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Markov Decision Processes with Application to Budgetary Radioactive Waste Treatment

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***Abstract**: Reliable methods for budgetary planning of radioactive waste treatment are crucial to cater for waste handling from its generation to its storage, from treatment to disposal and subsequent monitoring. This can ensure financial sustainability of radioactive waste treatment programs. In this study, we present a markov decision process model that can assist radioactive waste treatment plants to optimally allocate funds for radioactive waste treatment. We formulate this decision problem as a discrete-time, discrete-state markov decision process where states of a markov chain represent possible states of radioactivity under a finite period planning horizon. The waste treatment cost represents the long run measure of performance for the markov decision process problem. We consider generated waste at two waste treatment plants in Uganda; and using monthly equal intervals, the decisions of whether or not to allocate additional funds for radioactive waste treatment are made using dynamic programming over a finite period planning horizon. We test the developed model to determine the optimal decision for allocating additional funds and the corresponding radioactive waste treatment costs. The study considers stationary radioactivity transition probabilities for easier computational purposes. We follow this theoretical part of the study to demonstrate the applicability of our model; where stochastic states of radioactivity are put into consideration. A numerical example presented shows how the decision to allocate or not to allocate additional funds are impacted by the stochastic levels of radioactivity. Results indicate the existence of an optimal state-dependent decision for allocating additional funds and the corresponding costs for the radioactive waste treatment plants considered in this study.*

Keywords—budgetary; Markov; modeling; radioactive; waste

1. INTRODUCTION

Radioactive waste treatment costs billions of dollars per year in several nations. This is a huge cost; which can extend for several decades into the future. The financial cost to taxpayers is therefore high as well as the cost to the environment, national borders and future generations. Budgetary planning for radioactive waste treatment is still a great challenge for waste management; when radioactive levels follow a stochastic trend. It requires a good understanding of the environment in which the waste treatment plant is operating and the development of a vision regarding future decisions for allocating funds. Proper allocation of funds must therefore ensure that the requirements of waste treatment activities that are not hindered by lack of funds or the treatment plants do not have idle, excess funds that are unutilized. Two major problems are usually encountered:

(i) Determining the most desirable period during which to allocate additional funds
(ii) Determining the optimal radioactive waste treatment costs corresponding to the fund allocation decision given a periodic review system for funding under stochastic levels of radioactivity

It is essential for the waste treatment plant to gauge and know its reasonably or optimum funds available as no single budgetary criterion is applicable to the

It is essential for the waste treatment plant to gauge and know its reasonably or optimum funds available as no single budgetary criterion is applicable to the. radioactive waste treatment cycle. In this paper, a Markov decision model is proposed whose goal is to optimize the fund allocation decisions for waste treatment given the stochastic nature of radioactivity levels.

The paper is organized as follows. After reviewing the previous work done in §2, a mathematical model is proposed in §3, where consideration is given to the process of estimating model parameters. The model is then solved in §4 and applied to a special case study in §5. The conclusions and final remarks follow in §6.

2. LITERATURE REVIEW

Waste management fund is vital to collect contributions and revenues from nuclear utilities, other major waste producers and possibly from state budgets IAEA [1]. In this way, money is made available for satisfying cashflow needs throughout the cycle of repository; by financing future liabilities from existing resources. Financial risks however; need to be taken into consideration before establishing a fund to manage financial resources. It

has been observed that as the number of nuclear power plants continues to increase, the problem of nuclear waste disposal is becoming more and more serious Liu & Dai [2]. Considering nuclear waste treatment, people initially temporarily deposit this waste or damp it directly. It is necessary to analyze the current characteristics of nuclear waste and its pollution status in order to find a better nuclear waste treatment and management method. It is evident that radioactive waste management policies have limitations in funding structures of nuclear fuel cycle (2020). Policies change from time to time; as well as the role of responsible agencies. At present, the budget of the Department of energy in USA is about $30 billion Feldman & Spogli [3]. On that budget, almost $12 billion is for the nuclear weapons programs. This; therefore leaves $18 billion to use for all things related to energy. Tax payers pay $6 billion every year; a huge cost that will be incurred for several decades into the future. The federal government must take the possession of and permanently dispose of radioactive waste

