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# On Artin's characters table of the group ($Q_{2m} \times C_3$) When m= $2^h.p_1^{r_1}.p_2^{r_2}.....p_n^{r_n}$ ,h,r ∈ $Z^+$ and p is prime Number

**Rajaa Hasan Abbas**

University of Kufa, College of Education for Girls, Department of Mathematics

Email: rajaah.alabidy@uokufa.edu.iq

***Abstract—*** *The main purpose of this paper is to find the general form of Artin's characters table of the group ($Q_{2m} \times C_3$ ) When m is an even number where $Q_{2m}$ is the Quaternion group of order 4m and $C_3$ is the Cyclic group of order3 this table depends on Artin's characters table of a quaternion group of order 4m when m is an even number. which is denoted by Ar($Q_{2m} \times C_3$ ).*

**Keywords— Group; Quaternion group; Artin; Artin's characters table.**

## INTRODUCTION

Representation theory is a branch of the Mathematics that studies abstract algebraic structures by representing their elements as linear transformations of vector spaces and studies modules over these abstract algebraic structures .So that representation theory is a power full tool because it reduces problems in abstract algebra to problems in a linear algebra which is a very well understood theory . For a finite group G, let $\overline{R}$ (G) denotes the abelian group generated by Z - valued characters of G under operation of point wise addition. Inside this group we have a subgroup generated by Artin characters ( The characters induced form the principal characters of cyclic subgroups of G), which will be denoted by T(G).The factor group $\overline{R}$ (G) /T (G) is called the Artin Cokernel of G denoted by AC(G).

A well known theorem dues to Artin asserted that T(G) has a finite index in $\overline{R}$ (G) i.e, [ $\overline{R}$ (G):T(G)] is finite so AC(G) is a finite abelian group.

The exponent of AC(G) is called Artin exponent of G and denoted by A(G). In 1967 T.Y. lam [9] proved a sharp form of Artin's theorem ,he determines the `least positive integer A(G) such that [ $\overline{R}$ (G):T(G)]=A(G).In 1976 I. M.Isaacs[3] studied Character Theory of Finite Groups. In 1996 K.K Nwabuez [5] studied A(G) of p-groups. In 2009 S.J.Mahmood [8] studied the general from of Artin's characters table Ar($Q_{2m}$ ) when m is an even number. The aim of this paper is to find the general from of the Artin's characters table of the group ($Q_{2m} \times C_3$ ) When m= $2^h.p_1^{r_1}.p_2^{r_2}.....p_n^{r_n}$ ,h,r ∈ $Z^+$ and p is prime Number

## 1 Preliminaries

This section introduce some important definitions and basic concepts of the induced character, the Artin's characters tables, the Artin's characters table of $C_{p^s}$ , the Artin's characters table of the Quaternion group $Q_{2m}$ when m is an even number and the Group ($Q_{2m} \times C_3$).

### 1.1 Definition: [7]

Two elements of G are said to be **Γ- conjugate** if the cyclic subgroups they generate are conjugate in G , this defines an equivalence relation on G. It is classes are called Γ- classes.

**1.2 Example:**

Consider a cyclic group $C_3$=< x > such that:

1 is Γ- conjugate 1

Then the Γ- class [1] = {1}

< x > = < $x^2$>

Then x and $x^2$ are Γ- conjugate , and [x] = { x , $x^2$}

So that there are two Γ- classes of $C_3$ :[1] and [x]

*In general for $C_{p^s}$ where p is any prime number, so that are s+1 distinct*

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*Γ- classes Which are* [1], [x], [$x^p$], … ,[$x^{p^{s-1}}$] .

**1.3Definition:[3]**

Let H be a subgroup of G and let $\varphi$ be a class function on H, ***the induced class function on G***, is given by :

$$\varphi'(g) = \frac{1}{|H|}\sum_{h\in G}\varphi^{\circ}(hgh^{-1})$$

where $\varphi^{\circ}$ is defined by:

$$\varphi^{\circ}(x) = \begin{cases}\varphi(x) & if \quad x\in H\\ 0 & if \quad x\notin H\end{cases}$$

**1.4Proposition :[6]**

Let H be a subgroup of G and $\varphi$ be a character of H, then $\varphi'$ is a character of G.

