

Photoelastic Properties of Lithium Tantalate Crystals

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Abstract: Studying the acoustic, electro-optical, and photoelastic properties of crystals used as working elements of lasers and acousto-optic devices is of great interest. In this work, the effective photoelastic constants and the anisotropy of the acousto-optic quality factor in lithium tantalate crystals are determined by the Dixon-Cohen method. The maximum values of the velocity of quasi-longitudinal (6685 m/s) and quasi-transverse (4297 m/s) acoustic waves and the maximum value of the acousto-optic quality factor M_2 ($8 \cdot 10^{-15} \text{ s}^3/\text{kg}$) have been determined in these crystals.

Keywords: Acoustic wave, polarization, acousto-optic quality factor, Bragg light diffraction, lithium tantalate, acoustic wave velocity.

I. INTRODUCTION

In addition to a high acoustic quality factor, ferroelectric lithium tantalate crystals exhibit a relatively high refractive index and photoelastic constant with a high light diffraction efficiency by acoustic waves [1, 4]. Results of measurements of lithium tantalate crystal photoelastic constants are presented in this work. By measuring the intensity of light diffracted by hypersonic waves, photoelastic constants were determined. The diffracted light intensity of the studied crystal and the reference crystal was compared according to Dixon and Cohen [3].

Acoustic-optic methods based on light diffraction by acoustic waves are universal methods for studying materials' elastic and photoelastic properties. In Bragg diffraction, the relative intensity of diffracted light is determined by the following equation at small deformations: [6,13]

$$\frac{I_1}{I_0} = \left(\frac{1}{4} \cdot \frac{\Delta \varepsilon}{\varepsilon_0} \cdot \frac{k \cdot L}{\cos \theta} \right)^2 \quad (1)$$

where L is the width of the beam of acoustic waves in the diffraction plane, k is the wave number of the diffracted light, θ is the Bragg diffraction angle, ε_0 is the unperturbed value of the permittivity.

The expression for the maximum value of the intensity of light diffracted to the first order with respect to the incident light wave can be written as [8,13]:

$$\frac{I}{I_0} = \frac{\pi^2}{2} \cdot \frac{n^6 \cdot p_{eff}^2}{\rho \cdot V^3} \cdot \frac{P_a \cdot L^2}{\lambda_0^2 \cdot \cos^2 \theta} \quad (2)$$

where P_a is the applied acoustic power, p_{eff} is the effective photoelastic constant, n is the refractive indices, ρ and V are the density and phase velocity in the crystal, respectively.

Using the usual formulas for transforming the fourth-rank tensor components, one can obtain the effective

photoelastic constant p_{eff} in the formulas for the diffraction intensity. The effective photoelastic constants can be expressed as (3) and (4) by simple transformations: [8,13]:

$$p_{eff}^2 = (\chi_1 P_2 - \chi_2 P_1)^2 + (\chi_1 P_3 - \chi_3 P_1)^2 + (\chi_2 P_3 - \chi_3 P_2)^2 \quad (3)$$

where χ_i are the direction cosines of the wave vector of diffracted light, P_i are the values of the light polarization components, which are calculated by the formula:

$$P_i = P_{iklm} \beta_k \gamma_l \kappa_m \quad (4)$$

where β_k are the direction cosines of the incident light polarization, γ_l, κ_m are the direction cosines of the polarization and wave vector of the acoustic wave [13].

We can determine the effective value of the photoelastic constants by knowing the propagation and polarization directions of the acoustic wave, the polarization of the incident light, and the direction of the diffracted light. Lithium tantalate crystals are widely used in acousto-optic devices due to their anisotropic photoelastic properties. For transverse acoustic waves propagating in the crystallographic plane of (100) at arbitrary angles to the Z axis, anisotropy of the effective photoelastic constant p_{eff} was calculated based on experimental values of the photoelastic tensor components of these LiTaO₃ crystals.

LiTaO₃ crystals, widely used in acousto-optics [6–8], are studied for their acoustic and acousto-optical properties. Unlike the studies in the literature, we investigated the dependence of the acousto-optic quality factor M_2 on the direction of propagation of longitudinal acoustic waves. It was introduced by Dixon as a characteristic that determines the intensity of diffracted light in a material regardless of the size of the piezoelectric transducer and the power of the acoustic wave [8, 13]:

$$M_2 = \frac{n_1^3 n_2^3 p_{eff}^2}{\rho V^3}, \quad (5)$$

where n_1 and n_2 are the refractive indices of the incident and diffracted light, respectively, ρ is the density of the crystal, V is the velocity of the acoustic wave [13].

II. SAMPLES AND RESEARCH METHODS

The studied LiTaO₃ samples were oriented along the [001] and [100] crystallographic axes. In the experiments, X-cut quartz piezoelectric transducers were used to excite acoustic waves with frequencies of 400–1200 MHz to conduct Bragg light diffraction measurements at room temperature [9, 13]. Light was provided by a helium-neon laser ($\lambda_0=632.8$ nm). Using a polarization analyzer, the direction of polarization of the light beam incident on the sample was determined relative to the wave vector and polarization of the acoustic wave.