The federal government must also dispose of radioactive waste resulting from federal activities in manufacturing nuclear weapons. The government must establish a repository for nuclear waste Cawley [4]. In this Act, it requires planning efforts for several decades into the future. To date, discussion has centered on whether sites can be found and whether storage methods can be made sound enough to prevent accidental leakage of toxic waste into the environment as Grossman and Cassedy[5] notes. So far, no proposed disposal technology nor site has appeared acceptably safe from an environmental standpoint. Liau and Dai[6] emphasize the importance of principles and systems to follow to ensure that nuclear waste is avoided as much as possible during the entire process of production and disposal. The fund management can be established to collect, manage and pay on behalf of the state and selection of a professional company for spent fuel management through contract. According to NEA report [7], both decommissioning and radioactive waste management can be technically complex as they constitute highly capital-intensive long-term endeavors; and requires provision of large amounts of funds in advance. On the cost side, the funds required for different technological options or different time horizons can differ. Appunn [8] gives a distinction between the costs of the nuclear phase out. These are divided into expenses for decommissioning and waste disposal. Storage costs include the search for a final repository, the building, operating and loading of the facility, the transport of nuclear waste and finally the sealing of the repository. Jacoby [9] also affirms the problem piled nuclear waste. Tens of thousands of metric tons of radioactive spent nuclear fuel sit in steel and concrete storage cases at nuclear power plants across the world as they wait permanent disposal. All these wastes can remain dangerously radioactive for many thousand years. For that reason, they must be disposed of permanently. When it comes to storing the nation's nuclear waste in United states, the price is $38 billion and rising as Dixon [10] points out. That is how much tax payers will wind up spending, and the final price will be higher unless the government starts collecting the waste by 2020; which almost nobody who tracks the issue expects. The costs of inaction don't just include dollars. The lack of a final resting place for the waste means that each nuclear plant has to stock pile its own.

3. MODEL DEVELOPMENT

In this paper, two waste treatment plants are considered with a common goal of optimizing fund allocation decisions and waste treatment costs incurred. At the beginning of each period, a major decision has to be made, namely: whether to *allocate additional funds* or *not to allocate additional funds* in order to facilitate the radioactive waste treatment cycle at least costs. In this paper, two waste treatment plants are considered with a common goal of optimizing fund allocation decisions and waste treatment costs incurred. At the beginning of each period, a major decision has to be made, namely: whether to *allocate additional funds* or *not to allocate additional funds* in order to facilitate the radioactive waste treatment cycle at least costs.

3.1 Notation and Assumptions

i,j = states of radioactive waste
N^Z_{ij} = Observed radioactivity levels
A = Very high-level state
B = High level state
C = Intermediate level state
D = Low-level state
E = Very low-level state
P^Z_{ij} = Waste treatment costs
n,N = Stages
V^Z_i = Expected waste treatment costs
a^Z_i = Accumulated waste treatment costs
Q^Z = Radioactivity transition matrix
Q^Z_{ij} = Radioactivity transition probability
Z = Fund allocation decision
w = Waste treatment plant

i,j ε {A,B,C,D,E} Z ε {0,1} w={1,2} n=1,2,…..N

The representation assumes the correspondence between the radioactivity level and the states of the chain in Table 1.

We consider a set of plants for radioactive waste treatment whose levels of radioactivity during a given period over a fixed period planning horizon is classified as *Very High* (state A), *High* (state B), *Intermediate (*state C), *Low* (state D) or *Very low* (state E). The level of radioactivity of any such period and plant is assumed to depend on the radioactivity level of the preceding period

Table 1
Percentage levels and states of radioactivity

Level of radioactivity (%)	State of radioactivity	State code
85-100	Very high	A
70-84	High	B
55-69	Moderate	C
40-54	Low	D
0-39	Very low	E

The transition probabilities over the planning horizon from one radioactivity state to another may be described by means of a Markov chain.

