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Intopological Digraph Space and Related Som Properties

Hussein A. Neamah[1] Khalid Sh. Al'Dzhabri[2]

[1]*Department of Mathematics, University of Al-Qadisiyah , College of Education,* Iraq , Al Diwaniyah
edu-math.post6@qu.edu.iq
[2]*Department of Mathematics, University of Al-Qadisiyah , College of Education,* Iraq , Al Diwaniyah
khalid.aljabrimath@qu.edu.iq

Abstract: *The intopological digraph space, a novel topological constraint imposed by a subbasis, is introduced in this work .a subbasis* $\overrightarrow{I}_{ẹ}^{v}$ *be a set contains one vertice such that the edge ẹ is indegree of it. As a result, various theorems have been established. A characterization and some examples are provided to describe of this new structure.*

Keywords— digraph, topology, Alexandroff topology, incident edges system.

1. INTRODUCTION:

For two reasons, graph theory is recognized as a fundamental idea in independent mathematics and is a helpful mathematical tool in many different contexts. Initially, graphs are selected theoretically from a theoretical standpoint. Graphs can perform topological space, collection objects, and many other mathematical groups even if they are merely simple relational combinations. The second justification is that using graphs to represent some ideas makes them more applicable in practical contexts. Regarding the relationship between graph theory and topology, one of the graph's tools, such as transforming a set of edges or a set of vertices to topological space and exploring other topological ideas of this space, can be used to express topological concepts. Topology is one of the most well-known and contemporary topics that has occupied a wide area of mathematicians. Several earlier studies on the subject of topological graphs are included below. Evans and Harary [1] first proposed the concept of topology on digraphs in 1967. Between the collection of all topologies with n vertices, they discovered only one relationship. Bhargava and Ahlborn [2] looked at the topological space connected to digraphs in 1968. They expanded the previous finding to encompass infinite graphs. In 1983, Majumdar [3] created graph topology from continuous multivalued functions that was connected between a dense subset of topology. A domination set of a graph and a dense subset of topology were linked by Subramanian in 2001[4]. A novel idea in topology on a signed graph and topology on transitive products of a signed graph was researched by Subbiah[5] in 2007. Karanakaram [6] established topology T g on a graph G from a collection of spanning subgraphs of G in the same year. Thomas [7] investigated topology in 2013 and determined the topological numbers of several graphs using set indexers. By using two fixed vertices and determining vertex and edge incidence depending on the distance between them, shokry [8] described a new technique for creating graph topology in 2015. When applying topology to a digraph in 2018, Abdu and Kilicman [9]"associated two topologies with the set of edges dubbed compatible and incompatible edges topologies."In furthermore, Khalid Al'Dzhabri [10] presented new topological space structures connected to digraphs in 2020 by combining new topologies with digraphs that were generated from particular open sets known as DG-topological space. A few more kinds of open sets linked to graphs were also introduced in 2020 by Khalid Al'Dzhabri[11].

2. PRELIMINARIES:

In this work , some basic notions of graph theory [12], and topology [13] are presented.

Definition 2.1: A digraph Ɗ is a triple consisting of a vertex set (Ṿ(Ɗ), Ẹ(Ɗ)) , an edge set , and a relation that associates with each edge two vertices (not necessarily distinct) called it`s end point and we express a graph to arranged pairs $Ɗ = (Ṿ, Ẹ)$ or $Ɗ = (Ṿ(Ɗ), Ẹ(Ɗ))$.

Definition 2.2: Let $Ɗ = (Ṿ, Ẹ)$ be a digraph, we call H is a sub digraph from $\mathcal{D}$ if $Ṿ(H) \subseteq Ṿ(Ɗ)$, $Ẹ(H) \subseteq Ẹ(Ɗ)$, in this case we would write $H \subseteq Ɗ$.

Definition 2.3 : Let $Ɗ = (Ṿ, Ẹ)$ be a digraph, we say that two vertices ṿ and w of a graph (resp., digraph Ɗ) are a djacent if there is an edge of the form ṿw (rsep., $\overrightarrow{wṿ}$ or $\overrightarrow{ṿw}$) joining them, and the vertices v and w are then incident with such an edge.

Definition 2.4 : If Y is non-empty set, a collection $\tau \subseteq P\ (Y)$ is called topology on Y if the following holds:

(1) , $\emptyset \in \tau$.

(2) The intersection of a finite number of elements in , $is\ in$ τ .

