

# Diagnosing Learner Errors in Euclidean Geometry Problem-Solving: A Case of Grade 11 Learners in Amathole East District

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**Abstract:** *Geometry is an important topic in the school curriculum, yet a number of studies have reported that many learners exhibit errors while solving geometry problems. Acknowledging this problem, this study which aims to analyse learners' errors in solving geometry problems based on Van Hiele thinking level in conjunction with Newman's Error Analysis (NEA). This study is descriptive in nature. The Grade 11 secondary school learners in the Amathole East district of South Africa completed a geometry test and an interview that produced the data used in this study. Thus, Learners at the visualisation level made errors during the reading and comprehension steps. Learners who performed at the analysis level made errors in the processes of comprehension and transform. While the learners encountered errors in three stages—comprehension error, transform error, and conceptual mistake—in the levels of informal and formal deduction. Learners make the most errors in comprehension followed by transformation. This study found that learners lack basic concepts, hence they did not comprehend the questions and were unsure of how to proceed.*

**Keywords—Errors; Euclidean geometry; Van Hiele theory; Newman's Error Analysis (NEA)**

## INTRODUCTION

One area of mathematics where most learners consistently score poorly is geometry (Mamiala, Mji & Simelane-Mnisi, 2021). Most secondary school learners in South Africa, according to studies, have insufficient basic geometry understanding, which causes them to make significant errors when solving geometry tasks (Luneta, 2015; Makhubele, 2014). Errors committed by learners stems from different sources as identified by different researchers. In Jojo's (2016) and Siyepu's (2005) study, in which learners' geometric conceptual understanding was investigated, they found out that vast majority of learners start Further Education and Training (FET) level that is Grade 10 -12 classes, with limited geometric knowledge. This implies that lower classes are not equipping learners to cope with Grade 10 – 12 geometry. This is supported by Kutama (2009) who found that learners in Grade 8 – 9 could hardly interpret their thoughts in writing and drawing during geometric activities given. For learning to occur new information must be integrated into schemas if it is consistent with these particular schemas (Jojo, 2011). Learning does not occur when the information is contradictory to the schemas instead it is memorised (Masilo, 2018; Pankin, 2013). Learning by memorisation imposes limitations in terms of students' development of conceptual and problem-solving skills (Siyepu & Mtonjeni, 2014). Ngirishi (2015) added to the list by saying that mistakes are caused by students not grasping a variety of geometric concepts.

Studying learners' errors is necessary for teachers to become proficient at interpreting and effectively correcting learners' errors because they reveal the depth of their learning. This is supported by Fang (2010), who argues that incorrect responses might lead to a cultural pedagogy that turns mistakes in given tasks into sources for teaching and acquiring logical and analytical practices from a young age. The study conducted by Ndlovu and Mji (2012) argues that through learners' errors teachers can derived some insights which may

stimulate learning. According to Keith and Frese (2008), errors committed by learners are clues that teachers should capitalise on to discover learners' current knowledge. Teachers should therefore embrace errors instead of treating them as 'toxic'. Errors have the potential to empower learners, as such when teachers teach should use them as a stepping stone contributing to success (Brodie, Shalem, Sapire & Manson, 2010). This article holds the view that teachers may benefit from having a deep understanding of learners' errors in geometry because they will inevitably occur. As a result, they might develop pedagogical strategies to help minimise these errors.

Newman's Error Analysis (NEA) is one method that teachers can use to explore and understand errors learners construct when responding to geometry problems. The NEA is map out as a basic analysis process. Abdullah, Abidin and Ali (2015) stated the stages of Newman Error Analysis, namely, reading error, comprehension error, transformation error, process skill error, and encoding error. Furthermore, the stages of Newman Error Analysis are as follows: (1) reading error – the learners misread the given information pertaining to geometry question; (2) comprehension error – (a) the learners hardly write what is known from the question; (b) the learners write what is known but not exactly what is known in the questions; (2) transformation error – (a) the learners writing the correct statement with incorrect reason or vice versa; (b) the learner write answers that are partially correct (3) process error – (a) the learners do not write down the stages in calculating (b) learners pay less attention to different sizes of angles; (5) encoding error – the learners fail to use the statements of logic to draw a conclusion after writing the correct statements and reasons for example, if  $x=y$  and  $y = z$  therefor  $x = z$ .

The researcher's intention to perform a study that examines the causes of various student errors was motivated by the effectiveness of NEA in identifying the types of errors that students encounter. Because to the lack of studies on error

analysis in geometry problem solving, in the Amathole East district of the Eastern Cape Province of South Africa.