Suppose one is interested in determining the optimal course of action; namely to allocate additional funds (a decision denoted by Z=1) or not to allocate additional funds (a decision denoted by Z=0) during each period over the planning horizon where Z is a binary decision variable. Optimality is defined such that the minimum expected waste treatment costs are accumulated at the end of N consecutive time periods spanning the planning horizon. In this paper, two waste treatment plants (w=2) and a two-period (N=2) planning horizon are considered.

3.2 Finite Dynamic Programming Formulation

Recalling that the level of radioactivity can be in state A, B, C, D or E, the problem of finding and optimal fund allocation decision can be expressed as a finite period dynamic programming model.

Let $g_n(i,w)$ denote the expected waste treatment costs accumulated at waste treatment plant *w* during periods *n,n+1,……..N* given that the state of the system at the beginning of period n is i. The recursive equation relating g_n and g_{n+1} is

$$\begin{aligned} g_n(i,w) = {} & min_Z[Q^Z_{iA}(w)O^Z_{iA}(w) + g_n(A,w)] \\ & +min_Z[Q^Z_{iB}(w)O^Z_{iB}(w) + g_n(B,w)] \\ & +min_Z[Q^Z_{iC}(w)O^Z_{iC}(w) + g_n(C,w)] \\ & +min_Z[Q^Z_{iD}(w)O^Z_{iD}(w) + g_n(D,w)] \\ & +min_Z[Q^Z_{iE}(w)O^Z_{iE}(w) + g_n(E,w)] \end{aligned}$$

together with the conditions

$g_{N+1}(A)=0$ $g_{N+1}(B)=0$ $g_{N+1}(C)=0$ $g_{N+1}(D)=0$ $g_{N+1}(E)=0$

This recursive equation may be justified by noting that the cumulative total waste treatment costs $O^Z_{ij}(w) + g_{n+1}(i,w)$ resulting from reaching state j ε {A,B,C,D,E} at the start of period *n+1* from state i ε {A,B,C,D,E} at the start of period *n* occurs with probability $Q^Z_{ij}(w)$.

Clearly, $V^Z(w) = Q^Z(w)[O^Z(w)]^T$, Z ε {0,1} where "T" denotes matrix transposition, and hence the dynamic programming recursive equations

$$\begin{aligned} g_n(i,w) = {} & min_Z[V_i^Z(w) + Q^Z_{iA}(w)g_{n+1}(A) + Q^Z_{iB}(w)g_{n+1}(B)] \\ & +min_Z[V_i^Z(w) + Q^Z_{iC}(w)g_{n+1}(C) + Q^Z_{iD}(w)g_{n+1}(D)] \\ & +min_Z[V_i^Z(w) + Q^Z_{iE}(w)g_{n+1}(E)] \end{aligned}$$

$$g_N(i,w) = min[V_i^Z(w)] \qquad (2)$$

i ε {A,B,C,D,E} ,n=1,2,……N-1,
Z ε {0,1} result.

3.3 Computing $Q^Z(w)$

The transition probability for radioactive level from state i ε {A,B,C,D,E} to state j ε {A,B,C,D,E} given fund allocation decision Z ε {0,1} may be taken as the number of radioactive waste levels observed at waste treatment plant *w* when the radioactivity level is initially in state i and later changing to state j, divided by the number of observed radioactivity levels over all states.

That is.

$$\begin{aligned} Q^Z_{ij}(w) = {} & N^Z_{ij}(w)/[N^Z_{iA}(w) + N^Z_{iB}(w) \\ & +N^Z_{iC}(w) + N^Z_{iD}(w) + N^Z_{iE}(w)] \end{aligned}$$

i ε {A,B,C,D,E} w={1,2} Z ε {0,1} (3)

4. COMPUTING AN OPTIMAL FUND ALLOCATION DECISION

The optimal fund allocation decision for radioactive waste treatment is found in this section for each period separately. We recall that the model assumes two treatment plants over a two-period planning horizon.

