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Implicative AT-ideals of AT-Algebra

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Abstract— *We consider the of implicative, positive implicative and commutative on AT − algebras, and investigate some related properties. We give conditions implicative AT − ideal, positive implicative AT − ideal and commutative AT − ideal on AT − algebras .*

Keywords— *AT − algebras*, implicative, implicative, commutative, implicative *AT − ideal*, positive implicative *AT − ideal*, commutative *AT − ideal*.

1- Introduction

Iseki [4] *introduced the* notion *of a BCI − algebra which* is a *generalization of BCK − algebra*. The *notions of ideals* in *BCK − algebras* and positive implicative (implicative) ideals *in BCK − algebras* were *introduced* and *investigated some related* properties. Mostafa *and et al* [5 − 7] *introduced the notion of KU − ideals of KU − algebras* and then *they investigated several* basic properties *which are related to KU − ideals*. *The idea of sub implicative ideal was introduced* , *they established the concepts* of *sub − implicative ideals* and *sub − commutative* ideals *in KU − algebras and investigated some of their properties*. *The goal of this* paper *is to introduce the* notions *of implicative*, positive implicative, commutative *AT − ideals* on *AT − algebras and investigate some their related properties*.

2- Preliminaries

Now, *we* will recall some known concepts related to AT-algebra from the literature which will be helpful in further study of this article.

DEF. 2.1[1-3]. An ***AT − algebra*** *is* a *nonempty set X with* a *constant* (0) *and a binary operation* ($*$) *satisfying the following* axioms: for all $x, y, z \in X$,

(i) $(x * y) * ((y * z) * (x * z)) = 0$,
(ii) $0 * x = x$,
(iii) $x * 0 = 0$.

In X we *can define* a *binary relation* ($\leq$) by : $x \leq y$ *if and only if* , $y * x = 0$.

In AT-algebra $(X; *, 0)$, *the* following *properties are satisfied*: for all $x, y, z \in X$,

(i') $(y * z) * (x * z) \leq (x * y)$,
(ii') $0 \leq x$ . .

PROP. 2.2 [3]. In any AT-algebra $(X; *, 0)$, *the* following properties *holds*: for all $x, y, z \in X$;

a) $x * x = 0$,
b) $z * (x * z) = 0$,
c) $y * ((y * z) * z) = 0$,
d) $x * y = 0$ *implies that* $x * 0 = y * 0$,
e) $x = 0 * (0 * x)$,
f) $0 * x = 0 * y$ *implies that* $x = y$.

PROP. 2.3[1-3]. In any AT-algebra $(X; *, 0)$, the following properties holds: for all $x, y, z \in X$;

a) $x \leq y$ *implies that* $y * z \leq x * z$,
b) $x \leq y$ *implies that* $z * x \leq z * y$,
c) $z * x \leq z * y$ *implies that* $x \leq y$ (left cancellation law).
d) $x * y \leq z$ *imply* $z * y \leq x$.

DEF. 2.4 [1-3]. A nonempty subset S of an AT-algebra $(X;*,0)$ is called **an AT-subalgebra of AT-algebra X** if $x*y \in S$, whenever $x, y \in S$.

DEF. 2.5 [3]. A nonempty subset I of an AT-algebra $(X;*,0)$ is called **an ideal of AT-algebra X** if it satisfies the following conditions: for all $x, y, z \in X$;

1) $0 \in I$;

2) $x * y \in I$ and $x \in I \implies y \in I$.

DEF. 2.6 [2-4]. A nonempty subset I of an AT-algebra $(X;*,0)$ is called **an AT-ideal of AT−algebra X** if it satisfies the following conditions: for all $x, y, z \in X$;

AT_1) $0 \in I$;

AT_2) $x * (y * z) \in I$ and $y \in I \implies x * z \in I$.

3. Commutative, positive implicative and implicative ideals of AT − algebras

In this section, we discuss the notions of sub − implicative, positive implicative, sub commutative AT − ideals of AT −algebra and we give some characterizations of these concepts.