The study aimed to address the following questions in particular:

1. What are the root causes of errors that are seen at various levels?

**Method of the study**

The research approach employed in the study was descriptive qualitative. The study's purpose was to gather data on error analysis using the Van Hiele theory and Newman techniques when responding to the geometry test. The geometry test was dispersed among Van Hiele's levels. Question 1 covered the level of visualisation, Question 2 the level of analysis, and Questions 3 and 4 the levels of informal and formal deductive reasoning, respectively. To determine the discrimination index and select the appropriate question for the ultimate levels of reasoning, the test was piloted and its questions were analysed. Additionally, to validate the tool and determine the proper levels for each test question, specialists' professional assistance was requested. Examining was done on the learners' responses. Following that, interviews with five learners who had been chosen based on their test responses were held to ascertain the causes of the errors they had shown up in the test scripts. To learn the reasons for their errors in responding to the geometric test, the researcher interviewed those five learners.

**Participants of the Study**

This study consisted of 38 Grade 11 mathematics learners. Eighteen of the participants were considered for the pilot testing and the rest took the final test where the samples were taken. Errors experienced by twenty learners at each level of Van Hiele were carried out, that is visualisation level, analysis level, informal deduction level and formal deduction level.

**Presentation and Analysis of Data**

Table 1 summarises the responses given to questions 1 through 4 by the 20 test-takers. Table 1 displays the errors that learners at each van Hiele level made.

Table 1: The errors made by learners to test questions

Error type	Question 1	Question 2	Question 3		Question 4	Total	Percentage
	1.1	2.1.1	3.1.1	3.1.2	4.1		
	Number of errors committed by learners at visualisation level	Number of errors committed by learners at analysis level	Number of errors committed by learners at informal level		Number of errors committed by learners at formal level		
Reading error	1	0	0	0	0	1	1%
Comprehension error	2	10	4	5	3	24	24%

Transformation error		5	4	6	7	22	22%
Encoding error						0	0%
Process/procedural error						0	0%
Conceptual error			8	6	7	21	21%

According to the test description and the results of the learner interviews, it was discovered that the following errors were made by the learners when responding to item 1.1, which requires them to identify a radius from given parts of a circle. Reading error made as a result of the learner's quick glance at the figure before thoroughly reading the question. The learner was excited to complete. The student's reading error level, as determined by the calculation of the error level percentage, is 1%. The learner committed reading error because he/she is not used to reading mathematics questions. The learner who failed to write down what was known and what the test asked about made a comprehension error. The majority of learners struggled to apply the test material, which prevented them from correctly solving the test's problems. The calculating result of the error level percentage yielded a learner error level of 24% for this type of error. Due to the learners' inability to comprehend geometric concepts, a comprehension error was made. Learners who write the correct statement with the erroneous reason or vice versa are committing transformation error. Most of the learners here gave answers that were only half accurate. According to the calculation of the error level percentage, the learners' error level for this particular sort of transformation error is 22%. Learners made a transformation error by applying rules they had previously memorised and only remembering a portion of the material. Conceptual error brought on by learners' poor command of fundamental facts, concepts, and abilities. Learners were unable to correctly employ the properties of geometric principles to reduce geometry problems. Learners provided an explanation that had nothing to do with a problem. This is a sign that they misapplied the concepts that were taught since they did not understand them. The calculating result of the error level percentage yielded a student error level of 21% for this type of conceptual error. Since few learners chose not to tackle the questions, it was difficult to spot their errors. However, it can be concluded that they lacked basic conceptual comprehension of the concept of geometry because encoding and process errors were both 0 percent.

The findings showed that the visualisation level learners made errors at both the reading and comprehension levels. The analysis level learners encountered errors in both the comprehension and transform stages. Learners at the informal deduction and formal deduction levels made errors at three different levels: comprehension, transform, and conceptual. Further analysis of the results showed that whereas transformation and conceptual errors were more commonly made at the levels of formal and informal deduction,

comprehension errors were more frequently made at the levels of visualisation and analysis.

### Conclusion

The study's findings are based on information obtained from the test tool and an actual interview.

Only one percent of learners in Grade 11 made a reading error because the majority of them had reading skills.

Learners at the visualisation level made errors during the reading and comprehension steps. Learners who performed at the analysis level made errors in the processes of comprehension and transform. While the learners encountered errors in three stages—comprehension error, transform error, and conceptual mistake—in the levels of informal and formal deduction.

Few learners did not attempt the questions, so it was not easy to identify their errors. It can however be assumed that since encoding and process errors were at zero percent, they lacked conceptual understanding of the concept of geometry.

The majority of learners struggled with many of the fundamental geometry concepts. The majority of errors included conceptual, indicating that learners did not comprehend the problems and were unable to proceed as a result.

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