International Journal of Academic Information Systems Research (IJAISR)
ISSN: 2643-9026
Vol. 6 Issue 10, October - 2022, Pages: 1-5

N-Ary Relation Soft set

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Abstract: *In this work, we present a new idea of n-ary relation soft set by joining the characteristics of n-ary relation with the parametrization of the soft set, and constructs the new type named n-ary relation soft set, which is more functional to make practical and theoretical ideas or studies in the soft set theory. We center around the fundamental operations for the n-ary relation soft set and study their properties, and applied examples of n-ary relation soft set were presented. This new idea advantageous and contributes in some scientific applications, I will introduce these applications in next articles.*

Keywords : N-Ary Relation ,soft set , **N-Ary Relation Soft set**

Introduction:

Molodtsov [1] was first started or initiated the soft theory which is the advantage Mathematical instrument. Molodtsov was talked around the functionality of the idea of soft sets for applications in some sort of various and different ways. Maji was examined some theoretical investigation of soft-sets such as; soft sub set, not set, equality involving two soft sets. And they discussed some main applications on soft sets, for example, the 'union', the 'intersection', the 'and' and 'or' operations. In [2] Haci Aktas et.al imported some initial important soft sets components in addition to compare soft sets towards the related ideas associated with rough-sets and fuzzy sets. Ahmad and Kharal [3] presented the main notations on mapping of soft classes and also presented some main properties of images of soft sets in which these can be applied in some main problems of medical diagnosis. In [4] Das and Borgahain was studied around fuzzy soft set and that may be applied on a multi criteria multi observer decision making problems and difficulties.

Recently, the researches on the soft set theory has been so active, and rapidly and great progress has been achieved [5–9]. It is worth noting that all of those works are based on the classical soft set theory.

In the present idea, our motivation is to present a new type of soft set contains the concept of n-ary relation on many sets of parameters which enable us to apply in one of the important scientific fields.

This article consists of 3 sections. In Section 1, we will mention some main definitions and concepts of soft sets which we are need in this our work. In Section 2, we introduce a new definition of soft set (n-ary relation soft set) with example. In Section 3 introduce the important and main operations on n-ary relation soft sets, with illustrative example.

1. Preliminaries

In the following some concepts and definitions about soft sets which we are need in this paper.

Definition 1.1[1]: Let $\mathfrak{X}$ be an initial universe and $\mathcal{E}$ be a set of parameters. Let $\mathcal{P}(\mathfrak{X})$ denotes the power set of $\mathfrak{X}$ and $\mathcal{A}$ be a non-empty subset of $\mathcal{E}$. A pair $(\mathscr{s}, \mathcal{A})$ is said to be a soft set over $\mathfrak{X}$, where s is a mapping given by $\mathscr{s}: \to \mathcal{P}(\mathfrak{X})$.

In other words, a soft set over $\mathfrak{X}$ is a parametrized family of subsets of the universe $\mathfrak{X}$. For $e \in \mathcal{A}$, $\mathscr{s}(e)$ may be considered as the set of e −approximate elements of the soft set $(\mathscr{s}, \mathcal{A})$, and if $e \notin \mathcal{A}$, then $\mathscr{s}(e) = \emptyset$,

i.e.,$(\mathscr{s}, \mathcal{A}) = \{(e, \mathscr{s}(e)) : e \in \mathcal{A} \subseteq \mathbb{E}, \mathscr{s}: \mathcal{A} \to \mathcal{P}(\mathfrak{X})\}$, Clear that, a soft set is n$\mathscr{s}$ot a set.