DEF. 3.1. an $AT-$ algebra $(X;*,0)$ is said to be **positive implicative**, if it satisfies:

$(z*x)*(z*y)=z*(x*y)$, for all $, y, z \in X$.

TH.3.2. Let $(X;*,0)$ be an AT-algebra. X is positive implicative if and only if

$y*x=y*(y*x)$.

PR.: Clear.

DEF. 3.3. an AT-algebra $(X;*,0)$ is said to be ***implicative***, if it satisfies: $x=(x*y)*x$, for all $x, y \in X$.

DEF. 3.4. An AT-algebra $(X;*,0)$ is said to be **commutative** if it satisfies: $\forall x, y \in X$, $(y*x)*x=(x*y)*y$.

TH.3.5. For an AT-algebra $(X;*,0)$, the following are equivalent: $\forall x, y \in X$

(a) X is commutative,

(b) $(y*x)*x \leq (x*y)*y$,

(c) $\big((x*y)*y\big)*((y*x)*x)=0$.

PR.:

(a) ⇔ (b) Suppose that X is commutative, then $(y*x)*x=\big((x*y)*y\big)*x)*x \leq (x*y)*y$ that is (b) holds.

Conversely, the inequality $(y*x) \leq \big((x*y)*y\big)*x)$ holding. Next by (b) $\big((x*y)*y\big)*x \leq \big((y*x)*x\big)*x=y*x$. Hence $y*x=\big((x*y)*y\big)*x$ n and X is commutative.

(b) ⇔ (c) Since $(y*x)*x \leq (x*y)*y \Leftrightarrow \big((x*y)*y\big)*((y*x)*x)=0$.

DEF. 3.6. An AT−algebra $(X;*,0)$ is said to be **n-fold *implicative*** if it satisfies $x=(x^n*y)*x, \forall x, y \in X$, and $n \in Z^+$.

DEF. 3.7. An AT-algebra $(X;*,0)$ is said to be **n-fold positive implicative** if it satisfies $x^n*y=x^{n+1}*y, \forall x, y \in X$, and $n \in Z^+$.

DEF. 3.8. An AT-algebra $(X;*,0)$ is said to be **n-fold commutative** if it satisfies $y*x=\big((x^n*y)*y\big)*x, \forall x, y \in X$, and $n \in Z^+$.

EX. 3.9. Let $X=\{0, a, b, c, d\}$ in which the operation $*$ is given by:

*	0	a	b	c	d
0	0	a	b	c	d
a	0	0	a	a	b

b	0	0	0	a	a
c	0	0	a	0	b
d	0	0	0	0	0

Then $(X;*,0)$ is an AT-algebra, it is easy to verify that X is 2-fold commutative, but not commutative since (d * c)* c = c ≠ a = (c * d)* d . And X is neither 2-fold positive implicative nor 2-fold implicative, since a^2 *b = b ≠ a = a^3 *b and (a^2 *b)* a = 0 ≠ a.

PROP. 3.10. Let $(X;*,0)$ be an AT-algebra. If X is n-fold implicative then X is n-fold commutative but the inverse is false.

PR.: Suppose that X is n-fold implicative, then for any $, y \in X$, there exists a natural number n such that $x = (x^n * y) * x$,

$(y * x) * x = (y * x) * \left((x^n * y) * x\right) \leq (x^n * y) * y$ and so X is n-fold commutative by Theorem (3.5). From EX. (3.9), the inverse is not true.

PROP. 3.11. Let $(X;*,0)$ be an AT-algebra. If X is n-fold implicative then X is n-fold positive implicative.

PR.: Suppose that X is n-fold implicative, and putting $u = (x^n * y)$, since X is n-fold implicative, then there exists a natural number n' such that u $= \left(u^{n'} * x\right) * u$. Because

$u * x = (x^n * y) * x$, we have $u^{n'} * x = x$. Then $u = \left(u^{n'} * x\right) * u = x * u$, i.e., $x^n * y = x^{n+1} * y$. Therefore X is n-fold positive implicative.

