International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 5 Issue 8, August - 2021, Pages: 31-34

The Rational Characters table of the group ($Q_{2m} \times C_3$) When m= $3^h.p_1^{r_1}.p_2^{r_2}.\ldots.p_n^{r_n}$,h,r ∈ Z^+ and p is prime Number

Rajaa Hasan Abbas

University of Kufa, College of Education for Girls, Department of Mathematics

Email: rajaah.alabidy@uokufa.edu.iq

Abstract— *The main purpose of this paper is to find the general form of the rational valued characters table* $\overset{*}{\equiv}(Q_{2m}\times C_3)$ *When m=* $3^h.p_1^{r_1}.p_2^{r_2}.\ldots.p_n^{r_n}$ *,h,r* ∈ Z^+ *and p is prime Number where* Q_{2m} *is the Quaternion group of order 4m and* C_3 *is the Cyclic group of order3 this table depends on Artin's characters table of a quaternion group of order 4m when m=* $3^h.p_1^{r_1}.p_2^{r_2}.\ldots.p_n^{r_n}$ *. which is denoted by* $\overset{*}{\equiv}(Q_{2m}\times C_3)$*.*

Keywords— Group; Quaternion group; Rational; Rational's characters table**.**

INTRODUCTION

Let *G* be a finite group, two elements of *G* are said to be r-conjugate if the cyclic subgroups they generate are conjugate in G. This process defines an equivalence relation on G; its classes are called r-classes.

Let $\equiv^*(G)$ denotes the r×r matrix which the rows corresponds to the θ_i 's and the columns correspond to the r-classes of *G*. The matrix expressing $\overline{R}(G)$ basis in terms of the *cf(G,Z)* basis is $\equiv^*(G)$.In 1959, M.J. Hall[6] is found" The rational valued characters table of finite group" In 1981 C.W. Curits and I. Reiner[2] studied Methods of Representation Theory with Application to Finite Groups. The aim of this paper is to find the general from of The rational valued characters table $\overset{*}{\equiv}(Q_{2m}\times C_3)$ when m is an odd number and finding the cyclic decomposition of Artin cokernel $AC(Q_{2m}\times C_3)$ when m is an odd number

1 Preliminaries

In this section studied some important definitions and basic concepts of a rational valued characters and a rational valued characters table**.**

Definition (2.1):[3]A rational valued characterθ of G is a character whose values are in Z, which is θ(g)∈ Z, for all g∈G.

Corollary (2.2):[4]The rational valued characters $\theta_i = \sum_{\sigma \in Gal(Q(\chi_i)/Q)} \sigma(\chi_i)$ form the basis for $\overline{R}$(G), where χ_i are the irreducible characters of G and their numbers are equal to the number of conjugacy classes of cyclic subgroup of G .

Theorem (2.3):[1] Let T_1: $G_1 \rightarrow GL(n,F)$ and T_2: $G_2 \rightarrow GL(m,F)$ be two irreducible representations of the groups G_1 and G_2 with characters χ_1 and χ_2 respectively then:$T_1 \otimes T_2$ is irreducible representation of the group $G_1 \times G_2$ with the character $\chi_1 \cdot \chi_2$.

Proposition (2.4):[2]The number of all rational valued characters of a finite group G is equal to the number of all distinct Γ-classes on G.

Definition (2.5): [4] The complete information about rational valued characters of a finite group G is displayed in a table called **rational valued characters table of G**. We refer to it by $\overset{*}{\equiv}$(G) which is n×n matrix whose columns are Γ-classes and rows which are the values of all rational valued characters of G, where n is the number of Γ-classes.