Definition 1.2 [10]: The complement of a soft set $(\mathscr{s}, \mathcal{E})$ is denoted by $(\mathscr{s}\ \ )^c$and is defined by $(\mathscr{s})^c = (\mathscr{s}^c)$ where $\mathscr{s}^c: \mathcal{E} \to \mathcal{P}(\mathfrak{X})$ is a mapping given by $\mathscr{s}^c(e) = \mathfrak{X} - \mathscr{s}(e)$ for all $e \in \mathcal{E}$.
Definition 1.3 [10]: Let Y be a non-empty subset of $\mathcal{X}$, then Y denotes the soft set (Y, E) over $\mathcal{X}$ for which $Y(e) = Y$, for all $e \in \mathcal{E}$. In particular, $(\mathfrak{X}, \mathcal{E})$ will be denoted by $\mathfrak{X}$.
Definition 1.4 [10]: Let $x \in \mathfrak{X}$. Then $(x, \mathcal{E})$ denotes the soft set over $\mathfrak{X}$ for which $\mathscr{s}(e) = \{x\}$, for all $e \in \mathbb{E}$.
Definition 1.5 [10]: Let $(\mathscr{s}, \ \mathcal{E})$ be a soft set over $\mathfrak{X}$ and $x \in \mathfrak{X}$. We say that
$x \in (\mathscr{s}, \mathcal{E})$ read as x belongs to the soft set $(\mathscr{s}, \ \mathcal{E})$ whenever $x \in \mathscr{s}(e)$ for all $e \in \mathcal{E}$. Note that for any $x \in \mathfrak{X}$, $x \not\in (\mathscr{s}, \mathcal{E})$, if $x \not\in \mathscr{s}(e)$ for some $e \in \mathcal{E}$.

Definition 1.6 [10]: For two soft sets $(\mathcal{s},\mathcal{A})$ and $(\mathcal{g},\mathcal{B})$ over a common universe $\mathfrak{X}$, we say that $(\mathcal{s},\mathcal{A})$ is a soft subset of $(\mathcal{g},\mathcal{B})$ if

(1) $\mathcal{A} \subseteq \mathcal{B}$ and

(2) $\mathcal{s}$ (e) $\subseteq$ $\mathcal{g}$ (e), for all e $\in$ $\mathcal{A}$.

$(\mathcal{s},\mathcal{A})$ is said to be a soft super set of $(\mathcal{g},\mathcal{B})$, if $(\mathcal{g},\mathcal{B})$ is a soft subset of $(\mathcal{s},\mathcal{A})$.

Definition 1.7 [10]: The Union of two soft sets $(\mathcal{s},\mathcal{E})$ and $(\mathcal{g}, \mathcal{E})$ over the common universe $\mathcal{X}$ is the soft set $(\mathcal{H},\mathcal{E})$, where $\mathcal{H}(\text{e}) = \mathcal{s}(\text{e}) \cup \mathcal{g}(\text{e})$ for all e $\in \mathcal{E}$. We express $(\mathcal{s},\mathcal{E}) \cup (\mathcal{g},\mathcal{E}) = (\mathcal{H},\mathcal{E})$.

Definition 1.8 [10]: The intersection of two soft sets $(\mathcal{s},\mathcal{E})$ and $(\mathcal{g},\mathcal{E})$ over the common universe $\mathcal{X}$ is the soft set $(\mathcal{H},\mathcal{E})$ where $\mathcal{H}(\text{e}) = \mathcal{s}(\text{e}) \cap \mathcal{g}(\text{e})$ for all e $\in \mathcal{E}$. We express $(\mathcal{s},\mathcal{E}) \cap (\mathcal{g},\mathcal{E}) = (\mathcal{H},\mathcal{E})$.

Definition 1.9 [11]: The ordered *n*-tuple (shortly *n*-tuple), **indicated as** $(a_1, \ldots, a_n)$, is the ordered collection of items with a_1 as its first member, a_2 as its second member,…, and a_n as its n^{th} member. Two *n*-tuples are equal, if each corresponding pair of their elements are equivalent..

Let $A_1, \ldots, A_n$be finite sets. The Cartesian product of the sets $A_1, \ldots, A_n$ indicated by$A_1 \times A_2 \ldots \times A_n$ is the set of *n*-tuples $(a_1, \ldots, a_n)$ in which a_1 belongs to A_1 for $i = 1, \ldots, n$ symbols, $A_1 \times A_2 \ldots \times A_n = \{(a_1, \ldots, a_n) | a_i \in A_i \; for \; i = 1, \ldots, n\}$
Now, the definition of an *n*-ary relation may follow:
An ***n*-ary relation** on sets $A_1, \ldots, A_n$, represented by R, is any subset of their Cartesian product, ie. $R \subseteq A_1 \times \ldots \times A_n$. The sets $A_1, \ldots, A_n$ are the domains of R where n is its degree. In the particular scenario where case $n = 2$, denoting $A_1 = A, A_2 = B$, we speak about a **binary relation between** A and B and if moreover $A = B$ about a relation on A;
In case $n = 3$ also the word **ternary** is used. For our next consideration the generalization of the concept of functionality of a binary relation plays a key role. A binary relation R from A to B is functional, if for every $a \in A$ there is at most one $b \in B$ such that $(a, b) \in R$.