(3) The union of a finite or infinite number of elements of sets in $\tau\ belong\ to$. Then (,) is called a topological space. Any element in (Y, τ) is called open set and it is complement is called closed set .

Definition 2.5 : Let Y is a non-empty set and let τ is the collection of every subsets from Y. Then τ is named the discrete topology on the set Y. The topological space (, τ) is called a discrete space. If τ = { , $\emptyset$}. Then τ is named indiscrete topology and the topological space (, τ) is named an indiscrete topological space .

Definition 2.6 : Let $(Y\ , \tau)$ be a topological space, $A \subseteq Y$. The closure of A symbolized by $l\ (A)$ is defined as the smallest closed set that includes A.It is thus the intersection of every closed sets that include A .

Definition 2.6 :"Let (Y, τ) be a topological space, $A \subseteq Y$. The interior of A symbolized by $Int(A)$ is defined as the largest open set included in A. It is thus the union of every open sets included in A .

Definition 2.7 : Let (Y, τ) be a topological space, $A \subseteq Y$ is said dense if $\overline{A} = Y$.

3. INTOPOLOGICAL DIGRAPH SPACE.

In this section, we offer our novel subbasis family of a digraph Ɗ = (Ṿ, Ẹ) to build a topology on the set of vertices:

Definition 3.1 : Let Ɗ = (Ṿ, Ẹ) be a digraph we defined the $\overrightarrow{I_{e}}^{v}$ be a set contains one vertice such that the edge ẹ is indegree of it . Also defined $\overrightarrow{S_{D}}^{v}$ as follows $\overrightarrow{S_{D}}^{v} = V(D) \cup \{\overrightarrow{I_{e}}^{v} \mid e \in E\}$, Hence $\overrightarrow{S_{D}}^{v}$ forms a subbasis for a topology $\overrightarrow{\tau_{D}}^{v}$ on Ṿ called intopological digraph space $\overrightarrow{\tau_{D}}^{v}$ (briefly intop.digsp.) of Ɗ.

Example 3.2 : Let Ɗ = (Ṿ,Ẹ) be digraph in figure (1) such that $V=\{v_1, v_2, v_3, v_4\}$, $E = \{e_1, e_2, e_3, e_4, e_5\}$.

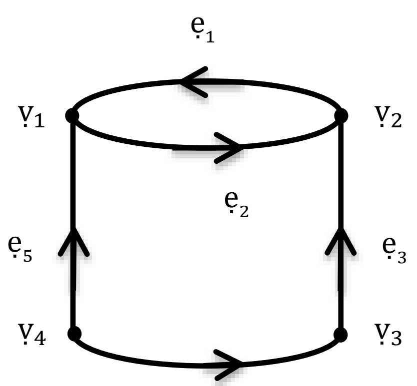


Figure (1)

We have
$\overrightarrow{I_{e_1}}^{v} = \{v_1\}$, $\overrightarrow{I_{e_2}}^{v}=\{v_2\}$, $\overrightarrow{I_{e_3}}^{v}=\{v_2\}$, $\overrightarrow{I_{e_4}}^{v}=\{v_3\}$, $\overrightarrow{I_{e_5}}^{v}=\{v_1\}$ and $\overrightarrow{S_{D}}^{v} = \{V(D), \{v_1\}, \{v_2\}, \{v_3\}\}$

By taking finitely intersection the basis obtained is :
$\{V(D), \emptyset, \{v_1\}, \{v_2\}, \{v_3\}\}$.

Then by taking all unions the intop.digsp. can be written as:
$\overrightarrow{\tau_{D}}^{v} = \{V(D), \emptyset, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_1, v_3\}, \{v_2, v_3\}, \{v_1, v_2, v_3\}\}$

Definition 3.3 : Let Ɗ = (Ṿ, Ẹ) be a digraph then $\overrightarrow{E_{v}}$ is the set of all edges that indgree to the vertice ṿ.

Example 3.4 : According to example 3.2 **,** we get
$\overrightarrow{E_{v_1}} = \{e_1, e_5\}$, $\overrightarrow{E_{v_2}} = \{e_2, e_3\}$, $\overrightarrow{E_{v_3}} = \{e_4\}$, $\overrightarrow{E_{v_4}} = \{\emptyset\}$.

Proposition 3.5 : Let $\overrightarrow{\tau_{D}}^{v}$ be intop.digsp. of the digraph Ɗ = (Ṿ,Ẹ) If $\overrightarrow{E_{v}} \neq \emptyset$, then $\{v\} \in \overrightarrow{\tau_{D}}^{v}$ for every $v \in V$.