**1.5Definition:[4]**

The character $\varphi'$ of G is called ***induced character*** on G.

**1.6Example**:

$C_3$ is cyclic subgroup of $D_3$,

the character $\varphi$ on $C_3$ is defined as follow :

$\varphi(1) = 1$, $\varphi(r) = \omega$, $\varphi(r^2) = \omega^2$

where $\omega = e^{2\pi i/3}$

$$\varphi'(1) = \frac{1}{|H|}\sum_{h\in D_3}\varphi^{\circ}(h.1.h^{-1}) = \frac{1}{|H|}\sum_{h\in D_3}\varphi(1)$$

$$= \frac{1}{3}(1+1+1+1+1+1) = \frac{1}{3}\,.\,6 = 2$$

$$\varphi'(r) = \frac{1}{|H|}\sum_{h\in D_3}\varphi^{\circ}(h.r.h^{-1}) = \frac{1}{|H|}(\varphi(r)+\varphi(r)+\varphi(r)) + (\varphi(r^2)+\varphi(r^2)+\varphi(r^2))$$

$$= \frac{1}{3}.3(\varphi(r)+\varphi(r^2)) = \omega+\omega^2$$

Since $S\notin C_3$ then $\varphi'(S) = 0$.

and $\varphi'$ is a character of $D_3$ .

**1.7Theorem:[2]**

Let H be a cyclic subgroup of G and $h_1, h_2, \dots, h_m$ are chosen representative for m-conjugate classes, then :

1- $\varphi'(g) = \frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i)$ if $h_i \in H\cap CL(g)$

2- $\varphi'(g) = 0$ if $H\cap CL(g) = \phi$ .

**1.8Definition:[3]**

Let G be a finite group, all characters of G induced from a principal character of cyclic subgroups of G are called ***Artin's characters of G.***

**1.9Example:**

To find the Artin's character of $C_3$,

there are two cyclic subgroups of $C_3$, which are {1} and $C_3 = \langle x\rangle$ and let $\varphi$ be principal character, then :

by using theorem (1.7)

$$\varphi'(g) = \frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) \qquad \text{if} \quad h_i \in H \cap CL(g)$$

$$\varphi'(g) = 0 \qquad \text{if} \quad H \cap CL(g) = \phi .$$

if $H = \{1\}$ and $G = C_3$

since $H \cap CL(1) = \{1\}$, then

$\varphi'_1(1) = \frac{3}{1} . \varphi(1) = 3.1=3$

since $H \cap CL(x) = \phi$, then

$\varphi'_1(x) = 0$

since $H \cap CL(x^2) = \phi$, then

$\varphi'_1(x^2) = 0$

if $H = C_3$

since $H \cap CL(1) = \{1\}$, then

$\varphi'_2(1) = \frac{3}{3} . \varphi(1) = 1$

since $H \cap CL(x) = \{x\}$, then

$\varphi'_2(x) = \frac{3}{3} . \varphi(x) = \frac{3}{3} .1=1$

since $H \cap CL(x^2) = \{x^2\}$, then

$\varphi'_2(x^2) = \frac{3}{3} . \varphi(x^2) = \frac{3}{3} .1=1$

then we get the two Artin's characters $\varphi'_1$ and $\varphi'_2$.

**1.10Proposition:[2]**

The number of all distinct Artin's characters on a group G is equal to the number of Γ-classes on G. furthermore, Artin's characters are constant on each Γ-classes.

**1.11Definition:[1]**

Artin's characters of finite group G can be displayed in table ***called Artin's characters table of G*** which is denoted by Ar (G).

The first row is the Γ- conjugate classes, the second row is the number of elements in each conjugate classes, the third row is the size of the centralize $|C_G(CL_\alpha)|$ and the rest row contain the values of Artin's characters.