Proposition (2.6):[5]The **rational valued characters table of cyclic group** C_{p^s} of rank s+1 where p is prime number which is denoted by ($\overset{*}{\equiv}(C_{p^s})$), is given by:

TABLE1

RATIONALCHARACTERS TABLE OF CYCLIC GROUP C $_{p^s}$

Γ -classes	[1]	$[x^{p^{s-1}}]$	$[x^{p^{s-2}}]$	$[x^{p^{s-3}}]$	$[x^{p^2}]$	$[x^p]$	$[x]$
θ_1	$p^{s-1}(p-1)$	p^{s-1}-	0	0	0	0	**0**
θ_2	$p^{s-2}(p-1)$	$p^{s-2}(p-1)$	$-p^{s-2}$	0	0	0	**0**
θ_3	$p^{s-3}(p-1)$	$p^{s-3}(p-1)$	$p^{s-2}(p-1)$	$-p^{s-3}$	0	0	**0**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
θ_{s-1}	$p(p-1)$	$p(p-1)$	$p(p-1)$	$p(p-1)$	$p(p-1)$	$-p$	**0**
θ_s	$p-1$	$p-1$	$p-1$	$p-1$	$p-1$	$p-1$	**-1**
θ_{s+1}	1	1	1	1	1	1	**1**

Where its rank s+1 which represents the number of all distinct Γ-classes.

Lemma (2.7):[6] The rational valued characters table of Q_{2m} when m is $3^h.p_1^{r_1}.p_2^{r_2}.....p_n^{r_n}$ is given as follows:

$\overset{*}{\equiv}(Q_{2m})=$

TABLE2

RATIONALCHARACTERS TABLE OF GROUP Q_{2m}

	Γ-classes of C_{2m}		
	x^{2r}	x^{2r+1}	**[y]**
θ_1	$\equiv^*(C_m)$	$\equiv^*(C_m)$	**0**
⋮			**⋮**
$\theta_{(l/2)-1}$	1 1 ⋯ 1	1 1 ⋯ 1	**0**
$\theta_{(l/2)}$			**1**
$\theta_{(l/2)+1}$	$\equiv^*(C_m)$	H	**0**
⋮			**⋮**
θ_{l-1}	1 1 ⋯ 1	1 1 ⋯ 1	**0**
θ_1			**-1**
θ_{l+1}	2 ⋯ 2 -2	-2 ⋯ -2	**0**

Where $0 \le r \le m-1$, I is the number of Γ-classes of C_{2m}, θ_j such that$1 \le j \le I+1$ are the rational valued characters of the group Q_{2m} and if we denoted C_{ij} the elements of $\overset{*}{\equiv}(C_m)$ and h_{ij} the elements of H as defined by:

$$h_{ij} = \begin{cases} C_{ij} & if \quad i = 1 \\ C_{ij} & if \quad i \neq 1 \end{cases}$$

And where I is the number of Γ-classes of C_{2m}.

2 THE MAIN RESULTS

In this section we find the rational valued character table of the group ($Q_{2m} \times C_3$) and AC($Q_{2m} \times C_3$) when m is an odd number.

Proposition(3.1): The rational valued characters table of the group ($Q_{2m}\times C_3$) when m is $3^h.p_1^{r_1}.p_2^{r_2}.....p_n^{r_n}$ is given as follows:

$\overset{*}{\equiv}(Q_{2m}\times C_3)=\overset{*}{\equiv}(Q_{2m})\otimes\overset{*}{\equiv}(C_3)$.