Prove : Let D be digraph since $\overrightarrow{E_{v}} \neq \emptyset$ we get $\bigcap_{e \in \overrightarrow{E_{v}}} \overrightarrow{I_{e}}^{v} = \{v\}$ [because $\overrightarrow{I_{e}}^{v} = \{v\}, \forall e \in \overrightarrow{E_{v}}$]

Now by the definition of intop.digsp. $\overrightarrow{\tau_{D}}^{v}$. $\{v\}$ is element in the basis of intop.digsp. $\overrightarrow{\tau_{D}}^{v}$. Hence $\{v\} \in \overrightarrow{\tau_{D}}^{v}$.

Remark 3.6 : Let Ɗ = (Ṿ, Ẹ) discrete to be a digraph , then the intop.digsp. $\overrightarrow{\tau_{D}}^{v}$ is not necessary to be discrete topology in general .

Example 3.7 : Let C_6 be cyclic digraph such that edges are not all in the same direction, show in figure (2).

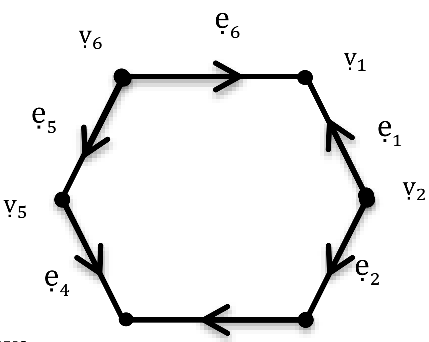


Figure (2) C_6 digraph

We have
$\overrightarrow{I_{e_1}}^{v} = \{v_1\}$, $\overrightarrow{I_{e_2}}^{v}=\{v_3\}$, $\overrightarrow{I_{e_3}}^{v}=\{v_4\}$, $\overrightarrow{I_{e_4}}^{v}=\{v_4\}$, $\overrightarrow{I_{e_5}}^{v}=\{v_5\}$, $\overrightarrow{I_{e_6}}^{v}=\{v_1\}$

And $\overrightarrow{S_{D}}^{v} = \{V(D), \{v_1\}, \{v_3\}, \{v_4\}, \{v_5\}\}$

$\overrightarrow{\tau_{D}}^{v} = \{ V(D), \emptyset, \{v_1\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_1, v_5\}, \{v_3, v_4\}, \{v_3, v_5\}, \{v_4, v_5\}, \{v_1, v_3, v_4\}, \{v_1, v_3, v_5\}, \{v_1, v_4, v_5\}, \{v_3, v_4, v_5\}, \{v_1, v_3, v_4, v_5\}$.

then we get the intop.digsp. $\overrightarrow{\tau_{D}}^{v}$ of C_6 is not discrete .

Corollary 3.8 : Let Ɗ = (Ṿ, Ẹ) be a digraph then .

(i) If $\overrightarrow{E_{v}} \neq \emptyset$ for all $v \in V$ then $\overrightarrow{\tau_{D}}^{v}$ is discrete topology.

(ii) If Ɗ = (Ṿ, Ẹ) is reflexive then $\overrightarrow{\tau_{D}}^{v}$ is discrete topology.

(iii) If Ɗ = (Ṿ, Ẹ) is equivalent then $\overrightarrow{\tau_{D}}^{v}$ is discrete topology.

(iv) If Ɗ = (Ṿ, Ẹ) is null digraph then $\overrightarrow{\tau_{D}}^{v}$ is indiscrete topology.

Prove : clear .

Definition 3.9 : In any digraph Ɗ = (Ṿ, Ẹ) since $(V, \overrightarrow{\tau_{D}}^{v})$ is Alexandroff space , for each $v \in V$, the intersection of all open set containing ṿ is the smallest open set containing ṿ and denoted by U_v ,

Also the family $\overrightarrow{M_{D}}^{v} = \{U_v \mid v \in V\}$ is the minimal basis for the intop.digsp. $(V, \overrightarrow{\tau_{D}}^{v})$.

Proposition 3.10 : In any digraph Ɗ = (Ṿ, Ẹ) , $U_v = \bigcap_{e \in \overrightarrow{E_{v}}} \overrightarrow{I_{e}}^{v}$ for every $v \in V$.

