International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 5 Issue 8, August - 2021, Pages: 40-44

The method AI(MSuSu)) to Evaluate triple integrals

Safaa M. Aljassas

safaam.musa@uokufa.edu.iq

Iraq\ University of Kufa \ College of Education for girls \ Mathematics Dep.

Abstract: *The main goal of this research to calculate the triple integrals which has continuous functions numerically by method is* $AI\,(MSuSu)$ *that obtained by Aitken's acceleration with composite rule from a suggested method on the first dimension X and the second dimension Y with the Mid-point method on the third dimension Z and, which is denoted by* $MSuSu$ *. where the number of divisions on these three dimensions is exactly the Sam. we obtained high accuracy results in a relatively few sub intervals and short time.*

Kewords: Triple Integrals; Mid-Point Rule; Suggested Method; Romberg Accelerating and Aitken's acceleration

1. Introduction

Finding the values of triple integrals analytically is not easy at all times, so it became urgent to find approximate values for these integrals, given their importance in finding volumes, intermediate centers, and the inertia moment of volumes, for example, the size inside $x^2+y^2=4x$, above $z=0$ and below $x^2+y^2=4z$. The volume located inside the cylinder $\rho=4\cos(\theta)$ is determined from the top of the ball $p^2+z^2=16$ and from the bottom to the plane $z=0$. Calculating the average position of the volume below $z^2=xy$ and above the triangle $y=x, y=0, x=4$. Also calculate the moment of inertia of the volume located inside $x^2+y^2=9$ and above the plane $z=0$ and below the plane $x+z=4$. The importance of triple integrals is evident in finding variable density blocks such as a thin wire piece and a thin sheet of metal, Frank Ayers [6]

many authers was interested in evaluating the triple integrals such as Dheyaa [3], in 2009, he introduced numerical composite method ($RMRM\,(RS)$, $RMRM\,(RM)$, $RMRS\,(RM)$ and $RMRS\,(RS)$). These methods have obtained from Romberg acceleration method with Midpoint method (RM) on the third dimension(Z). RS(RS), RS(RM), RM(RM), RM(RS) on the second dimension (Y) and first dimension (X) Has got good results.

In 2010, Eghaar [5], introduced numerical method to calculate the value of triple integrals by Romberg acceleration method on the resulting values from applying Midpoint method on three dimensions X ,Y and Z. She got good results in terms of accuracy and a relatively few sub intervals.

Mohammed et al. [1] presented in 2013 numerical method to evaluate the value of triple integrals with continuous functions by RSSS method that obtained from Romberg acceleration with Simpson's rule on three dimensions X,Y and Z as the same approach of Eghaar [5]. In 2015, Aljassas [11] introduced a numerical method $RM\,(RMM)$ to calculating triple integrals with continuous functions by using Romberg acceleration with Mid-point rule on the three dimensions when the number of divisions on the first dimension is equal to the number of divisions on the second dimension, but both of them are deferent from the number of divisions on the third dimension and she got a high accuracy in the results in a little sub-intervals relatively and a short time.

Also in 2015, Sarada et al. [9] use the generalized Gaussian Quadrature to evaluate triple integral and got a good results.
In our research, , we will compute the triple integrals with continuous integrand numerically by using the composite rule MSuSu. To improve results, we used the mentioned rule with Aitken's acceleration. (denoted it by AI(MSuSu))

2. Mid-Point Rule

Suppose that the integral H defined as the following $H=\int_r^s k(\chi)d\chi = M(p)+\varpi_M(p)+R_M$

…(1)

such that $M(p)$ is Mid-Point rule to evaluate the value of integral H and $\varpi_M(p)$ is correction terms for M(p) and R_M is the reminder that related by truncation from $\varpi_M(p)$ after using specified terms from $\varpi_M(p)$. The general formula for the Mid-Point rule $M(p)$ is:

$$M(p) = p\sum_{e=1}^{n} k\left(r + \frac{(2e-1)}{2}p\right) \qquad \ldots(2)$$

Fox[7] and Eghaar [5]

3. Suggested method (Su)

Suggested method which is introduced by Mohammed et al. [2], it considers one of the singular integral method. The general form is

$$Su(p) = \frac{p}{4}\left[k(r) + k(s) + 2k(r + (n-0.5)p) + 2\sum_{e=1}^{n-1}(k(r + (e-0.5)p) + k(r + ep))\right] \qquad \ldots(3)$$