PROP. 3.12. In AT-algebra $(X; *,0)$. X is n-fold implicative ⇔ X is both n −fold positive implicative and n-fold commutative.

PR.: It suffices to prove the part "⇐ ".

Let $x, y \in X$ and $u = (x^n * y)$. Since X is n- fold positive implicative, $x^n * u = u$. So by X is n-fold commutative and by Theorem (3.5), we have
$((x^n * y) * x) * x = (u * x) * x = (x^n * u) * u = u * u = 0$ n n . Likewise $((x^n * y) * x) \leq x$. Hence $\left((x^n * y) * x\right) = x$, then X is n-fold implicative.

4. $N-$ *fold of commutative, positive implicative and implicative ideals of AT − algebras*

In this section, we discuss the notions of n-fold commutative, positive implicative, implicative AT-ideals, and then we give some characterizations of these concepts.

DEF. 4.1. A non empty subset I of an AT−algebra $(X; *,0)$ is called a **commutative AT-ideal of X** , if $\forall x, y, z \in X$
(1) $0 \in I$,
(2) if $y * (z * x) \in I$ and $z \in I$, imply $((x * y) * y) * x) \in I$.

TH.4.2. An $AT-$ideal I of an AT-algebra $(X; *,0)$ is commutative if and only if, for all $x, y \in X$, $y * x \in I$ implies $((x * y) * y) * x) \in I$ …..(B).
PR. : (⇒) If an AT-idael I is commutative and $y * x \in I$, then $(y * (0 * x)) \in I$ and $0 \in I$ by the Definition of AT-ideal, we have $((x * y) * y) * x) \in I$.
Conversely, let an AT-ideal I satisfies (B), if $y * (z * x) \in I$ and $z \in I$,then by the definition of AT-ideals, we obtain $y * x \in I$ and It follows from (B) that
$((x * y) * y) * x) \in I$. This means that I is a commutative AT-ideal.

DEF. 4.3. A non empty subset I of an AT-algebra $(X; *,0)$ is called **an n-fold commutative AT-ideal of** , if $\forall x, y \in X$
(i) $0 \in I$
(ii) $y * (z * x) \in I$ and $z \in I$, imply $\left((x^n * y) * y\right) * x \in I$.

LM. 4.4. An AT −ideal I of an AT-algebra $(X; *,0)$ is an n-fold commutative AT-ideal if and only if, for all $x, y \in X$, $y * x \in I$ implies $\left((x^n * y) * y\right) * x \in I$.
PR.: the proof is similar to that Theorem (4.2).

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DEF. 4.5. Let $(X; *,0)$ be a$n\ AT-alge$bra, a non$empty\ su$bset I of X is sai$d\ to\ be\ a$ $\boldsymbol{pos}$**itive implic$\boldsymbol{ative}$ AT-ideal** if it satis$fies, f$or all $,y,z \in X$,
(1) $0 \in I$,
(2) $*(x * y) \in I\ and\ z * x \in I\ imply\ z * y \in I$.

DEF. 4.6. A no$n\ empty\ s$ubset I of an AT-al$gebr$a $(X; *,0)$ is sa$id\ to\ be$ $\boldsymbol{n-fold\ positive}$ **implicative AT-ideal of X** , if $\forall x,y,z \in X$
(i) $0 \in I$,
(ii) $x^{n+1} * (z * y) \in I\ and\ z \in I, imply\ \ x^{n} * y \in I$.

DEF. 4.7. A no$n\ emp$ty subset I of an AT-algebra $(X; *,0)$ is said to be **an $\boldsymbol{implic}$ative AT-ideal of X** , if $\forall x,y,z \in X$
(i) $0 \in I$,
(ii) $((x * y) * (z * x)) \in I\ and\ z \in I, imply\ \ x \in I$.