Prove : since $\overrightarrow{S_{D}}^{v}$ is the subbasis of $\overrightarrow{\tau_{D}}^{v}$ and U_v the intersection of all open set containing ṿ, we have $U_v = \bigcap_{e \in A} \overrightarrow{I_{e}}^{v}$ for some subset A of E, by definition of U_v then $v \in U_v$ and since $U_v = \bigcap_{e \in A} \overrightarrow{I_{e}}^{v}$ implies $v \in \bigcap_{e \in A} \overrightarrow{I_{e}}^{v}$ then $v \in \overrightarrow{I_{e}}^{v}$ for all $e \in A$, since $\overrightarrow{I_{e}}^{v}$ contain one vertex then $\overrightarrow{I_{e}}^{v} = \{v\}$ for all $e \in A$,this leads to $e \in \overrightarrow{E_{v}}$ for each $e \in A$. Hence $A \subseteq \overrightarrow{E_{v}}$ and so $v \in \bigcap_{e \in \overrightarrow{E_{v}}} \overrightarrow{I_{e}}^{v} \subseteq$

$\bigcap_{e\in A} \overrightarrow{I_e}^{v}$ and hence $v \in \bigcap_{e\in\overrightarrow{E_v}} \overrightarrow{I_e}^{v} \subseteq U_v$. from the definition of U_v the prove of complete.

Remark 3.11 : Let $D = (V, E)$ be a digraph, For any $v \in V$

(i) If $\overrightarrow{E_v} \neq \emptyset$ then by proposition 3.10 $U_v = \bigcap_{e\in\overrightarrow{E_v}} \overrightarrow{I_e}^{v} = \{v\}$

(ii) If $\overrightarrow{E_v} = \emptyset$ then by proposition 3.10 $U_v = \bigcap_{e\in\overrightarrow{E_v}} \overrightarrow{I_e}^{v} = V$.

Theorem 3.12 : For any $u, v \in V$ in a digraph $D = (V, E)$, we have $u \in U_v$ if and only if $\overrightarrow{E_v} = \emptyset$, i.e $U_v = \{u \in V |\ \overrightarrow{E_v} = \emptyset$.

Prove : $\Longrightarrow$ let $u \in U_v$ to prove $\overrightarrow{E_v} = \emptyset$, if $\overrightarrow{E_v} \neq \emptyset$ by remark 3.11 (i) implies $U_v = \{v\} \Rightarrow u \notin U_v$ is contradiction with hypothesis , then $\overrightarrow{E_v} = \emptyset$ and hence $\overrightarrow{E_v} = \emptyset\}$.

$\Longleftarrow$ if $\overrightarrow{E_v} = \emptyset$ and by remark 3.11 (ii) we get $U_v = V$ and hence $u \in U_v$.

Corollary 3.13 : For any $u, v \in V$ in a digraph $D = (V, E)$, we have $u \in U_v$ if and only if $\overrightarrow{E_v} \subseteq \overrightarrow{E_u}$.

Prove : By theorem 3.12 since this inequality $\overrightarrow{E_v} \subseteq \overrightarrow{E_u}$ is imposable and it is correct if $\overrightarrow{E_v} = \emptyset$.

4. PROPERTIES OF INTOPOLOGICAL DIGRAPH.

In this section, some properties of our new structure we investigated.

Proposition 4.1 : Let $\overrightarrow{\tau_D}^{v}$ be intop.digsp. of the a digraph $D = (V, E)$ then we have following :

(i). If $H = \{v \in V |\ \overrightarrow{E_v} \neq \emptyset\}$, then $H \in \overrightarrow{\tau_D}^{v}$.

(ii). If $K = \{v \in V |\ \overrightarrow{E_v} = \emptyset\}$, then K is closed in $\overrightarrow{\tau_D}^{v}$.

Prove : (i) Let $v \in H$ since $\overrightarrow{E_v} \neq \emptyset$ then by remark 3.11 (i) , $U_v = \{v\}$

As result $v \in U_v \subseteq H$ and so v is interior point of H, Hence $H \in \overrightarrow{\tau_D}^{v}$.

(ii). By assumption $K = \bigcup_{v\in k}\{v\}$ and so , $\overline{K} = \overline{\bigcup_{v\in k}\{v\}} = \bigcup_{v\in k}\overline{\{v\}}$ by proposition of closure . let $\in \overline{K}$, then $u \in \overline{\{v\}}$ for some $v \in K$. by corollary 3.13 , $\overrightarrow{E_u} \subseteq \overrightarrow{E_v}$ since $\overrightarrow{E_v} = \emptyset$ and $\overrightarrow{E_u} \subseteq \overrightarrow{E_v}$, then $\overrightarrow{E_u} = \emptyset$, and so $u \in K$ hence $\overline{K} \subseteq K$, and the prove complete.