4. The Numerical Rule MSuSu

Let $k(\chi,\gamma,\varsigma)$ is continuous function and derivable at each point of the region $[r,s]\times[t,u]\times[v,w]$, then the approximate value of $H = \int_v^w\int_t^u\int_r^s k(\chi,\gamma,\varsigma)\,d\chi\, d\gamma\, d\varsigma$ can be evaluated by the following rule:

$$\begin{aligned}
MSuSu = \frac{p^3}{16}\sum_{\ell=1}^{n}\Big[& k(r,t,\varsigma_\ell) + k(r,u,\varsigma_\ell) + k(s,t,\varsigma_\ell) + k(s,u,\varsigma_\ell) + 2(k(r,t+(n-0.5)p,\varsigma_\ell) + k(s,t+(n-0.5)p,\varsigma_\ell) + \\
& + k(r+(n-0.5)p,t,\varsigma_\ell) + k(r+(n-0.5)k,u,\varsigma_\ell) + 2k(r+(n-0.5)p,t+(n-0.5)p,\varsigma_\ell)) \\
& + 2\sum_{j=1}^{n-1}\Big(k(r,\gamma_j,\varsigma_\ell) + k(r,t+(j-0.5)p,\varsigma_\ell) + k(s,\gamma_j,\varsigma_\ell) + k(s,t+(j-0.5)p,\varsigma_\ell) + 2k(r+(n-0.5)p,\gamma_j,\varsigma_\ell) \\
& + 2k(r+(n-0.5)p,t+(j-0.5)p,\varsigma_\ell) + 2\sum k(\chi_e,t,\varsigma_\ell) + k(\chi_e,u,\varsigma_\ell) + k(r+(e-0.5)p,t,\varsigma_\ell) + k(r+(e-0.5)p,u,\varsigma_\ell) \\
& + 2k(\chi_j,t+(n-0.5)p,\varsigma_\ell) + 2k(r+(e-0.5)p,t+(n-0.5)p,\varsigma_\ell) + 2\sum_{e=1}^{n-1}\Big(k(\chi_e,\gamma_j,\varsigma_\ell) + k(\chi_e,t+(j-0.5)p,\varsigma_\ell) \\
& + k(r+(e-0.5)p,\gamma_j,\varsigma_\ell) + k(r+(e-0.5)p,t+(j-0.5)p,\varsigma_\ell))\ \Big]
\end{aligned}$$

$$\gamma_j = t + (j-0.5)p\ ,\quad j = 1,2,\ldots,n$$

5-Aitken's delta – Squared Process

In 1926, Alexander Aitken (1985-1926) found a new approach to accelerate the sequence convergence rate. To explain this method, we suppose the sequence $\{x_n\}$ such that $\{x_n\} = \{x_1, x_2, \ldots x_k \ldots\}$ linearly convergence to a certain final value β, so $\beta - x_{i+1} = C_i(\beta - x_i)$, Ralston [4], such that $|C_i| < 1$ and $C_i \to C$.

We can see that C_i will be approximately steady and we can write

$$\beta - x_{i+1} \approx \bar{C}\,(\beta - x_i) \qquad \ldots(4)$$

Such that $\left|\bar{C}\right| = C$

We also can see that

$$\frac{\beta - x_{i+2}}{\beta - x_{i+1}} \approx \frac{\beta - x_{i+1}}{\beta - x_i} \qquad \ldots(5)$$

i.e, $$\beta \cong \frac{x_i x_{i+2} - x_{i+1}^2}{x_{i+2} - 2x_{i+1} + x_i} = x_{i+2} - \frac{(\Delta x_{i+1})^2}{\Delta^2 x_i} \qquad \ldots(6)$$

such that $\Delta x_i = (x_{i+1} - x_i)$ and $\Delta^2 x_i = x_i - 2x_{i+1} - x_{i-2}$
when using u from elements of the sequence $\{x_u\}$, we can get u-2 of another sequence {S} Approaching faster than $\{x_u\}$

$$s_{i+2} = x_{i+2} - \frac{(\Delta x_{i+1})^2}{\Delta^2 x_i} \qquad \ldots(7)$$

where $i = 1, 2, \ldots, u-2$
This process is accelerating the convergence to the final value β.

6. Examples and Results:

Example(1): The integral $\int_0^{0.5}\int_0^{0.5}\int_0^{0.5} (1.2\chi + 0.4\varsigma) e^{\chi+\gamma+\varsigma} d\chi d\gamma d\varsigma$ which its analytical value is 0.1182655120 (Rounded to 10 decimal places) with integrand is defined for all $(x, y, z) \in [0, 0.5] \times [0, 0.5] \times [0, 0.5]$, The table (1) show the above integral value numerically by, $AI\left(MSuSu\right)$:-

We can note from table (1) where $u = 64$ the value is correct for six decimal places using $MSuSu$, Then applying $Al(MSuSu)$ method, we obtained similar value to the analytical value (Rounded to 10 decimal places) .

Example(2): The integral $\int_0^{\frac{\pi}{4}}\int_0^{\frac{\pi}{4}}\int_0^{\frac{\pi}{4}} \chi \sin(4.1\chi - 3.2\gamma - 1.5\varsigma) d\chi d\gamma d\varsigma$ which its analytical value is 0.0344452991 (Rounded to 10 decimal places) with integrand is defined for all $(\chi, \gamma, \varsigma) \in \left[0, \frac{\pi}{4}\right] \times \left[0, \frac{\pi}{4}\right] \times \left[0, \frac{\pi}{4}\right]$, The table (2) show the above integral value numerically by, $AI\left(MSuSu\right)$

From table (2) We can note where $n = 64$ the value is correct for four decimal places using $MSuSu$, Then applying $Al(MSuSu)$ method, we obtained similar value to the analytical value (Rounded to 10 decimal places) .