2. Main Notions of N-Ary Relation Soft Set

In this section we introduce the state of a n-ary relation soft set and its operations. These operations are equality, union, intersection and subset.

2.1. Definition of N-Ary Relation Soft Set

Let $\mathfrak{X}$ be an initial universe set and $\mathcal{E}_1, \ldots, \mathcal{E}_n$ be a different sets of parameters. Let $\mathcal{P}(\mathfrak{X})$ denotes the power set of $\mathfrak{X}$.
A pair $(\mathcal{s}, \mathcal{E}_1)$ is said to be a soft set over $\mathfrak{X}$, where $\mathcal{s}$ is a mapping given by $\mathcal{s}: \mathcal{E}_1 \to \mathcal{P}(\mathfrak{X})$. i.e.,$(\mathcal{s}, \mathcal{E}_1) = \{(\text{e}, \mathcal{s}(\text{e})) : \text{e} \in \mathcal{E}_1, \mathcal{s}: \mathcal{E}_1 \to \mathcal{P}(\mathfrak{X})\}$. (typically like the GAMMA- soft-set in [12])
A pair $(\mathcal{s}, \mathcal{E}_1 \times \mathcal{E}_2)$ is said to be a **binary relation soft set** over $\mathfrak{X}$, where $\mathcal{s}$ is a mapping given by $\mathcal{s}: \mathcal{E}_1 \times \mathcal{E}_2 \to \mathcal{P}(\mathfrak{X})$.
i.e.,$(\mathcal{s}, \mathcal{E}_1 \times \mathcal{E}_2) = \{\left((e_i, e_j), \mathcal{s}(e_i, e_j)\right) : e_i \in \mathbb{E}_1$ and $e_j \in \mathcal{E}_2, \mathcal{s}: \mathcal{E}_1 \times \mathcal{E}_2 \to \mathcal{P}(\mathfrak{X})\}$.

A pair $(\mathcal{s}, \mathcal{E}_1 \times \mathcal{E}_2 \times \mathcal{E}_3)$ is said to be a **ternary relation soft set** over $\mathfrak{X}$, where $\mathcal{s}$ is a mapping given by $\mathcal{s}: \mathcal{E}_1 \times \mathcal{E}_2 \times \mathcal{E}_3 \to \mathcal{P}(\mathfrak{X})$.
i.e.,$(\mathcal{s}, \mathcal{E}_1 \times \mathcal{E}_2 \times \mathcal{E}_3) = \{\left((e_i, e_j, e_k), \mathcal{s}(e_i, e_j, e_k)\right) : e_i \in \mathcal{E}_1 , e_j \in \mathcal{E}_2$ and $e_k \in \mathcal{E}_3 , \mathcal{s}: \mathcal{E}_1 \times \mathcal{E}_2 \times \mathcal{E}_3 \to \mathcal{P}(\mathfrak{X})\}$.

And A pair $(\mathcal{s}, \mathcal{E}_1 \times \ldots \times \mathcal{E}_n)$ is said to be a **n-ary relation soft set** over $\mathfrak{X}$, where $\mathcal{s}$ is a mapping given by $\mathcal{s}: \mathcal{E}_1 \times \ldots \times \mathcal{E}_n \to \mathcal{P}(\mathfrak{X})$.
i.e., $(\mathcal{s}, \prod_{i=1}^{n} \mathcal{E}_i = \mathcal{E}_1 \times \ldots \times \mathcal{E}_n) = \{\left((e_1, \ldots, e_n), \mathcal{s}(e_1, \ldots, e_n)\right) : e_i \in \mathcal{E}_i, \; \mathcal{s}: (\prod_{i=1}^{n} \mathcal{E}_i) = \mathcal{E}_1 \times \ldots \times \mathcal{E}_n \to \mathcal{P}(\mathfrak{X})\}$.