Example 4.2: according to example 3.7 C_6 then we get $H = \{v_1, v_3, v_4, v_5\}$, $K = \{v_2, v_6\}$.

We note that $\{v_1, v_3, v_4, v_5\} \in \overrightarrow{\tau_D}^{v} \Longrightarrow H \in \overrightarrow{\tau_D}^{v}$ and $\{v_2, v_6\} \notin \overrightarrow{\tau_D}^{v} \Longrightarrow K$ *is* closed in $\overrightarrow{\tau_D}^{v}$ since $\{v_2, v_6\}^c = \{v_1, v_3, v_4, v_5\} \in \overrightarrow{\tau_D}^{v}$.

Proposition 4.3 : Let $D = (V, E)$ be a digraph , then $(V, \overrightarrow{\tau_D}^{v})$ is a compact intop.digsp. if and only if V is finite.

Prove : Let $(V, \overrightarrow{\tau_D}^{v})$ is a compact intop.digsp. suppose that V is infinite then $\overrightarrow{M_D}^{v} = \{U_v |\ u \in V\}$ is an open covering of $(V, \overrightarrow{\tau_D}^{v})$ which has no finite sub cover. therefore, $(V, \overrightarrow{\tau_D}^{v})$ is incongruous since it is not compact. for the converse, it follows directly that $(V, \overrightarrow{\tau_D}^{v})$ is compact because the number of open subsets on the finite space is finite.

Definition 4.4 : Let $D = (V, E)$ a digraph if the number of components of D increases by the remove of a vertex v and every indegree and outdegree edges to the v is said a cut vertex if $D - C$ has more than one component such that $C \subseteq V(D)$ and D is connected ,then C is called a vertex cut , if every proper subset of the vertex cut C of D is not a vertex cut , then C is called a minimal vertex cut .

Remark 4.5 : Let $D = (V, E)$ digraph (not necessary connected) and v is cut vertex we note that $\overrightarrow{E_v} \neq \emptyset$.

Prove : we will prove that by contradiction if $\overrightarrow{E_v} = \emptyset$ then the deletion of a vertex of $\overrightarrow{E_v} = \emptyset$ and the out edges of it , does not increase the number of component of a digraph D this contradiction (since v is cut vertex), thus $\overrightarrow{E_v} \neq \emptyset$, the following example is applied to show this remark.

Example 4.6 : Let $D = (V, E)$ be digraph in figure (3) such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{e_1, e_2, e_3, e_4, e_5\}$

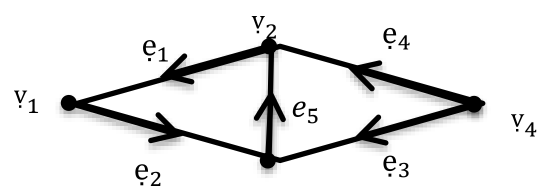


Figure (3)

We note that v_1, v_2, v_3 are cut vertices, due to that vertex's dilatation, we obtain three components. and v_4 is not cut vertex , since the dilation of v_4 is not increasing the component and so we note.

$$\overrightarrow{E_{v_1}} \neq \emptyset\ ,\ \overrightarrow{E_{v_2}} \neq \emptyset\ ,\ \overrightarrow{E_{v_3}} \neq \emptyset\ \ but\ \ \overrightarrow{E_{v_4}} = \emptyset$$

Lemma 4.7 : Let $\overrightarrow{\tau_D}^{v}$ be intop.digsp. in any a digraph $D = (V, E)$ [not necessary connected] and v is cut vertex then $\{v\} \in \overrightarrow{\tau_D}^{v}$.

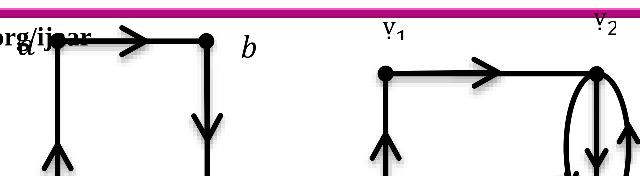
Prove: in any a digraph $D = (V, E)$ not necessary connected ,if v is cut vertex then by remark 4.5 $\overrightarrow{E_v} \neq \emptyset$. consequently by proposition 3.5 we get $\{v\} \in \overrightarrow{\tau_D}^{v}$.