**1.12Example:**

In the Artin's character table of $C_3$ there are two Γ- classes, [1], [x] then, from proposition (1.10) they obtain two distinct Artin's characters and From example (1.9) we obtain the values of Artin's characters, then the table of it as follows:

Ar ($C_3$) =

| Γ- classes | [1] | [x] |
|---|---|---|
| $\lvert CL_\alpha \rvert$ | 1 | 1 |
| $\lvert C_{C_3}(CL_\alpha) \rvert$ | 3 | 3 |
| $\varphi'_1$ | 3 | 0 |
| $\varphi'_2$ | 1 | 1 |

Table (1)

**1.13Theorem:[1]**

The general form of Artin's character table of $C_{p^s}$ when p is a prime number and n is an integer number is given by:

Ar($C_{p^s}$)=

| Γ-classes | [1] | $[x^{p^{s-1}}]$ | $[x^{p^{s-2}}]$ | $[x^{p^{s-3}}]$ | … | $[x^p]$ | $[x]$ |
|---|---|---|---|---|---|---|---|
| $\|CL_\alpha\|$ | 1 | 1 | 1 | 1 | … | 1 | 1 |
| $\|C_{p^s}(CL_\alpha)\|$ | $P^s$ | $p^s$ | $P^s$ | $p^s$ | … | $p^s$ | $P^s$ |
| $\varphi'_1$ | $p^s$ | 0 | 0 | 0 | … | 0 | 0 |
| $\varphi'_2$ | $P^{s-1}$ | $P^{s-1}$ | 0 | 0 | … | 0 | 0 |
| $\varphi'_3$ | $P^{s-2}$ | $P^{s-2}$ | $P^{s-2}$ | 0 | … | 0 | 0 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋱ | ⋮ | ⋮ |
| $\varphi'_s$ | P | P | P | P | … | P | 0 |
| $\varphi'_{s+1}$ | 1 | 1 | 1 | 1 | … | 1 | 1 |

Table (2)

**1.14Example:**

Consider the cyclic group $C_8$ ,

To find the Artin's character table we use theorem (1.13) as follows:

The group $C_8 = C_{2^3}$ then:

Ar ($C_{2^3}$) =

| Γ- classes | [1] | $[x^{2^2}]$ | $[x^2]$ | [x] |
|---|---|---|---|---|
| $\|CL_\alpha\|$ | 1 | 1 | 1 | 1 |
| $\|C_{2^3}(CL_\alpha)\|$ | $2^3$ | $2^3$ | $2^3$ | $2^3$ |
| $\varphi'_1$ | $2^3$ | 0 | 0 | 0 |
| $\varphi'_2$ | $2^2$ | $2^2$ | 0 | 0 |
| $\varphi'_3$ | 2 | 2 | 2 | 0 |
| $\varphi'_4$ | 1 | 1 | 1 | 1 |

Table (3)

**1.15 Theorem: [8]**

The Artin's characters table of the Quaternion group $Q_{2m}$ when m is an even number is given as follows :

Ar $(Q_{2m})$ =

| Γ- classes | Γ- classes of $C_{2m}$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | [1] | $[x^m]$ | | | | | [y] | [xy] |
| $\lvert CL_\alpha \rvert$ | 1 | 1 | 2 | 2 | … | 2 | m | m |
| $\lvert C_{Q_{2m}}(CL_\alpha) \rvert$ | 4m | 4m | 2m | 2m | … | 2m | 4 | 4 |
| $\Phi_1$ | **2Ar($C_{2m}$)** | | | | | | 0 | 0 |
| $\Phi_2$ | | | | | | | 0 | 0 |
| ⋮ | | | | | | | ⋮ | ⋮ |
| $\Phi_l$ | | | | | | | 0 | 0 |
| $\Phi_{l+1}$ | m | m | 0 | 0 | … | 0 | 2 | 0 |
| $\Phi_{l+2}$ | m | m | 0 | 0 | … | 0 | 0 | 2 |

Table(4)

where $l$ is the number of Γ- classes of $C_{2m}$ and $\Phi_j$ ; $1 \le j \le l+2$ are the Artin characters of the Quaternion group $Q_{2m}$.