Clear that, This kind of soft sets is not a set.

Example 2.2. Let us consider a ternary relation soft set $(\mathcal{s}, \mathcal{E}_1 \times \mathcal{E}_2 \times \mathcal{E}_3)$.
Let $\mathfrak{X}$ be an initial universe sets of objects under consideration representing motor-cycles = $\{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4, \varkappa_5, \varkappa_6, \varkappa_7, \varkappa_8, \varkappa_9, \varkappa_{10}, \varkappa_{11}, , \varkappa_{12}, , \varkappa_{13}, \varkappa_{14}, , \varkappa_{15}\}$, and $\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3$ be a different sets of parameters. Let $\mathcal{E}_1$ be a set of cost parameters given by;
$\mathcal{E}_1 = \{\zeta_1$=expensive, ζ_2=cheap$\}$, let $\mathcal{E}_2$ be a set of color parameters given by;
$\mathcal{E}_2 = \{\eta_1$=white, η_2 = blue$\}$, and let $\mathcal{E}_3$ be a set of size parameters given by;
$\mathcal{E}_3 = \{\rho_1$=big, ρ_2=small$\}$

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ISSN: 2643-9026
Vol. 6 Issue 10, October - 2022, Pages: 1-5

Suppose Mr. $\mathcal{H}$ is considering for purchase a big and cheap motor-cycle regardless of the color. Then the soft set is considered such as:

$(\mathcal{s}, \mathcal{E}_1 \times \mathcal{E}_2 \times \mathcal{E}_3) = \{\left(\left(\zeta_i, \eta_j, \rho_k\right), \mathcal{s}\left(\zeta_i, \eta_j, \rho_k\right)\right) : \zeta_i \in \mathcal{E}_1, \eta_j \in \mathcal{E}_2 \text{ and } \rho_k \in \mathcal{E}_3, \mathcal{s}: \mathcal{E}_1 \times \mathcal{E}_2 \times \mathcal{E}_3 \to \mathcal{P}(\mathfrak{X})\}$. Where $\mathrm{i, j, k} = 1,2$.

And suppose that $(\zeta_1, \eta_1, \rho_1) = \{\varkappa_1, \varkappa_3\}$, $\mathcal{s}(\zeta_2, \eta_1, \rho_1) = \{\varkappa_{12}, \varkappa_5\}$, $\mathcal{s}(\zeta_1, \eta_2, \rho_1) = \{\varkappa_8, \varkappa_9\}$

$\mathcal{s}(\zeta_1, \eta_1, \rho_2) = \{\varkappa_4, \varkappa_7\}$, $\mathcal{s}(\zeta_2, \eta_2, \rho_1) = \{\varkappa_6, \varkappa_{10}\}$, $\mathcal{s}(\zeta_2, \eta_1, \rho_2) = \{\varkappa_2, \varkappa_{15}\}$, $\mathcal{s}(\zeta_1, \eta_2, \rho_2) = \{\varkappa_{13}\}$, $\mathcal{s}(\zeta_2, \eta_2, \rho_2) = \{\varkappa_{14}\}$

Thus $(\mathcal{H}, \mathcal{E}_1 \times \mathcal{E}_2 \times \mathcal{E}_3) = \{\mathcal{s}(\zeta_2, \eta_1, \rho_1) = \{\varkappa_{12}, \varkappa_5\}, \mathcal{s}(\zeta_2, \eta_2, \rho_1) = \{\varkappa_6, \varkappa_{10}\}\ \}$, i.e. there are four motor-cycles $\{\varkappa_{12}, \varkappa_5, \varkappa_6, \varkappa_{10}\}$ can be considered for purchase.

3. Operations on N-Ary Relation Soft Set

Definition 3.1. A n-ary relation soft set $(\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ over $\mathfrak{X}$ is said to be a null n-ary relation soft set if $\mathcal{s}(e_1, \dots, e_n) = \emptyset$ for all $e_i \in \mathcal{E}_i\, ; i = 1, \dots, n$, we write $(\emptyset, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ for the empty n-ary relation soft set over $\mathfrak{X}$.