4.1 Optimization during period 1

When radioactivity level is very high (state A), the optimal fund allocation decision during period 1 is

$$Z = \begin{cases} 1 & if \quad V_A^1(w) < V_A^0(w) \\ 0 & if \quad V_A^1(w) \geq V_A^0(w) \end{cases}$$

The associated waste treatment costs are

$$g_1(A,w) = \begin{cases} V_A^1(w) & if \quad Z = 1 \\ V_A^0(w) & if \quad Z = 0 \end{cases}$$

When radioactivity level is high (state B), the optimal fund allocation decision during period 1 is

$$Z = \begin{cases} 1 & if \quad V_B^1(w) < V_B^0(w) \\ 0 & if \quad V_B^1(w) \geq V_B^0(w) \end{cases}$$

and the associated total waste treatment costs are

$$g_1(B,w) = \begin{cases} V_B^1(w) & if \quad Z = 1 \\ V_B^0(w) & if \quad Z = 0 \end{cases}$$

When radioactivity level is moderate (state C), the optimal fund allocation decision during period 1 is

$$Z = \begin{cases} 1 & if \quad V_C^1(w) < V_C^0(w) \\ 0 & if \quad V_C^1(w) \geq V_C^0(w) \end{cases}$$

and the associated total waste treatment costs are

$$g_1(C,w) = \begin{cases} V_C^1(w) & if \quad Z = 1 \\ V_C^0(w) & if \quad Z = 0 \end{cases}$$

When level of radioactivity is moderate (state D), the optimal fund allocation decision during period 1 is

$$Z = \begin{cases} 1 & if \quad V_D^1(w) < V_D^0(w) \\ 0 & if \quad V_D^1(w) \geq V_D^0(w) \end{cases}$$

and the associated total waste treatment costs are

$$g_1(D,w) = \begin{cases} V_D^1(w) & if \quad Z = 1 \\ V_D^0(w) & if \quad Z = 0 \end{cases}$$

When level of radioactivity is very low (state E), the optimal fund allocation decision during period 1 is

$$Z = \begin{cases} 1 & if \quad V_E^1(w) < V_E^0(w) \\ 0 & if \quad V_E^1(w) \geq V_E^0(w) \end{cases}$$

and the associated total waste treatment costs are

$$g_1(E,w) = \begin{cases} V_E^1(w) & if \quad Z = 1 \\ V_E^0(w) & if \quad Z = 0 \end{cases}$$

3.2 Optimization during period 2

Using the dynamic programming recursive equation in (2), and recalling that $a^Z_i(w)$ denotes the already accumulated waste treatment costs at the end of period 1 as a result of decisions made during that period, it follows that

$$\begin{aligned} a_i^Z(w) = V_i^Z(w) &+ Q_{iA}^Z(w) min[V_A^1(w), V_A^0(w)] \\ &+ Q_{iB}^Z(w) min[V_B^1(w), V_B^0(w)] \\ &+ Q_{iC}^Z(w) min[V_C^1(w), V_C^0(w)] \\ &+ Q_{iD}^Z(w) min[V_D^1(w), V_D^0(w)] \\ &+ Q_{iE}^Z(w) min[V_E^1(w), V_E^0(w)] \end{aligned}$$

Therefore, when level of radioactive waste is very high (state A), the optimal fund allocation decision during period 2 is

$$Z = \begin{cases} 1 & if \quad a_A^1(w) < a_A^0(w) \\ 0 & if \quad a_A^1(w) \geq a_A^0(w) \end{cases}$$

and the associated accumulated waste treatment costs are

$$g_2(A,w) = \begin{cases} a_A^1(w) & if \quad Z = 1 \\ a_A^0(w) & if \quad Z = 0 \end{cases}$$

When level of radioactivity is high (state B), the optimal fund allocation decision during period 2 is

$$Z = \begin{cases} 1 & if \quad a_B^1(w) < a_B^0(w) \\ 0 & if \quad a_B^1(w) \geq a_B^0(w) \end{cases}$$

and the associated accumulated waste treatment costs are

$$g_2(B,w) = \begin{cases} a_B^1(w) & if \quad Z = 1 \\ a_B^0(w) & if \quad Z = 0 \end{cases}$$

When level of radioactive is moderate (state C), the optimal fund allocation decision during period 2 is

$$Z = \begin{cases} 1 & if \quad a_C^1(w) < a_C^0(w) \\ 0 & if \quad a_C^1(w) \geq a_C^0(w) \end{cases}$$

and the associated accumulated waste treatment costs are

$$g_2(C,w) = \begin{cases} a_C^1(w) & if \quad Z = 1 \\ a_C^0(w) & if \quad Z = 0 \end{cases}$$