Proof :-

Since

TABLE3

CHARACTERS TABLE OF THE GROUP C_3

	h_1'	h_2'
χ_1'	**3-1**	**-1**
χ_2'	**1**	**1**

TABLE4

RATIONALCHARACTERS TABLE OF GROUP C_p

$\overset{*}{\equiv}(C_3)=$

	h_1'	h_2'
θ_1'	**3-1**	**-1**
θ_2'	**1**	**1**

Then,

$\chi_1'(h_1')=\theta_1'(h_1')$ =3-1

$\chi_1'(h_2')=\theta_1'(h_2')$ = -1

$\chi_2'(h_1')=\chi_2'(h_2')=\theta_2'(h_1')=\theta_2'(h_2')=1$

From the definition of $Q_{2m}\times C_3$,

and Theorem(2.3) we have

$\equiv(Q_{2m}\times C_p)=(\equiv Q_{2m})\otimes(\equiv C_3)$

Each element in $Q_{2m}\times C_3$

$h_{ng}=h_n\cdot h_g'\ \ \forall h_n\in Q_{2m}$, $h_g'\in C_3$,

$n=1,2,3,\ldots,4m$, $g=1,2$

And each irreducible character of $Q_{2m}\times C_3$ is

$\chi_{(i,j)}=\chi_i.\chi_j'$

where χ_i is an irreducible character of Q_{2m} and χ_j' is the irreducible character of C_3 ,then

$$\chi_{(i,j)}(h_{ng})=\begin{cases}(3-1)\chi_i(h_n) & if\quad j=1\ \ and\ \ g=1\\ -\chi_i(h_n) & if\quad j=1\ \ and\ \ g=2\\ \chi_i(h_n) & if\quad j=2\ \ and\ \ g=2\end{cases}$$

From Corollary (2.2)

$$\theta_{(i,j)}=\sum_{\sigma\in Gal(Q(\chi_{(i,j)})/Q)}\sigma(\chi_{(i,j)})$$

where $\theta_{(i,j)}$ is the rational valued character of $Q_{2m}\times C_3$

Then,

$$\theta_{(i,j)}(h_{ng})=\sum_{\sigma\in Gal(Q(\chi_{(i,j)}(h_{ng}))/Q)}\sigma(\chi_{(i,j)}(h_{ng}))$$

(I) (a) If j=1 and g=1

$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_{i(h_n}))/Q)} \sigma(\chi_i(h_n)) = \theta_i(h_n).(3-1) =$

$= \theta_i(h_n) . \theta'_j(h'_g)$

(b) If j=1 and g=2

$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_{i(h_n}))/Q)} \sigma(-\chi_i(h_n)) = -\sum_{\sigma \in Gal(Q(\chi_{i(h_n}))/Q)} \sigma(\chi_i(h_n)) = \sum_{\sigma \in Gal(Q(\chi_{i(h_n}))/Q)} \sigma(\chi_i(h_n)).-1 = \theta_i(h_n).-1 = \theta_i(h_n) . \theta'_j(h'_g)$

(II) if j=2 and g=2

$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_{i(h_n}))/Q)} \sigma(\chi_i(h_n)) = \sum_{\sigma \in Gal(Q(\chi_{i(h_n}))/Q)} \sigma(\chi_i(h_n)) = \sum_{\sigma \in Gal(Q(\chi_{i(h_n}))/Q)} \sigma(\chi_i(h_n)).1 = \theta_i(h_n).1 = \theta_i(h_n) . \theta'_j(h'_g)$

where θ_i is the rational valued character of Q_{2m} .

From [I] and [II] we have

$\theta_{(i,j)} = \theta_i . \theta'_j$.

Then $\equiv^*(Q_{2m} \times C_3) = \equiv^*(Q_{2m}) \otimes \equiv^*(C_3)$.

3. References

[1] C.Curits and I. Reiner, " Methods of Representation Theory with Application Finite Groups and Order ", John wily & sons, New York, 1981.

[2] J. P. Serre, "Linear Representation of Finite Groups", Springer- Verlage, 1977.

[3] K.Knwabusz, " Some Definitions of Artin's Exponent of Finit Group ", USA.National foundation Math.GR.1996.

[4] K. Sekigvchi, " Extensions and the Irreducibilities of The Induced Characters of Cyclic p-Group ", Hiroshima math Journal, p165-178 ,2002.

[5] L. E. Sigler, " Algebra ", Springer- verlage, 1976.

[6] S.J. Mahmood "On Artin cokernel of the Quaternion Group Q_{2m} when m is an Even Number" M.Sc. thesis, University of Kufa,2009.