Definition 3.2. A n-ary relation soft set $(\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ over $\mathfrak{X}$ is said to be a universal n-ary relation soft set if $\mathcal{s}(e_1, \dots, e_n) = \mathfrak{X}$ for all $e_i \in \mathcal{E}_i\, ; i = 1, \dots, n$, we write $(\mathfrak{X}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ for the universal n-ary relation soft set over $\mathfrak{X}$.

Definition 3.3. The complement of n-ary relation soft set $(\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ over $\mathfrak{X}$ is denoted by $(\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)^{\mathrm{c}}$ and is defined by $(\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)^{\mathrm{c}} = (\mathcal{s}^{\mathrm{c}}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ where $\mathcal{s}^{\mathrm{c}}: \mathcal{E}_1 \times \dots \times \mathcal{E}_n \to \mathcal{P}(\mathcal{X})$ is a mapping given by $\mathcal{s}^{\mathrm{c}}(e_1, \dots, e_n) = \mathfrak{X} - \mathcal{s}(e_1, \dots, e_n)$ for all $e_i \in \mathcal{E}_i\, ; i = 1, \dots, n$.

Definition 3.4. Let $\varkappa \in \mathfrak{X}$. Then $(\varkappa, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ denotes the n-ary relation soft set over $\mathcal{X}$ for which $\mathcal{s}(e_1, \dots, e_n) = \{\varkappa\}$, for all $e_i \in \mathcal{E}_i\, ; i = 1, \dots, n$.

Definition 3.5. Let $(\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ be n-ary relation soft set over $\mathfrak{X}$ and $\varkappa \in \mathfrak{X}$. We say that $\varkappa \in (\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ read as $\varkappa$ belongs to the n-ary relation soft set $(\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ whenever $\varkappa \in \mathcal{s}(e_1, \dots, e_n)$ for all $e_i \in \mathcal{E}_i\, ; i = 1, \dots, n$. Note that for any $\varkappa \in \mathcal{X}$, $\varkappa \notin (\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$, if $\varkappa \notin \mathcal{s}(e_1, \dots, e_n)$ for some $e_i \in \mathcal{E}_i\, ; i = 1, \dots, n$.

Definition 3.6. For two n-ary relation soft sets $(\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ and $(\mathcal{g}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ over a common universe $\mathfrak{X}$, we say that $(\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ is n-ary relation soft subset of $(\mathcal{g}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ denoted by $(\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n) \,\widetilde{\subseteq}\, (\mathcal{g}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ if $\mathcal{s}(e_1, \dots, e_n) \subseteq \mathcal{g}(e_1, \dots, e_n)$, for all $e_i \in \mathcal{E}_i\, ; i = 1, \dots, n$.

$(\mathcal{g}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ is said to be n-ary relation soft super set of $(\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$, if $(\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ is n-ary relation soft subset of $(\mathcal{g}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$.

Two n-ary relation soft sets $(\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ and $(\mathcal{g}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ are said to be *equal*, denoted $(\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n) = (\mathcal{g}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$, if $(\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n) \,\widetilde{\subseteq}\, (\mathrm{G}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ and $(\mathcal{g}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n) \,\widetilde{\subseteq}\, (\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$.

Definition 3.7. The Union of two n-ary relation soft sets $(\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ and $(\mathcal{g}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ over the common universe $\mathfrak{X}$ is the n-ary relation soft set $(\mathcal{H}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$, where $\mathcal{H}(e_i, \dots, e_n) = \mathcal{s}(e_i, \dots, e_n) \cup \mathcal{g}(e_i, \dots, e_n)$ for all $e_i \in \mathcal{E}_i\, ; i = 1, \dots, n$. We write $(\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n) \cup (\mathcal{g}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n) = (\mathcal{H}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$.

Definition 3.8. The intersection of two n-ary relation soft sets $(\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ and $(\mathcal{g}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ over the common universe $\mathfrak{X}$ is the n-ary relation soft set $(\mathcal{H}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ where $\mathrm{H}(e_i, \dots, e_n) = \mathcal{s}(e_i, \dots, e_n) \cap \mathcal{g}(e_i, \dots, e_n)$ for all $e_i \in \mathcal{E}_i\, ; i = 1, \dots, n$. We write $(\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n) \cap (\mathcal{g}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n) = (\mathcal{H}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$.