When level of radioactivity is low (state D), the optimal fund allocation decision during period 2 is

$$Z = \begin{cases} 1 & if \quad a_D^1(w) < a_D^0(w) \\ 0 & if \quad a_D^1(w) \geq a_D^0(w) \end{cases}$$

and the associated total waste treatment costs are

$$g_2(D,w) = \begin{cases} a_D^1(w) & if \quad Z = 1 \\ a_D^0(w) & if \quad Z = 0 \end{cases}$$

When level of radioactivity is very low (state E), the optimal fund allocation decision during period 2 is

$$Z = \begin{cases} 1 & if \quad a_E^1(w) < a_E^0(w) \\ 0 & if \quad a_E^1(w) \geq a_E^0(w) \end{cases}$$

5. A CASE STUDY AT KAWEMPE WASTE TREATMENT PLANTS

5.1 Case Description

In order to demonstrate the use of the model in §3-4, a real case application to Kawempe waste treatment plants in Uganda is presented in this section. The radioactivity levels for waste fluctuates on a weekly basis. The plant wants to avoid unnecessary allocation of funds, and hence seeks decision support in terms of an optimal fund allocation decision for radioactive waste treatment and the associated waste treatment costs over a two-week planning horizon.

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5.2 *Data Collection*

A sample of 40 observations depicting the level of radioactivity was considered at two separate waste treatment plants and the associated total treatment costs were noted when additional funds were allocated (Z=1) versus when additional funds were not allocated (Z=0). When additional funds were allocated (Z=1), the observations of radioactivity levels and waste treatment costs are as follows:

Waste treatment plant 1:

$$N^1(1) = \begin{bmatrix} 20 & 10 & 5 & 3 & 2 \\ 10 & 15 & 8 & 4 & 3 \\ 8 & 6 & 20 & 4 & 2 \\ 6 & 8 & 4 & 15 & 7 \\ 9 & 6 & 5 & 2 & 18 \end{bmatrix}$$

$$P^1(1) = \begin{bmatrix} 50 & 25 & 10 & 17 & 13 \\ 18 & 40 & 10 & 18 & 15 \\ 15 & 19 & 18 & 14 & 18 \\ 12 & 14 & 18 & 20 & 11 \\ 8 & 10 & 12 & 18 & 20 \end{bmatrix}$$

Waste treatment plant 2

$$N^1(2) = \begin{bmatrix} 18 & 12 & 6 & 2 & 2 \\ 7 & 18 & 6 & 4 & 5 \\ 9 & 7 & 16 & 5 & 3 \\ 8 & 6 & 5 & 14 & 7 \\ 10 & 5 & 8 & 2 & 15 \end{bmatrix}$$

$$P^1(2) = \begin{bmatrix} 42 & 28 & 12 & 14 & 13 \\ 15 & 45 & 7 & 18 & 18 \\ 20 & 22 & 12 & 16 & 20 \\ 13 & 10 & 20 & 17 & 11 \\ 11 & 8 & 14 & 18 & 16 \end{bmatrix}$$

When additional funds we're *not* allocated (Z=0), the observations of radioactivity levels and waste treatment costs are as follows:

Waste treatment plant 1:

$$N^0(1) = \begin{bmatrix} 22 & 3 & 8 & 5 & 2 \\ 9 & 16 & 5 & 7 & 3 \\ 8 & 5 & 18 & 6 & 3 \\ 7 & 10 & 3 & 16 & 4 \\ 10 & 5 & 3 & 4 & 18 \end{bmatrix}$$

$$P^0(1) = \begin{bmatrix} 38 & 20 & 12 & 16 & 20 \\ 10 & 40 & 8 & 10 & 12 \\ 6 & 20 & 10 & 16 & 18 \\ 8 & 12 & 20 & 18 & 12 \\ 8 & 10 & 12 & 24 & 20 \end{bmatrix}$$

