + Sh[y(\cos t)] = \frac{-1}{(p+1)^2+1}$$

$$2 - (p+1)Sh[x(\cos t)] + Sh[y(\cos t)] = \frac{-1}{(p+1)^2+1} \qquad ...(11)$$

We take Shaaban transform for both sides equation (10)

$$Sh[\cos t\ \ y'(\cos t) + x(\cos t)] = Sh[\cos(\ln\cos t)]$$

Using the linear equation , we get

$$Sh[\cos t\ \ y'(\cos t)] + Sh[x(\cos t)] = Sh[\cos(\ln\cos t)]$$

$$y(1)-(p+1)Sh[y(\cos t)]+Sh[x(t)]=\frac{p+1}{(p+1)^2+1}$$

$$-(p+1)sh[y(\cos x)]+sh[x(\cos x)]=\frac{p+1}{(p+1)^2+1} \qquad \text{... (12)}$$

Multiply equation (11) by $(p+1)$,

$$2(p+1)-(p+1)^2Sh[x(\cos t)]+(p+1)Sh[y(\cos t)]=\frac{-(p+1)}{(p+1)^2+1} \qquad \text{... (13)}$$

Sum the two equations (11) ,(12) , we get

$$2(p+1)-(p+1)^2Sh[x(\cos t)]+Sh[x(\cos t)]=0$$

$$-[(p+1)^2-1]sh[x(\cos t)]=-2(p+1)$$

$$Sh[x(\cos t)]=\frac{2(p+1)}{[(p+1)^2-1]}$$

We take Shaaban inverse transformation

$$Sh^{-1}[sh[x(\cos t)]]=Sh^{-1}\left[\frac{2(p+1)}{[(p+1)^2+36]}\right]$$

$$x(\cos t)=2\cos h(\ln\cos t)$$

And

By compensating equation (11)in equation (12), we get

$$-(p+1)Sh[y(\cos x)]+\frac{2(p+1)}{[(p+1)^2-1]}=\frac{p+1}{(p+1)^2+1}$$

$$Sh[y(\cos x)]=\frac{2}{[(p+1)^2-1]}-\frac{1}{(p+1)^2+1}$$

We take Shaaban inverse transformation

$$Sh^{-1}[Sh[y(\cos t)]]=Sh^{-1}[\frac{2}{[(p+1)^2-1]}]-Sh^{-1}[\frac{1}{(p+1)^2+1}]$$

$$y(\cos t)=\sin(\ln\cos t)-2\sinh(\ln\cos t)$$

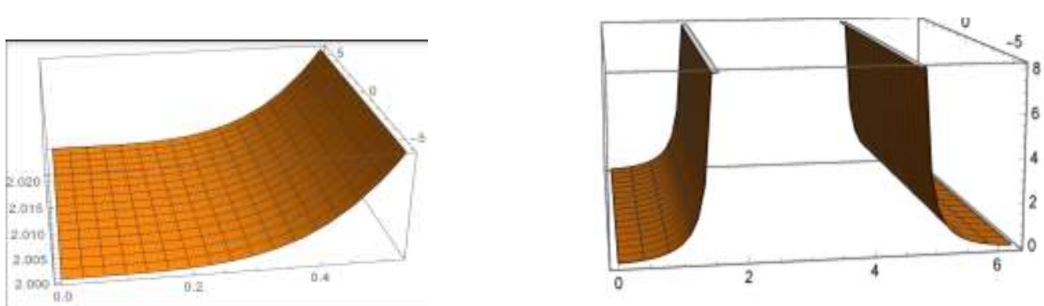


Figure (2) : **surfaces of the analytical solution of $x(cost), y(cost)$**

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