Definition 3.9. The difference $(\mathcal{H}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ of two $soft-$ sets $(\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ and $(\mathcal{g}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$ over the common universe $\mathcal{X}$, denoted by $(\mathcal{s}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n) \backslash (\mathcal{g}, \mathcal{E}_1 \times \dots \times \mathcal{E}_n)$, is defined as $\mathcal{H}(e_i, \dots, e_n) = \mathcal{s}(e_i, \dots, e_n) \backslash \mathcal{g}(e_i, \dots, e_n)$ for all $e_i \in \mathcal{E}_i\, ; i = 1, \dots, n$.

Example 3.10. $\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4$ be a different sets of parameters. Let $\mathfrak{X}$ be an initial universe set $\mathfrak{X} = \{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4, \varkappa_5\}$, $\mathcal{E}_1 = \{\zeta_1\}$, $\mathcal{E}_2 = \{\eta_1\}$, $\mathcal{E}_3 = \{\rho_1, \rho_2\}$, $\mathcal{E}_4 = \{\delta_1, \delta_2, \delta_3\}$

$(\mathcal{s}, \prod_{i=1}^{4} \mathcal{E}_i) = \{\left(\left(\zeta_i, \eta_j, \rho_k, \delta_{\mathrm{L}}\right), \mathcal{s}\left(\zeta_i, \eta_j, \rho_k, \delta_{\mathrm{L}}\right)\right) : \zeta_i \in \mathcal{E}_1, \eta_j \in \mathcal{E}_2 \text{ and } \rho_k \in \mathcal{E}_3, \delta_{\mathrm{L}} \in \mathcal{E}_4, \mathcal{s}: \mathcal{E}_1 \times \mathcal{E}_2 \times \mathcal{E}_3 \times \mathcal{E}_4 \to \mathcal{P}(\mathfrak{X})\}$. Where $\mathrm{i} = 1$, $\mathrm{j} = 1$, $\mathrm{k} = 1,2$, $L = 1,2,3$.

Let us consider a 4-ary relation soft sets $(\mathcal{s}, \prod_{i=1}^{4} \mathcal{E}_i)$ and $(\mathcal{g}, \prod_{i=1}^{4} \mathcal{E}_i)$, where; $\mathcal{s}(\zeta_1, \eta_1, \rho_1, \delta_1) = \{\varkappa_1, \varkappa_5\}$, $\mathcal{s}(\zeta_1, \eta_1, \rho_1, \delta_2) = \{\varkappa_2, \varkappa_5\}$, $\mathcal{s}(\zeta_1, \eta_1, \rho_1, \delta_3) = \{\varkappa_1\}$, $\mathcal{s}(\zeta_1, \eta_1, \rho_2, \delta_1) = \{\varkappa_4, \varkappa_5\}$, $\mathcal{s}(\zeta_1, \eta_1, \rho_2, \delta_2) = \{\varkappa_1, \varkappa_3, \varkappa_4\}$, $\mathcal{s}(\zeta_1, \eta_1, \rho_2, \delta_3) = \mathfrak{X}$

And

$g(\zeta_1,\eta_1,\rho_1,\delta_1)=\{\varkappa_1,\varkappa_3\}$, $g(\zeta_1,\eta_1,\rho_1,\delta_2)=\emptyset$, $g(\zeta_1,\eta_1,\rho_1,\delta_3)=\{\varkappa_2,\varkappa_3\}$, $g(\zeta_1,\eta_1,\rho_2,\delta_1)=\{\varkappa_4,\varkappa_5\}$, $g(\zeta_1,\eta_1,\rho_2,\delta_2)=\{\varkappa_3,\varkappa_4\}$, $g(\zeta_1,\eta_1,\rho_2,\delta_3)=\{\varkappa_2\}$,

Then we can perform the operations above on both 4-ary relation soft sets, as fellows;