Waste treatment plant 2:

$$N^0(2) = \begin{bmatrix} 20 & 7 & 4 & 6 & 3 \\ 10 & 12 & 8 & 9 & 1 \\ 7 & 6 & 20 & 2 & 5 \\ 6 & 4 & 8 & 14 & 8 \\ 12 & 7 & 4 & 1 & 16 \end{bmatrix}$$

$$P^0(2) = \begin{bmatrix} 35 & 18 & 16 & 10 & 8 \\ 12 & 42 & 8 & 12 & 11 \\ 4 & 22 & 8 & 10 & 12 \\ 6 & 14 & 18 & 16 & 14 \\ 10 & 12 & 14 & 20 & 18 \end{bmatrix}$$

4.3 Computation of Model Parameters

Using (3), the state transition matrices for waste treatment plants 1 and 2 are determined

Waste treatment plant 1:

$$Q^1(1) = \begin{bmatrix} 0.5 & 0.25 & 0.125 & 0.075 & 0.05 \\ 0.25 & 0.375 & 0.20 & 0.10 & 0.075 \\ 0.20 & 0.15 & 0.50 & 0.10 & 0.05 \\ 0.15 & 0.20 & 0.10 & 0.375 & 0.175 \\ 0.225 & 0.15 & 0.125 & 0.05 & 0.45 \end{bmatrix}$$

$$Q^0(1) = \begin{bmatrix} 0.55 & 0.20 & 0.075 & 0.125 & 0.05 \\ 0.225 & 0.40 & 0.125 & 0.175 & 0.075 \\ 0.20 & 0.125 & 0.45 & 0.15 & 0.075 \\ 0.175 & 0.25 & 0.075 & 0.40 & 0.10 \\ 0.25 & 0.125 & 0.075 & 0.10 & 0.45 \end{bmatrix}$$

Waste treatment plant 2

$$Q^1(2) = \begin{bmatrix} 0.45 & 0.30 & 0.15 & 0.05 & 0.05 \\ 0.175 & 0.45 & 0.15 & 0.10 & 0.125 \\ 0.225 & 0.175 & 0.40 & 0.125 & 0.075 \\ 0.20 & 0.15 & 0.125 & 0.35 & 0.175 \\ 0.25 & 0.125 & 0.20 & 0.05 & 0.375 \end{bmatrix}$$

$$Q^0(2) = \begin{bmatrix} 0.50 & 0.175 & 0.10 & 0.15 & 0.075 \\ 0.25 & 0.30 & 0.20 & 0.225 & 0.025 \\ 0.175 & 0.15 & 0.50 & 0.05 & 0.125 \\ 0.15 & 0.10 & 0.20 & 0.35 & 0.20 \\ 0.25 & 0.125 & 0.075 & 0.10 & 0.45 \end{bmatrix}$$

Considering $Q^Z(w)$ and $P^Z(w)$, the expected waste treatment costs are computed for $Z\varepsilon\{0.1\}$ and $w=\{1,2\}$ Results are summarized in Table 1.

The accumulated waste treatment costs are similarly calculated and results are summarized in Table 3.

Table 2: Fund allocation decisions, expected waste treatment costs (in USD), and states of radioactivity at waste treatment plants during week 1

Waste treatment plant (w)	State of radioactivity (i)	Expected waste treatment costs $V^Z_i(w)$	
		Z=1	Z=0
1	A	34.4	28.8
	B	24.4	20.9
	C	17.2	12.0
	D	15.8	14.3
	E	14.7	15.6
2	A	30.5	24.4
	B	27.9	20.2
	C	16.7	10.0
	D	14.5	14.3
	E	13.5	15.2

Table 3: Fund allocation decisions, accumulated waste treatment costs (in USD), and states of radioactivity at waste treatment plants during week 2

Waste treatment plant (w)	State of radioactivity (i)	Accumulated waste treatment costs $a^Z_i(w)$	
		Z=1	Z=0
1	A	53.33	52.24
	B	44.37	40.85
	C	34.26	29.02
	D	37.44	32.66
	E	33.15	33.50
2	A	50.43	44.29
	B	45.88	37.92
	C	32.53	24.70
	D	31.03	29.69
	E	29.90	32.08

5.4 *The Optimal Fund Allocation Decision*

5.4.1 Waste treatment plant 1 (Week 1)

Since 28.8 < 34.4, it follows that Z=0 is an optimal fund allocation decision with expected waste treatment costs of \$28.8 when level of radioactivity is Very High (state A). Since 20.9 < 24.4, it follows that Z=0 is an optimal fund allocation decision with expected waste treatment costs of \$20.9 when level of radioactivity is High (state B).