Proposition 4.8 : Let C is minimal vertex cut in connected digraph $D = (V, E)$ then $C \in \overrightarrow{\tau_D}^{v}$.

Prove : since C is minimal vertex cut in D ,then $\overrightarrow{E_v} \neq \emptyset$ for all $v \in C$. by proposition 3.10,As a result $v \in U_v \subseteq C$ and so v is interior point of C , Hence $C \in \overrightarrow{\tau_D}^{v}$.

Definition 4.9 : Two digraph $D_1 = (V_1, E_1)$ and $D_2 = (V_2, E_2)$ are said to be isomorphic to each other, and written $D_1 \cong D_2$ if there is a bijection $\mathcal{F} : V_1 \to V_2$ with $\{x, y\} \in E_1$ if and only if $\{\mathcal{F}(x), \mathcal{F}(y)\} \in E_2$ for all $x, y \in V_1$ the function $\mathcal{F}$ is called an isomorphism .

Example 4.10 : Let $D_1 = (V_1, E_1)$,$D_2 = (V_2, E_2)$ are be digraph in figure (4) such that $V_1 = \{a, b, c, d\}$,$V_2 = \{v_1, v_2, v_3, v_4\}$.



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Thin the digraph $Ɗ_1$, $Ɗ_2$ are isomorphic. since $a \to b \to c \to d$ and $Ṿ_4 \to Ṿ_1 \to Ṿ_2 \to Ṿ_3$ and put $\mathcal{F} : Ṿ_1 \to Ṿ_2$ such that $\mathcal{F}(a) = Ṿ_4$, $\mathcal{F}(b) = Ṿ_1$, $\mathcal{F}(c) = Ṿ_2$, $\mathcal{F}(d) = Ṿ_3$

Remark 4.11 : It is clear that the intop.digsp. $(Ṿ_1, \overrightarrow{\tau_{Ɗ_1}}^{v})$ and $(Ṿ_2, \overrightarrow{\tau_{Ɗ_2}}^{v})$ are homeomorphic , if the digraphs $Ɗ_1 = (Ṿ_1, Ẹ_1)$ and $Ɗ_2 = (Ṿ_2, Ẹ_2)$ are isomorphic but in general the opposite is not true, the following example is applied to show the opposite is not true.

Example 4.12: Le $Ɗ_1 = (Ṿ_1, Ẹ_1)$, $Ɗ_2 = (Ṿ_2, Ẹ_2)$ are be to digraph in figure (5) such that $Ṿ_1 = \{Ṿ_1, Ṿ_2, Ṿ_3, Ṿ_4\}$ and $Ṿ_2 = \{u_1, u_2, u_3, u_4\}$.

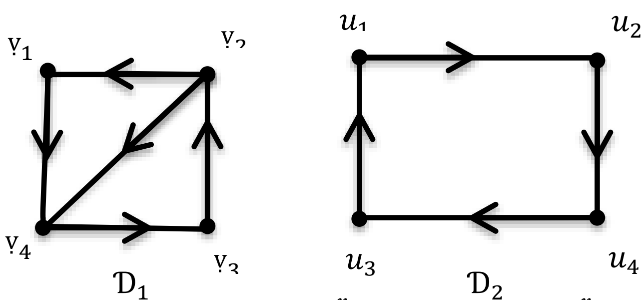

Figure (5)

The intop.digsp. ($Ṿ_1, \overrightarrow{\tau_{Ɗ_1}}^{v}$) and $(Ṿ_2, \overrightarrow{\tau_{Ɗ_2}}^{v})$ are homeomorphic (since both are discrete) . But they are not isomorphic digraph .

5.STIPULATIONS ON TOPOLOGICAL SPACE TO BE INTOPOLOGICAL DIGRAPH SPACE.

This section illustrates the prerequisite for topology space to be a intopological digraph space.

Definition 5.1: Any topological space(Ạ,$\mathcal{T}$) is called intop.digsp. if $\mathcal{T} = \overrightarrow{\tau_{Ɗ}}^{v}$ for some digraph Ɗ with vertex set Ạ .

Remark 5.2 :

(i) if is discrete topology on Ạ, then by corollary 3.8 , $\mathcal{T} = \overrightarrow{\tau_{Ɗ}}^{v}$ for some digraph Ɗ with vertex set Ạ , such that $\overrightarrow{Ẹ_{Ṿ}} \neq \emptyset$ for all Ṿ∈Ṿ . hence $\mathcal{T}$ is an intop.digsp. .