Let $(n,\prod_{i=1}^{4}\mathcal{E}_i)$ be a null 4-ary relation soft set, where;
$n(\zeta_1,\eta_1,\rho_1,\delta_1)=\emptyset$, $n(\zeta_1,\eta_1,\rho_1,\delta_2)=\emptyset$, $n(\zeta_1,\eta_1,\rho_1,\delta_3)=\emptyset$, $n(\zeta_1,\eta_1,\rho_2,\delta_1)=\emptyset$, $n(\zeta_1,\eta_1,\rho_2,\delta_2)=\emptyset$, $n(\zeta_1,\eta_1,\rho_2,\delta_3)=\emptyset$,

And let $(\mathcal{U},\prod_{i=1}^{4}\mathcal{E}_i)$ be a universal 4-ary relation soft set, where;
$\mathcal{U}(\zeta_1,\eta_1,\rho_1,\delta_1)=\mathfrak{X}$, $\mathcal{U}(\zeta_1,\eta_1,\rho_1,\delta_2)=\mathfrak{X}$, $\mathcal{U}(\zeta_1,\eta_1,\rho_1,\delta_3)=\mathfrak{X}$, $\mathcal{U}(\zeta_1,\eta_1,\rho_2,\delta_1)=\mathfrak{X}$, $\mathcal{U}(\zeta_1,\eta_1,\rho_2,\delta_2)=\mathfrak{X}$, $\mathcal{U}(\zeta_1,\eta_1,\rho_2,\delta_3)=\mathfrak{X}$,

The complement $(s,\prod_{i=1}^{4}\mathcal{E}_i)^{C}$ is ;
$s^C(\zeta_1,\eta_1,\rho_1,\delta_1)=\{\varkappa_2,\varkappa_3,\varkappa_4\}$, $s^C(\zeta_1,\eta_1,\rho_1,\delta_2)=\{\varkappa_1,\varkappa_3,\varkappa_4\}$,$s^C(\zeta_1,\eta_1,\rho_1,\delta_3)=\{\varkappa_2,\varkappa_3,\varkappa_4,\varkappa_5\}$, $s^C(\zeta_1,\eta_1,\rho_2,\delta_1)=\{\varkappa_1,\varkappa_2,\varkappa_3\}$, $s^C(\zeta_1,\eta_1,\rho_2,\delta_2)=\{\varkappa_2,\varkappa_5\}$, $s^C(\zeta_1,\eta_1,\rho_2,\delta_3)=\emptyset$.

The complement $(g,\prod_{i=1}^{4}\mathcal{E}_i)^{C}$ is ;
$g^C(\zeta_1,\eta_1,\rho_1,\delta_1)=\{\varkappa_2,\varkappa_4,\varkappa_5\}$, $g^C(\zeta_1,\eta_1,\rho_1,\delta_2)=\mathfrak{X}$, $g^C(\zeta_1,\eta_1,\rho_1,\delta_3)=\{\varkappa_1,\varkappa_4,\varkappa_5\}$, $g^C(\zeta_1,\eta_1,\rho_2,\delta_1)=\{\varkappa_1,\varkappa_2,\varkappa_3\}$, $g^C(\zeta_1,\eta_1,\rho_2,\delta_2)=\{\varkappa_1,\varkappa_2,\varkappa_5\}$, $g^C(\zeta_1,\eta_1,\rho_2,\delta_3)=\{\varkappa_1,\varkappa_3,\varkappa_4,\varkappa_5\}$.

And let $(\varkappa_1,\prod_{i=1}^{4}\mathcal{E}_i)$ be a 4-ary relation soft set, where;
$\mathcal{B}(\zeta_1,\eta_1,\rho_1,\delta_1)=\{\varkappa_1\}$, $\mathcal{B}(\zeta_1,\eta_1,\rho_1,\delta_2)=\{\varkappa_1\}$, $\mathcal{B}(\zeta_1,\eta_1,\rho_1,\delta_3)=\{\varkappa_1\}$, $\mathcal{B}(\zeta_1,\eta_1,\rho_2,\delta_1)=\{\varkappa_1\}$, $\mathcal{B}(\zeta_1,\eta_1,\rho_2,\delta_2)=\{\varkappa_1\}$, $\mathcal{B}(\zeta_1,\eta_1,\rho_2,\delta_3)=\{\varkappa_1\}$,