**1.16 Example:**

To construct Ar($Q_8$) by using theorem (1.15) we get the following table :

Ar($Q_8$)=Ar( $Q_{2^3}$ )=

| Γ- classes | [1] | $[x^4]$ | $[x^2]$ | [x] | [y] | [xy] |
|---|---|---|---|---|---|---|
| $\lvert CL_\alpha \rvert$ | 1 | 1 | 2 | 2 | 4 | 4 |
| $\lvert C_{Q_{2^3}}(CL_\alpha) \rvert$ | 16 | 16 | 8 | 8 | 4 | 4 |
| $\Phi_1$ | $2^4$ | 0 | 0 | 0 | 0 | 0 |
| $\Phi_2$ | $2^3$ | $2^3$ | 0 | 0 | 0 | 0 |
| $\Phi_3$ | $2^2$ | $2^2$ | $2^2$ | 0 | 0 | 0 |
| $\Phi_4$ | 2 | 2 | 2 | 2 | 0 | 0 |
| $\Phi_5$ | $2^2$ | $2^2$ | 0 | 0 | 2 | 0 |
| $\Phi_6$ | $2^2$ | $2^2$ | 0 | 0 | 0 | 2 |

Table (5)

**1.17The Group ($Q_{2m} \times C_3$ )**

The direct product group ($Q_{2m} \times C_3$ ) where $Q_{2m}$ is Quaternion group of order 4m with tow generators x and y is denoted by

$$Q_{2m} = \{x^k y^j : x^{2m} = y^4 = 1, yx^m y^{-1} = x^{-m}, 0 \leq k \leq 2m-1, j=0,1\}$$

and $C_3$ is acyclic group of order 3 consisting of elements $\{I,z,z^2\}$ .the generalized the group ($Q_{2m} \times C_3$ ) is denoted by

$$(Q_{2m} \times C_3) = \{(q,c) : q \in Q_{2m}, c \in C_3\} \text{ and } |Q_{2m} \times C_3| = |Q_{2m}|.|C_3| = 4m.3 = 12m$$

**2. The main results**

In this section is to find the general form of Artin's characters table of the group ($Q_{2m} \times C_3$ ) When m is an even number

**2.1Proposition:**

The general form of the Artin's characters table of the group ($Q_{2m} \times C_3$) when m is an even number is given as follows:

Ar($Q_{2m} \times C_3$)=

| Γ- classes of $(Q_{2m})\times\{I\}$ | | | | | | | Γ- classes of $(Q_{2m})\times\{z\}$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Γ-classes* | $[1,I]$ | $[x^m,I]$ | $\cdots$ | $[x\ ,I]$ | $[y,I]$ | $[xy,I]$ | $[1,z]$ | $[x^m,z]$ | $\cdots$ | $[x,z]$ | $[y,z]$ | $[xy,z]$ |
| $\lvert CL_\alpha\rvert$ | **1** | **1** | **…** | **2** | **m** | **m** | **1** | **1** | **…** | **2** | **m** | **m** |
| $\lvert C_{Q_{2m}\times C_3}(CL_\alpha)\rvert$ | **12m** | **12m** | **…** | **6m** | **12** | **12** | **12m** | **12m** | **...** | **6m** | **12** | **12** |
| $\Phi_{(1,1)}$ | **3Ar($Q_{2m}$)** | | | | | | **0** | | | | | |
| $\Phi_{(2,1)}$ | | | | | | | | | | | | |
| ⋮ | | | | | | | | | | | | |
| $\Phi_{(l,1)}$ | | | | | | | | | | | | |
| $\Phi_{(l+1,1)}$ | | | | | | | | | | | | |
| $\Phi_{(l+2,1)}$ | | | | | | | | | | | | |
| $\Phi_{(1,2)}$ | **Ar($Q_{2m}$)** | | | | | | **Ar($Q_{2m}$)** | | | | | |
| $\Phi_{(2,2)}$ | | | | | | | | | | | | |
| ⋮ | | | | | | | | | | | | |
| $\Phi_{(l,2)}$ | | | | | | | | | | | | |
| $\Phi_{(l+1,2)}$ | | | | | | | | | | | | |
| $\Phi_{(l+2,2)}$ | | | | | | | | | | | | |

Table (6)

***Proof :***

Let $g \in (Q_{2m}\times C_3)$ ;$g=(q,I)$ or $g=(q,z)$or $g=(q,z^2)$, $q \in Q_{2m}$ ,$I,z,z^2 \in C_3$

Case (I):

If H is a cyclic subgroup of $(Q_{2m}\times \{I\})$,then:

1-H=⟨(x, I)⟩ 2- H=⟨(y, I)⟩ 3- H=⟨(xy, I)⟩

And $\varphi$ the principal character of H, $\Phi_j$ Artin characters of $Q_{2m}$ where $1 \le j \le l+2$ then by using Theorem (1.7)