EX. 4.8. Let X = {0,a,b,c,d} in whic$h\ the\ o$peration $*is$ given by:

*	0	a	b	c	d
0	0	a	b	c	d
a	0	0	a	c	d
b	0	0	0	c	d
c	0	0	0	0	d
d	0	0	0	0	0

Then $(X; *, 0)$ is an AT-algebra. It is easy $to\ v$erify that I = {0,a,b,c} is an implicat$ive\ AT - ide$al of X .

DEF. 4.9. A non emp$ty\ subset\ I$ of an AT-algebra $(X; *,0)$ is called a **n-fold implicative AT-ideal of X** , if $\forall x,y,z \in X$
(i) $0 \in I$,
(ii) $((x^{n} * y) * z) * x \in I\ and\ z \in I, imply\ \ x \in I$.

LM. 4.10. A A$T - ide$al I of an AT-al$gebra$ $(X; *,0)$ is an n-f$old\ implicative$ AT-idea$l\ if$ and only if,for all $,y \in X$, $(x^{n} * y) * x \in I\ implies\ x \in I$.
PR.: Clear.

TH.4.11. Let $(X; *,0)$ be an AT-$algebra. If$ an AT- ideal I of X is n-fold $positive\ implicative$ AT-$ideal\ and$ n-fold commutative AT$-ideal\ of\ X$,$t$hen it is n-fold implicative AT-ideal of .
PR.: Su$ppos$e that I is $both\ n - fold$ positive implicative and n-fol$d\ commutative$ AT-ideals. Let $(x^{n} * y) * x \in I$,for all $x,y \in X$, since $x^{n} * \left((x^{n} * y) * y\right) = 0 \in I$ and I is n-fold $posi$tive implicative, then $(x^{n} (x^{n} * y)) * (x^{n} * y) \in I$, put $(x^{n} * y) = u$, $(x^{n} * u) * u \in I$, as I is n-fold commutative, applying Theorem (3.5), we obtain
$(u * x) * x \leq (x^{n} * u) * u \in I$, i.e., $(u * x) * x = x \in I$. Hence I is n-fold implicative AT-ideal.

5. $Sub - implicative\ AT - ideals$

$In\ this$ section, we $discuss\ the\ not$ions of $sub - commutative$, sub-p$ositive\ implicative$, sub-im$plicative$ ideals, and $we\ give\ som$e chara$cterizations\ of\ these\ concepts$.

DEF. 5.1. A no$n - em$pty subset I of an AT-algebra $(X; *,0)$ i$s\ cal$led **a sub-imp$\boldsymbol{licative}$ AT-ideal of X , if $\boldsymbol{x,y,z \in X}$,**
(1) $0 \in I$,
(2) $z * \left((x * y) * \left((y * x) * x\right)\right) \in I\ and\ z \in I, imply\ (x * y) * y \in I$.
EX. 5.2. Let X = {0,1,2,3,4} in which the operation $*is$ given by:

International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 6 Issue 10, October - 2022, Pages: 12-18

*	0	1	2	3	4
0	0	1	2	3	4
1	0	0	1	3	4
2	0	0	0	3	4
3	0	0	0	0	4
4	0	0	0	0	0

Then $(X;*,0)$ is an AT-algebra. It is easy to verify that $I = \{0,1,2,3\}$ is an sub $-$ implicative AT$-$ideal of X .

TH.5.3. Let I be an AT-ideal of an AT-algebra $(X;*,0)$. Then I is sub-implicative if and only if

$\left((x*y)*\left((y*x)*x\right)\right) \in I$ implies $(x*y)*y \in I$ …. (A).

PR.. Suppose that I is a sub-implicative AT-ideal of X. For any $x, y \in X$,

If $((x*y)*((y*x)*x)) \in I$, then $0*\left((x*y)*\left((y*x)*x\right)\right) \in I$ and $0 \in I$ by Definition (5.1). Hence (A) holds.