Since 12.0 < 17.2, it follows that Z=0 is an optimal fund allocation decision with expected waste treatment costs of \$12.0 when level of radioactivity is Moderate (state C). Since 14.3 < 15.8, it follows that Z=0 is an optimal fund allocation decision with expected waste treatment costs of \$14.3 when level of radioactive waste is Low (state D). Since 14.7 < 15.6, it follows that Z=1 is an optimal fund allocation decision with

expected waste treatment costs of $14.7 when level of radioactivity is Very Low (state E).

5.4.2 Waste treatment plant 2 (Week 1)

Since 24.4 < 30.5, it follows that Z=0 is an optimal fund allocation decision with expected waste treatment costs of $24.4 when level of radioactivity is Very High (state A). Since 20.2 < 27.9, it follows that Z=0 is an optimal fund allocation decision with expected waste treatment costs of $20.2 when level of radioactivity is High (state B).

Since 10.0 < 16.7, it follows that Z=0 is an optimal fund allocation decision with expected waste treatment costs of $10.0 when level pf radioactivity is Moderate (state C). Since 14.3 < 14.5, it follows that Z=0 is an optimal fund allocation decision with expected waste treatment costs of $14.3 when level of radioactivity is Low (state D).

Since 13.5 < 15.2, it follows that Z=1 is an optimal fund allocation decision with expected waste treatment costs of $13.5 when level of radioactivity is Very Low (state E).

5.4.3 Waste Treatment plant 1 (week 2)

Since 52.24 < 53.33, it follows that Z=0 is an optimal fund allocation decision with accumulated waste treatment costs of $52.24 when level of radioactivity is Very High (state A). Since 40.85 < 44.37, it follows that Z=0 is an optimal fund allocation decision with accumulated waste treatment costs of $40.85 when level of radioactivity is High (state B).
Since 29.02 < 34.26, it follows that Z=0 is an optimal fund allocation decision with accumulated waste treatment costs of $29.02 when level of radioactive waste is moderate (state C). Since 32.66 < 33.44, it follows that Z=0 is an optimal fund allocation decision with accumulated waste treatment costs of $32.66 when level of radioactivity is Low (state D).

Since 33.15 < 33.50, it follows that Z=1 is an optimal fund allocation decision with accumulated waste treatment costs of $33.15 when level of radioactivity is Very low (state E).

5.4.4 Waste treatment plant 2 –week 2

Since 44.29 < 50.43, it follows that Z=0 is an optimal fund allocation decision with accumulated waste treatment costs of $44.29 when level of radioactivity is Very High (state A). Since 37.92 < 45.88, it follows that Z=0 is an optimal fund allocation decision with accumulated waste treatment costs of $37.92 when level of radioactivity is High (state B). Since 24.70 < 32.53, it follows that Z=0 is an optimal fund allocation decision with accumulated waste treatment costs of $24.70 when level of radioactivity is moderate (state C). Since 26.69 < 31.03, it follows that Z=0 is an optimal fund allocation decision with accumulated waste treatment costs of $26.69 when level of radioactivity is Low (state D).

Since 29.90 < 32.08, it follows that Z=1 is an optimal fund allocation decision with accumulated waste treatment costs of $29.90 when level of radioactivity is very low (state E).

6. CONCLUSIONS

Markov decision processes can be very useful in optimizing fund allocation decisions for radioactive waste treatment under stochastic levels of radioactivity. This is possible provided the problem is formulated as a multi-stage decision problem using dynamic programming over a finite period planning horizon. It would however be worthwhile to extend the research and examine the behavior of fund allocation decisions under stochastic non stationary levels of radioactivity. In the same spirit, special interest is sought in further extending the research by examining fund allocation decisions using risk sensitive Markov decision processes.

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