$\Phi_j$ (g) =

;

1- H=⟨(x, I)⟩

(i) If g=(1,I) and g∈H

$$\Phi_{(j,1)}((1,I)) = \frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g) = \frac{12m}{|C_H(I,1)|}.1 = \frac{3.4m}{|C_H(I,1)|}.1 = \frac{3|C_{Q_{2m}}(1)|}{|C_{\langle x\rangle}(1)|}.\varphi(1) = 3.\Phi_j(1)$$ since

H ∩CL(1,I)={(1,I)}

(ii) if g= $(x^m, I)$ and g∈H

$$\Phi_{(j,1)}(g) = \frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g) = \frac{12m}{|C_H(g)|}.1 = \frac{3.4m}{|C_H(g)|}.1 = \frac{3|C_{Q_{2m}}(x^m)|}{|C_{\langle x\rangle}(x^m)|}.\varphi(g) = 3.\Phi_j(x^m)$$

since H ∩CL(g)={g},$\varphi$(g)=1

(iii) if g= $(x^i, I), i \neq m$ and $i \neq 2m$ and g∈H

$$\Phi_{(j,1)}(g) = \frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1})) = \frac{6m}{|C_H(g)|}(1+1) =$$

$$\frac{3.2m}{|C_H(g)|}.(1+1) = \frac{3|C_{Q_{2m}}(q)|}{|C_{\langle x\rangle}(q)|}.(\varphi(g)+\varphi(g^{-1})) = 3.\Phi_j(q)$$

since H∩CL(g )={g,g$^{-1}$} and φ(g)=φ(g$^{-1}$)=1,g=(q,I),q∈ $Q_{2m}$ and q≠ $x^m$ ,q≠1

(iv) if g∉H

$$\Phi_{(j,1)}(g) = 3.0 = 3.\Phi_j(q)$$ Since H∩CL(g)= $\phi$

2- H=⟨(y, I)⟩ ={(1,I),(y,I),(y$^2$,I),(y$^3$,I)}

(i) If g=(1,I) H∩CL(1,I)={(1,I)}

$$\Phi_{(l+1,1)}(g) = \frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g) = \frac{12m}{4}.1 = 3m = 3.\Phi_{l+1}(1)$$

(ii) If g= ( $x^m, I$ )=(y$^2$,I) and g∈ H

$$\Phi_{(l+1,1)}(g) = \frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g) = \frac{12m}{4}.1 = 3m = 3.\Phi_{l+1}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) g=(y,I) or g=(y$^3$,I) and g∈H

$$\Phi_{(l+1,1)}(g) = \frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1})) = \frac{12}{4}.(1+1) = 3.2 = 3.\Phi_{l+1}(y)$$

since H∩CL(g )={g,g$^{-1}$} and φ(g)=φ(g$^{-1}$)=1

Otherwise

$$\Phi_{(l+1,1)}(g) = 0$$ since H∩CL(g)=∅

3- H=⟨(xy, I)⟩ ={(1,I),(xy,I),((xy)$^2$,I),((xy)$^3$,I)}

(i) If g=(1,I) H∩CL(1,I)={(1,I)}

$$\Phi_{(l+2,1)}(g) = \frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g) = \frac{12m}{4}.1 = 3m = 3.\Phi_{l+2}(1)$$

(ii) If g= ( $x^m, I$ )=((xy)$^2$,I)=(y$^2$,I) and g∈ H

$$\Phi_{(l+2,1)}(g) = \frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g) = \frac{12m}{4}.1 = 3m = 3.\Phi_{l+2}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) If g=(xy,I) or g=((xy)$^3$,I) =(xy$^3$,I) and g∈H

$$\Phi_{(l+2,1)}(g) = \frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1})) = \frac{12}{4}.(1+1) = 3.2 = 3.\Phi_{l+2}(xy)$$

since H∩CL(g )={g,g$^{-1}$} and φ(g)=φ(g$^{-1}$)=1

Otherwise

$$\Phi_{(l+2,1)}(g) = 0$$ since H∩CL(g)=∅

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Case (II):