(ii) If is not a discrete topology on Ạ , by definition intop.digsp. $\overrightarrow{\tau_{Ɗ}}^{v}$, all open set $U_i \in \overrightarrow{\tau_{Ɗ}}^{v}$ for some i that contain one element are the indegree edges of the digraph Ɗ and form a subbasis for the intop.digsp. $\overrightarrow{\tau_{Ɗ}}^{v}$ .there for, if there exist open sets $U_i \in \overrightarrow{\tau_{Ɗ}}^{v}$ for some i that include one component, making these open sets the indgree edges of a digraph Ɗ and form a subbasis for $\mathcal{T}$,then $\mathcal{T}$ is an intop.digsp. $\overrightarrow{\tau_{Ɗ}}^{v}$ on Ạ .

Example 5.3 : Let $Ạ = \{Ṿ_1, Ṿ_2, Ṿ_3, Ṿ_4\}$, such that

$\mathcal{T}_1 = \{\emptyset,$
$Ạ, \{Ṿ_1\}, \{Ṿ_2\}, \{Ṿ_3\}, \{Ṿ_1, Ṿ_2\}, \{Ṿ_1, Ṿ_3\}, \{Ṿ_2, Ṿ_3\}, \{Ṿ_1, Ṿ_2, Ṿ_3\}\}$
and
$\mathcal{T}_2 = \{\emptyset,$
$Ạ, \{Ṿ_1\}, \{Ṿ_2\}, \{Ṿ_1, Ṿ_2\}, \{Ṿ_3, Ṿ_4\}, \{Ṿ_1, Ṿ_2, Ṿ_4\}, \{Ṿ_2, Ṿ_3, Ṿ_4\}\}$.

According to this example $\mathcal{T}_1$ is an intopological since $\{Ṿ_1\}, \{Ṿ_2\}$ and $\{Ṿ_3\}$ are indegree edges of a digraph as in figure (6). And these indegree edges form asubbasis for $\mathcal{T}_1$ [because $\{Ṿ_1\}, \{Ṿ_2\}, \{Ṿ_3\}$ form subbasis] for $\mathcal{T}_1$.

But $\mathcal{T}_2$ is not an intopology because$\{Ṿ_2\}$ are not a subbasis for $\mathcal{T}_2$.

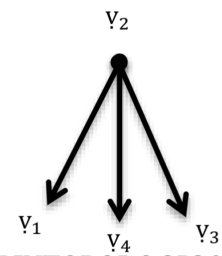

Figure (6)

6. DENSITY IN INTOPOLOGICAL DIGRAPH SPACE.

This section, examines several prerequisites for digraph-related dense subsets of the intopological digraph space. The only dense subset in (Ṿ, $\overrightarrow{\tau_{Ɗ}}^{v}$) of every digraph Ɗ = (Ṿ,Ẹ) , such that $\overrightarrow{Ẹ_{Ṿ}} \neq \emptyset$ for all Ṿ ∈ Ṿ is Ṿ since , $\overrightarrow{\tau_{Ɗ}}^{v}$ is a discrete topology .

Remark 6.1: It is known that in in (Ṿ, $\overrightarrow{\tau_{Ɗ}}^{v}$) the subset K ⊆ Ṿ is dense in Ṿ if and only if the complement of K has empty interior .

Proposition 6.2 : Let Ɗ = (Ṿ,Ẹ) be a digraph with at least one vertices Ṿ ∈ Ṿ such that $\overrightarrow{Ẹ_{Ṿ}} = \emptyset$ and n(Ẹ) ≥ 1, the set K = { Ṿ ∈ Ṿ| $\overrightarrow{Ẹ_{Ṿ}} \neq \emptyset$ } is dance in (Ṿ, $\overrightarrow{\tau_{Ɗ}}^{v}$)

Prove : by remark 6.1 , it is enough to prove that the complement of K has empty interior . for every Ṿ ∈ K^c , Ṿ is a vertices such that $\overrightarrow{Ẹ_{Ṿ}} = \emptyset$. then for {Ṿ}∉ $\overrightarrow{\tau_{Ɗ}}^{v}$, for every Ṿ ∈ K^c .

As a resulted , B ⊆ K^c , B can not be written as a union of finitely intersection of elements of $\overrightarrow{S_{D}}^{v}$. i.e B ∉ $\overrightarrow{\tau_{Ɗ}}^{v}$. hence int(K^c) = ∅ . and this means K is dense subset in (Ṿ, $\overrightarrow{\tau_{Ɗ}}^{v}$) .