Conversely, suppose that an AT-ideal I satisfies (A), For $, y, z \in X$,

if $z*((x*y)*((y*x)*x)) \in I$ and $z \in I$, (by the definition of AT-ideals), we obtain

$((x*y)*((y*x)*x)) \in I$, It follows from (A) that $(x*y)*y \in I$ .This mean that I is a sub-implicative AT-ideal. This completes the PR..

PROP. 5.4. Any sub-implicative AT-ideal is an ideal, but the converse is not true.

PR.. Suppose that I is sub-implicative AT-ideal of X and let x = y in DEF. (5.1), we get $z*\left((x*x)*\left((x*x)*x\right)\right) = z*\left((0*(0*x))\right) = z*x \in I$ and $z \in I$ imply $\in I$. This means that I is an ideal. The last part is shown by the following exmpale.

EX. 5.5. Let $X = \{0,1,2,3,4\}$ in which the operation $*$ is given by:

*	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	1	4
2	0	1	0	3	4
3	0	0	2	0	4
4	0	1	0	3	0

Then $(X;*,0)$ is an AT-algebra. It is easy to verify that $I = \{0\}$ is an ideal, but not sub $-$ implicative ideal of . Since, $0*((4*2)*((2*4)*4)) \in I, 0 \in I$, but $(4*2)*2 = 2 \notin I$.

DEF. 5.6. Let $(X;*,0)$ be an AT-algebra, a nonempty subset I of X is said to be a **positive implicative AT-ideal** if it satisfies, for all $, y, z \in X$,

1) $0 \in I$,

2) $*(x*y) \in I$ and $z*x \in I$ imply $z*y \in I$.

LM. 5.7. Any positive implicative AT-ideal is an ideal, but the converse is not true.

PR.: clear.

International Journal of Engineering and Information Systems (IJEAIS)
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Vol. 6 Issue 10, October - 2022, Pages: 12-18

Example. 5.8. Let $X = \{0,1,2,3,4\}$ in which the operation $*$ is given by the table in Example(5.5) Then $(X; *, 0)$ is an AT-Algebra. $\{0,1,3\}$,$\{0,1,2,3\}$ are positive implicative AT-ideals of X . $\{0\}$,$\{0,2\}$ and $\{0,2,4\}$ are ideals of X , but not positive implicative AT-ideals.

TH.5.9. Let $(X; *, 0)$ be an AT-algebra, if I is a positive implicative AT-ideal of , the following are equivalent :

(a) I is a positive implicative ideal of X ,

(b) I is an ideal and for any $x, y \in X$, $y * (y * x) \in I$ implies $y * x \in I$.

(c) I is an ideal and for any $x, y, z \in X$, $z * (y * x) \in I$ implies $(z * y) * (z * x) \in I$.

(d) $0 \in I$ and $z * (y * (y * x)) \in I$, $z \in I$ implies $y * x \in I$.

PR.. (a) ⇒(b) If A is a positive implicative AT-ideal of X , by Lemma (5.7) is an ideal. Suppose $y * (y * x) \in I$, since $y * y = 0 \in I$,by DEF. (5.6), we have

$(y * x) \in I$, (b) hold.

(b) ⇒ (c) Assume (b) and $z * (y * x) \in I$. Since $z * \left(\left(z * \left((z * y) * x\right)\right)\right) = z * (((z * y) * (z * x)) \leq z * (y * x) \in I$, it follows that $z * (z * ((z * y) * x)) \in I$, by (b) we have $(z * y) * (z * x) = z * ((z * y) * x) \in I$ And so (c) hold.

(c) ⇒(d) It clear that $0 \in I$. If $* ((y * (y * x)) \in I$, $z \in I$, then

$(y * (y * (z * x)) \in I$ by (c), we get $z * (y * x) = 0 * (z * (y * x) = (y * y) * (z * (y * x) \in I$. Since I is an ideal and $z \in I$, then $(y * x) \in I$, and so (d) hold .