Example(3): The integral $I = \int_2^3\int_2^3\int_2^3 \sqrt{\varsigma} \log(\chi + \gamma + \varsigma) d\chi d\gamma d\varsigma$ which its analytical value is 3.1805026272 (Rounded to 10 decimal places) with function is defined for all $(x, y, z) \in [2,3] \times [2,3] \times [2,3]$, The table (3), show the above integral value numerically by, $Al\left(MSuSu\right)$. We deduce from tables (3) where $n = 32$ the value is correct for five decimal places using $MSuSu$ Then applying $AI(MSuSu)$ method, we obtained similar value to the analytical value .

International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 5 Issue 8, August - 2021, Pages: 40-44

7-Conclusion

It can be seen from the tables:

When we evaluated the approximate value of triple integral with continuous integrand by using composite rule MSuSu gives us correct value (for several decimal places) comparing with the real value for integrals by using several sub intervals without using any acceleration method, while we got a correct value for 10 decimal places for all examples if Aitken acceleration was used.

n	MSuSu	AI(MSuSu)	AI(MSuSu)
1	0.1193858121		
2	0.1185488132		
4	0.1183365400	0.1183122858	
8	0.1182832817	0.1182625486	
16	0.1182699552	0.1182655124	0.1182655122
32	0.1182653122	0.1182655123	0.1182655120

Table (1) :-
evaluating the triple integral

$$I = \int_0^{0.5}\int_0^{0.5}\int_0^{0.5} (1.2\chi + 0.4\varsigma) e^{\chi+\gamma+\varsigma} d\chi d\gamma d\varsigma = 0.1182655120$$

n	MSuSu	AI(MSuSu)	AI(MSuSu)
1	0.0465299946		
2	0.0374587902		
4	0.0351966833	0.0342876832	
8	0.0346330013	0.0344391363	
16	0.0344922154	0.0344453149	0.0344452988
32	0.0344570275	0.0344452922	0.0344452991

Table (2) :-
evaluating the triple integral

$$I = \int_0^{\frac{\pi}{4}}\int_0^{\frac{\pi}{4}}\int_0^{\frac{\pi}{4}} \chi \sin(4.1\chi - 3.2\gamma - 1.5\varsigma) d\chi d\gamma d\varsigma = 0.0344452991$$

n	MSuSu	AI(MSuSu)	AI(MSuSu)
1	3.1823080434		
2	3.1809653098		
4	3.1806190566	3.1805546180	
8	3.1805317829	3.1805035931	
16	3.1805099192	3.1805024991	3.1805026254
32	3.1805044504	3.1805026312	3.1805026272

Table (3) :-
evaluating the triple integral

$$I = \int_2^3\int_2^3\int_2^3 \sqrt{\varsigma}\,\log(\chi+\gamma+\varsigma)\,d\chi\,d\gamma\,d\varsigma = 3.1805026272$$

References

[1] A. H. Mohammed, S. M. Aljassas and W. Mohammed, " Derivation of numerically method for evaluating triple integrals with continuous integrands and form of error(Correction terms)", Journal of Kerbala University, pp. 67-76, Vol.11, No.4, 2013.

[2] A. H Mohammed, A. N. Alkiffai and R. A. Khudair , Suggested Numerical Method to Evaluate Single Integrals, Journal of Kerbala university, 9, 201-206, 2011.

[3] A. M. Dheyaa, Some numerical methods for calculating single, double, and triple integrals using Matlab language, MSc Dissertation, University of Kufa, 2009.

[4] Anthony Ralston, A First Course in Numerical Analysis, Mc Graw–Hill Book Company, 1965.

[5] B.H. Eghaar, Some numerical methods for calculating double and triple integrals, MSc Dissertation, University of Kufa, 2010.

[6] F. J. R. Ayres, Schaum's Outline Series: Theory and Problems of Calculus, McGraw-Hill book-Company,1972.

[7] L. Fox, Romberg Integration for a Class of Singular Integrands , The Computer Journal, Vol. 10, pp. 87-93 , 1967.

[8] L. Fox and L. Hayes , On the Definite Integration of Singular Integrands, SIAM REVIEW , 12 , pp. 449-457 , 1970.

[9] Sarada Jayan and K.V. Nagaraja, A General and Effective Numerical Integration Method to Evaluate Triple Integrals Using Generalized Gaussian Quadrature, ELSEVIER , Procedia Engineering, 127, pp. 1041-1047, 2015.

[10] S. M. Aljassas, F .H. Alsharify and N. A. M. Al – Karamy, Driving Two Numerical Methods to Evaluate The Triple Integrals R(MSM),R(SMM) and Compare between them, International Journal of Engineering & Technology, pp. 303-309, Vol.7, No.3.27, 2018.

[11] S. M. Aljassas, Evaluation of triple integrals with continuous integrands numerically by Using Method RM(RMM), Journal of AL-Qadisiyah for computer science and mathematics, pp. 1-10, Vol.7 No.1 , 2015.