And let $(\mathcal{D},\prod_{i=1}^{4}\mathcal{E}_i)$ be a 4-ary relation soft set, where;
$\mathcal{D}(\zeta_1,\eta_1,\rho_1,\delta_1)=\{\varkappa_5\}$, $\mathcal{D}(\zeta_1,\eta_1,\rho_1,\delta_2)=\{\varkappa_5\}$, $\mathcal{D}(\zeta_1,\eta_1,\rho_1,\delta_3)=\emptyset$, $\mathcal{D}(\zeta_1,\eta_1,\rho_2,\delta_1)=\{\varkappa_4\}$, $\mathcal{D}(\zeta_1,\eta_1,\rho_2,\delta_2)=\{\varkappa_1,\varkappa_3\}$, $\mathcal{D}(\zeta_1,\eta_1,\rho_2,\delta_3)=\mathcal{X}$, then it is clear that, $(\mathcal{D},\prod_{i=1}^{4}\mathcal{E}_i)\,\widetilde{\subseteq}\,(s,\prod_{i=1}^{4}\mathcal{E}_i)$.

The intersection $(s,\prod_{i=1}^{4}\mathcal{E}_i)\cap(g,\prod_{i=1}^{4}\mathcal{E}_i)$= $(\mathcal{H},\prod_{i=1}^{4}\mathcal{E}_i)$, such that *:*
$\mathcal{H}(\zeta_1,\eta_1,\rho_1,\delta_1)=\{\varkappa_1\}$, $\mathcal{H}(\zeta_1,\eta_1,\rho_1,\delta_2)=\emptyset$, $g(\zeta_1,\eta_1,\rho_1,\delta_3)=\emptyset$, $\mathcal{H}(\zeta_1,\eta_1,\rho_2,\delta_1)=\{\varkappa_4,\varkappa_5\}$, $\mathcal{H}(\zeta_1,\eta_1,\rho_2,\delta_2)=\{\varkappa_3,\varkappa_4\}$, $\mathcal{H}(\zeta_1,\eta_1,\rho_2,\delta_3)=\{\varkappa_2\}$, and

The union $(s,\prod_{i=1}^{4}\mathcal{E}_i)\cup(g,\prod_{i=1}^{4}\mathcal{E}_i)$= $(\mathcal{A},\prod_{i=1}^{4}\mathcal{E}_i)$, such that :
$\mathcal{A}(\zeta_1,\eta_1,\rho_1,\delta_1)=\{\varkappa_1,\varkappa_3,\varkappa_5\}$, $\mathcal{A}(\zeta_1,\eta_1,\rho_1,\delta_2)=\{\varkappa_2,\varkappa_5\}$, $\mathcal{A}(\zeta_1,\eta_1,\rho_1,\delta_3)=\{\varkappa_1,\varkappa_2,\varkappa_3\}$, $\mathcal{A}(\zeta_1,\eta_1,\rho_2,\delta_1)=\{\varkappa_4,\varkappa_5\}$, $\mathcal{A}(\zeta_1,\eta_1,\rho_2,\delta_2)=\{\varkappa_1,\varkappa_3,\varkappa_4\}$, $\mathrm{H}(\zeta_1,\eta_1,\rho_2,\delta_3)=\mathfrak{X}$, and

The deference $(s,\prod_{i=1}^{4}\mathcal{E}_i)\,/(g,\prod_{i=1}^{4}\mathcal{E}_i)$= $(p,\prod_{i=1}^{4}\mathcal{E}_i)$, such that :
$p(\zeta_1,\eta_1,\rho_1,\delta_1)=\{\varkappa_5\}$, $p(\zeta_1,\eta_1,\rho_1,\delta_2)=\{\varkappa_2,\varkappa_5\}$, $p(\zeta_1,\eta_1,\rho_1,\delta_3)=\{\varkappa_1\}$, $p(\zeta_1,\eta_1,\rho_2,\delta_1)=\emptyset$, $p(\zeta_1,\eta_1,\rho_2,\delta_2)=\{\varkappa_1\}$, $p(\zeta_1,\eta_1,\rho_2,\delta_3)=\{\varkappa_1,\varkappa_3,\varkappa_4,\varkappa_5\}$, and

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