If H is a cyclic subgroup of ($Q_{2m}\times$ {z}),then:

1-$H=\langle(x,z)\rangle=\left\langle(x,z^2)\right\rangle$ 2- $H=\langle(y,z)\rangle=\left\langle(y,z^2)\right\rangle$ 3- $H=\langle(xy,z)\rangle=\left\langle(xy,z^2)\right\rangle$

And $\varphi$ the principal character of H, $\Phi_j$ Artin characters of $Q_{2m}$ where $1\le j\le l+2$ then by using Theorem (1.7)

$$\Phi_j(g)=\begin{cases}\dfrac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & if \quad h_i\in H\cap CL(g)\\ 0 & if \quad H\cap CL(g)=\phi\end{cases}$$

1 -$H=\langle(x,z)\rangle=\left\langle(x,z^2)\right\rangle$

(i) If g=(1,I) or g=(1,z)or g=(1,$z^2$) and g∈H

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(1,I)|}\cdot\varphi(g)=\frac{12m}{|C_H(1,I)|}.1=\frac{3.4m}{|C_{\langle(x,z)\rangle}(1,I)|}.1=\frac{3|C_{Q_{2m}}(1)|}{3|C_{\langle x\rangle}(1)|}.\varphi(1)=\Phi_j(1)$$

since H ∩CL(g)={(1,I),(1,z),(1,$z^2$)}

(i) If g=(1,I) or g=($x^m$,I) or g=($x^m$,z) or g=(1,z)or g=($x^m$,$z^2$ ) or g=(1,$z^2$) and g∈H

(a) if g=(1,I) or g=(1,z) or g=(1,$z^2$) and g∈H.

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{12m}{|C_H(g)|}.1=\frac{3.4m}{|C_{\langle(x,z)\rangle}(g)|}.1=\frac{3|C_{Q_{2m}}(1)|}{3|C_{\langle x\rangle}(1)|}.\varphi(1)=\Phi_j(1)$$ since H ∩CL(g)={g},$\varphi$(g)=1

(b) if g=($x^m$,I) or g=($x^m$,z) or g=($x^m$,$z^2$) and g∈H

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{12m}{|C_H(g)|}.1=\frac{3.4m}{|C_H(g)|}.1=\frac{3|C_{Q_{2m}}(x^m)|}{3|C_{\langle x\rangle}(x^m)|}.\varphi(x^m)=\Phi_j(x^m)$$

since H ∩CL(g)={g},$\varphi$(g)=1

(i) if g$=\{(x^i,I),(x^i,z),(x^i,z^2\}$ ,$i\neq m, i\neq 2m$ and g∈H

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{6m}{|C_H(g)|}(1+1)$$

$$\frac{3.2m}{|C_H(g)|}.(1+1)=\frac{3|C_{Q_{2m}}(q)|}{3|C_{\langle x\rangle}(q)|}.(\varphi(g)+\varphi(g^{-1}))=\Phi_j(q)$$

since H∩CL(g )={g,$g^{-1}$} and $\varphi$(g)=$\varphi$($g^{-1}$)=1,g=(q,z)=(q,$z^2$),q∈ $Q_{2m}$ and q≠ $x^m$ ,q≠1

(ii) if g∉H

$$\Phi_{(j,2)}(g)=0$$ Since H∩CL(g)= $\phi$

2- H=$\left\langle(y,z)\right\rangle$ ={(1,I),(y,I),($y^2$,I),($y^3$,I),(1,z),(y,z),($y^2$,z),($y^3$,z), (1,$z^2$),(y,$z^2$),($y^2$,$z^2$),($y^3$,$z^2$)}

(i) If g=(1,I) or g=(1,z)or g=(1,$z^2$ )and g∈H H∩CL(g)={(1,I),(1,z),(1,$z^2$)}

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{12m}{12}.1=m=.\Phi_{l+1}(1)$$

(ii) If g= ( $x^m, I$ )=($y^2$,I) or g= ($y^2$,z) or g=($y^2$,$z^2$) and g∈ H

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{12m}{12}.1=m=\Phi_{l+1}(x^m)$$

Since H∩CL(g)={g} ,$\varphi$(g)=1

(iii) g=(y,I)or g= (y,z)or g=(y,$z^2$) or g=($y^3$,$I_2$)or g= ($y^3$,z) or g= ($y^3$,$z^2$) and g∈H