Corollary 6.3: Let Ɗ = (Ṿ,Ẹ) be a digraph such that intop.digsp. $\overrightarrow{\tau_{Ɗ}}^{v}$ is not a discrete topology and n (Ẹ) ≥ 1 . then a subset B of Ṿ is dense in (Ṿ, $\overrightarrow{\tau_{Ɗ}}^{v}$) if and only if K ⊆ B such that K = { Ṿ ∈ Ṿ| $\overrightarrow{Ẹ_{Ṿ}} \neq \emptyset$ } .

Prove : ⟹ if B is dense in (Ṿ, $\overrightarrow{\tau_{Ɗ}}^{v}$) ,then by remark 6.1 , B^c has empty interior. By propostion 3.5, {Ṿ} ∈ $\overrightarrow{\tau_{Ɗ}}^{v}$ for every Ṿ∈ K and so K ∈ $\overrightarrow{\tau_{Ɗ}}^{v}$.

Hence K ⊆ B because B^c has empty interior .

⟸ *by* proposition 5.1 , $\overline{K}$ = Ṿ from a assumption , K ⊆ B . hence $\overline{B}$ = Ṿ and so B is dense in (Ṿ, $\overrightarrow{\tau_{Ɗ}}^{v}$) .

Remark 6.4: : Let Ɗ = (Ṿ,Ẹ) be a digraph such that intop.digsp. $\overrightarrow{\tau_{Ɗ}}^{v}$ is not a discrete topology and n (Ẹ) ≥ 1 . by corollary 6.2 , $\overrightarrow{\mathrm{I}_{ẹ}}^{v}$ in $\overrightarrow{\mathrm{S}_{Ɗ}}^{v}$ is dense such that of for every ẹ ∈ Ẹ if and only if K ⊆ $\overrightarrow{\mathrm{I}_{ẹ}}^{v}$, such that K = { ṿ ∈ Ṿ| $\overrightarrow{\mathrm{Ẹ}_{ṿ}} \neq \emptyset$ }.

The next proposition gives the topological property for topological space to be intop.digsp. $\overrightarrow{\tau_{Ɗ}}^{v}$.

proposition 6.5: let $\overrightarrow{\tau_{Ɗ}}^{v}$ be the Intopological of the a digraph Ɗ = (Ṿ,Ẹ) the topological space (Ṿ*,$\mathcal{T}$) is an intopology if it is homeomorphic to (Ṿ, $\overrightarrow{\tau_{Ɗ}}^{v}$).

Prove: suppose that $\mathcal{F}$: (Ṿ, $\overrightarrow{\tau_{Ɗ}}^{v}$) → (Ṿ*,$\mathcal{T}$) is a homeomorphism . since (Ṿ, $\overrightarrow{\tau_{Ɗ}}^{v}$) is an Alexandroff space and (Ṿ, $\overrightarrow{\tau_{Ɗ}}^{v}$) ≅ (Ṿ*,$\mathcal{T}$) , (Ṿ*,$\mathcal{T}$) is an Alexandroff space. To construct on Ṿ* adigraph Ɗ* = (Ṿ*, Ẹ*) we put { $\mathcal{F}(u), \mathcal{F}$(ṿ)} is indegree edges in Ẹ* if anf only if {u,ṿ} is indegree in Ẹ for every u,ṿ ∈ Ṿ. then we have $\mathcal{F}$({u,ṿ}) = { $\mathcal{F}(u), \mathcal{F}$(ṿ)} and so $\mathcal{T}$ = $\overrightarrow{\tau_{Ɗ}}^{v}$.

As resulted , $U^*_u = M_u$ such that U^*_u , M_u are the smallest open set containing u in (Ṿ*, $\overrightarrow{\tau_{Ɗ^*}}^{v}$) and (Ṿ*,$\mathcal{T}$) respectively. Since $\mathcal{F}$ is ahomeomorphism , $\mathcal{F}(U^*_u) = M_{\mathcal{F}(ṿ)}$ such that U^*_u is the smallest open set containing u in (Ṿ*, $\overrightarrow{\tau_{Ɗ^*}}^{v}$) . also $\mathcal{F}$ is an isomorphism between Ɗ and Ɗ*, then $\mathcal{F}(U_u) = U^*_{\mathcal{F}(u)}$.

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