(d) ⇒(a) First observe that if I satisfied (d), then I is an ideal of X . In fact suppose

$(y * x) \in I$ and $y \in I$, then $(y * (0 * (0 * x)) \in I, y \in I$, using (d), we obtain

$x = 0 * x \in I$, i.e., I is an ideal. Next, let $z * (y * x) \in I$ and $* y \in I$. As

$(z * y) * \left(z * (z * x)\right) \leq y * (z * x) = z * (y * x) \in I$, it follows that

$(z * y) * (z * (z * x)) \in I$. Combining $(y * x) \in I$ and using (d), we have

$(z * x) \in I$. This have proved I is a positive implicative AT-ideal of .

PROP. 5.10. Any sub−implicative AT-ideal is a positive implicative AT-ideal, but the converse does not hold.

PR.. Assume that I is a sub-implicative AT-ideal of X. It follows from Proposition (5.4) that I is an ideal. In order to prove that I is a positive implicative AT-ideal from Theorem (5.9(b)) it suffices to show that if $y * (y * x) \in I$ then $y * x \in I$, by Theorem (5.3), for any $u, v \in X$, we have $((u * v) * ((v * u) * u)) \in I$ implies

$(u * v) * v \in I$. Substituting $x = u, y * x = v$,then

$$\left((u * v) * (v * u) * u\right) = (((x * (y * x)) * ((y * x) * x) * x$$
$$= \left((y * x) * x\right) * (y * x) = ((y * ((y * x) * x) * x)$$
$$= y * (y * x).$$

Hence if $y * (y * x) \in I$, then $(u * v) * v \in I$, i.e.,

$$(((x * (y * x)) * (y * x) = ((y * (x * x)) * (y * x) = 0 * (y * x) = (y * x) \in I .$$

Therefore I is a positive implicative AT-ideal of .

Example. 5.11. Let $X = \{0,1,2,3,4\}$ in which the operation $*$ is given by the table in Example(5.5) Then $(X; *, 0)$ is an AT-Algebra. $\{0,1,3\}$ is positive implicative AT-ideal of X, but it is not positive implicative AT-ideal. This finishes the PR..

DEF. 5.12. A non − empty subset I of an AT-algebra $(X; *, 0)$ is called **a sub − commutative AT-ideal of ,** if

1) $0 \in I$,

2) $* \left(\left((y * x) * x\right) * y\right) * y \in I$ and $z \in I$ imply $(y * x) * x \in I$.

EX. 5.13. Let X ={0, 1, 2, 3} in which the operation $*$ is given by:

*	0	1	2	3
0	0	1	2	3
1	0	0	1	3
2	0	0	0	3
3	0	1	2	0

Then $(X; *, 0)$ is an AT-algebra. It is easy to verify that {0} and {0,3} are all sub-commutative AT-ideals of X.

PROP. 5.14. An ideal I of an AT-algebra $(X; *, 0)$ is sub-commutative if and only if $((y * x) * x) * y) * y \in I$, we have $(y * x) * x \in I$…. (C).

PR.. Suppose that I is a sub-commutative AT-ideal of X. For any $,y \in X$.

If $((y * x) * x) * y) * y \in I$, then $0 * ((y * x) * x) * y) * y \in I$ and $0 \in I$ by Definition (5.12). Hence (C) holds.

Conversely, suppose that an ideal I satisfies (C). For $,y,z \in X$, if

$z * ((y * x) * x) * y) * y) \in I$ and $z \in I$, by the Definition of ideals, we obtain

$((y * x) * x) * y) * y) \in I$, It follows from (C) that $(y * x) * x \in I$ .This mean that I is a sub − commutative AT-ideal. This completes the proof.

PROP. 5.15. Any sub − commutative AT-ideal is an ideal, but the converse does not hold.

PR.. Suppose that I is sub-commutative AT-ideal of an AT-algebra and let $x = y$ in Definition (5.12), $z * \left(\left((x * x) * x\right) * x\right) * x = z * x \in I, z \in I$, imply $x \in I$. This means that I is ideal. The last part is shown by Example (5.5), {0,1,3} is ideal, but not sub − commutative AT-ideal. This finishes the proof.

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