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{12}{12}.(1+1)=2=\Phi_{l+1}(y)$$

since H∩CL(g )={g,$g^{-1}$} and φ(g)=φ($g^{-1}$)=1

Otherwise

$\Phi_{(l+1,2)}(g)=0$ since H∩CL(g)=∅

3.H= $\langle(xy,z)\rangle$ ={(1,I),(xy,I),($(xy)^2$,I)=($y^2$,I),($(xy)^3$,I)=($xy^3$,I),(1,z),(xy,z), ($(xy)^2$,z)),($(xy)^3$,z), (1,$z^2$),(xy,$z^2$),($(xy)^2$,$z^2$)),($(xy)^3$,$z^2$)}

(i) If g=(1,I) or g=(1,z) or g=(1,$z^2$) H∩CL(g)={g}

$$\Phi_{(l+2,2)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{12m}{12}.1=m=\Phi_{l+2}(1)$$

(ii) If g= ( $x^m, I$ )=($(xy)^2$,I)=($y^2$,I)or g=($(xy)^2$,z)=($y^2$,z)or g=($(xy)^2$,$z^2$)=($y^2$,$z^2$) and g∈ H

$$\Phi_{(l+2,2)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{12m}{12}.1=m=\Phi_{l+2}(x^m)$$

Since H∩CL(g)={g} ,$\varphi$(g)=1

(iii) If g=(xy,I) or g= ($(xy)^3$,I) or g= (xy,z) or g= ($(xy)^3$,z) or g= (xy,$z^2$) or g= ($(xy)^3$,$z^2$)and g∈H

$$\Phi_{(l+2,2)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{12}{12}.(1+1)=2=\Phi_{l+2}(xy)$$

since H∩CL(g )={g,$g^{-1}$} and φ(g)=φ($g^{-1}$)=1

Otherwise

$\Phi_{(l+2,2)}(g)=0$ since H∩CL(g)=∅

**2.2Example:**

To construct Ar($Q_8\times C_3$) by using the theorem (2.1) we get the following table:

Ar($Q_{2^3}\times C_3$)=

| Γ- classes | [1,I] | [$x^4$,I] | [$x^2$,I] | [x,I] | [y,I] | [xy,I] | [1,z] | [$x^4$,z] | [$x^2$,z] | [x,z] | [y,z] | [xy,z] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\|CL_\alpha\|$ | 1 | 1 | 2 | 2 | 4 | 4 | 1 | 1 | 2 | 2 | 4 | 4 |
| $\|C_{Q_{2^3}\times C_3}(CL_\alpha)\|$ | 48 | 48 | 24 | 24 | 12 | 12 | 48 | 48 | 24 | 24 | 12 | 12 |
| $\Phi_{(1,1)}$ | 48 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\Phi_{(2,1)}$ | 24 | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\Phi_{(3,1)}$ | 12 | 12 | 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\Phi_{(4,1)}$ | 6 | 6 | 6 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\Phi_{(5,1)}$ | 12 | 12 | 0 | 0 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\Phi_{(6,1)}$ | 12 | 12 | 0 | 0 | 0 | 6 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\Phi_{(1,2)}$ | 16 | 0 | 0 | 0 | 0 | 0 | 16 | 0 | 0 | 0 | 0 | 0 |
| $\Phi_{(2,2)}$ | 8 | 8 | 0 | 0 | 0 | 0 | 8 | 8 | 0 | 0 | 0 | 0 |
| $\Phi_{(3,2)}$ | 4 | 4 | 4 | 0 | 0 | 0 | 4 | 4 | 4 | 0 | 0 | 0 |
| $\Phi_{(4,2)}$ | 2 | 2 | 2 | 2 | 0 | 0 | 2 | 2 | 2 | 2 | 0 | 0 |
| $\Phi_{(5,2)}$ | 4 | 4 | 0 | 0 | 2 | 0 | 4 | 4 | 0 | 0 | 2 | 0 |
| $\Phi_{(6,2)}$ | 4 | 4 | 0 | 0 | 0 | 2 | 4 | 4 | 0 | 0 | 0 | 2 |

Table (7)

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