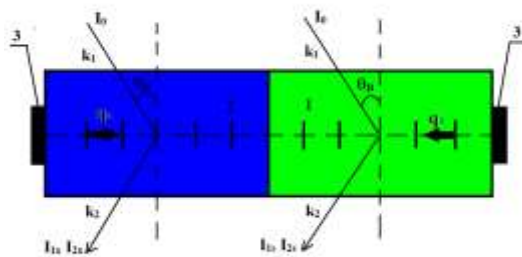


Figure 1. Scheme for determining photoelastic constants by the Dixon method.

Where 1-standard sample, 2- studied sample, 3 – piezoelectric transducer, q – acoustic wave vector, k_1 and k_2 – incident and diffracted light wave vectors, respectively, I_0 incident light intensity, θ_B – Bragg diffraction outer angle.

I_{1x} and I_{1e} are the intensities of light diffracted in the sample and reference, respectively, during sound propagation from the sample to the reference;

I_{2x} and I_{2e} are the same light intensities during sound propagation from the standard to the sample.

The acoustic wave velocities along the [100] and [001] directions were determined from the Bragg light diffraction angle on these waves with an accuracy of approximately 0.2% [9].

III. RESULTS AND DISCUSSION

The effective photoelastic constants p_{eff} in lithium tantalate crystals were determined on the base of measured values of the intensities of diffracted light for different directions and polarizations of light and acoustic waves and relation (3). Then the acousto-optic quality factors M_2 have been calculated by relation (4) and the experimental values of the velocity of acoustic waves and the data on density and refractive indices from [10].

To determine the anisotropy of the acousto-optic quality factor in LiTaO₃ crystals, it is first necessary to find the components of the polarization vector of longitudinal acoustic waves, which in the general case, deviates from the wave

vector. To solve this problem, we use the wave equation without taking into account the energy dissipation in an acoustic wave [5]:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = c_{ijkl} \frac{\partial^2 u_l}{\partial x_j \partial x_k} + e_{kij} \frac{\partial^2 \Phi}{\partial x_j \partial x_k} \quad (6)$$

where u_i are the components of the particle displacement vector, u_{kl} are the components of the strain tensor, x_j are the coordinates, c_{ijkl} are the components of the tensor of elastic constants, $E_k = -\frac{\Phi}{\partial x_k}$ quasi-static electric field. Equation

(6) is usually solved in the approximation of a plane harmonic wave written in the form of components: [8]

$$u_i = u_{0i} \exp[i(\omega t - q_k x_k)] \quad (7)$$

where u_0 is the wave amplitude, q_k are the wave vector components, ω is the circular frequency of the acoustic wave.

As a result, a system of algebraic Green–Christoffel equations is obtained [5,7].

$$[\Gamma_{ik} - \delta_{ik} \rho V^2] \gamma_k = 0 \quad (8)$$

where Γ_{ik} are the components of the Green-Christoffel tensor, δ_{ik} are the components of the Kronecker tensor, V is velocity of the acoustic wave, γ_k are the components of the polarization vector

The components of the Green-Christoffel tensor represent the convolution of the material tensor of elastic constants along the direction cosines of the unit wave normal κ [14].

$$\Gamma_{ik} = c_{ijkl} \kappa_j \kappa_l \quad (9)$$

Wave propagation in the YZ plane of lithium tantalate.

Lithium tantalate is a piezoelectric crystal of dot group 3m. Taking into account the piezoelectric tensor and the expression for the direction of propagation, which lies in the symmetry plane YOZ ($\kappa_1=0$, $\kappa_2=\cos\varphi$, $\kappa_3=\sin\varphi$), we obtain [4]:

$$\begin{aligned} \gamma_1 &= 0, \\ \gamma_2 &= e_{15} \cos^2 \varphi + (e_{15} + e_{31}) \sin \varphi \cos \varphi \\ \gamma_3 &= e_{15} \cos^2 \varphi + e_{33} \sin^2 \varphi \\ \varepsilon &= \varepsilon_{11} \cos^2 \varphi + \varepsilon_{33} \sin^2 \varphi \end{aligned} \quad (10)$$

Therefore, only the following components of the Christoffel tensor are not equal to zero:

$$\begin{aligned} \Gamma_{11} &= c_{66} \cos^2 \varphi + c_{44} \sin^2 \varphi + c_{14} \sin 2\varphi \\ \Gamma_{22} &= c_{11} \cos^2 \varphi + c_{44} \sin^2 \varphi - c_{14} \sin 2\varphi \\ \Gamma_{33} &= c_{44} \cos^2 \varphi + c_{33} \sin^2 \varphi \\ \Gamma_{23} &= \Gamma_{32} = (c_{13} + c_{44}) \cos \varphi \sin \varphi - c_{14} \cos^2 \varphi \end{aligned} \quad (11)$$

By substituting the values of the components of the Green-Christoffel tensor Γ_{ik} into equations (8) from relations (11), we determined the dependence of the phase velocity of longitudinal acoustic waves on the direction of the wave vector in the (100) plane of the crystal, LiTaO₃.

The following values of density and elastic constants were used in the calculation: $\rho = 7.46 \cdot 10^3 \text{ kg/m}^3$, $c_{11} = 23.8 \cdot 10^{10} \text{ N/m}^2$, $c_{12} = 4.1 \cdot 10^{10} \text{ N/m}^2$, $c_{13} = 8 \cdot 10^{10} \text{ N/m}^2$, $c_{14} = -2.2 \cdot 10^{10} \text{ N/m}^2$, $c_{33} = 28 \cdot 10^{10} \text{ N/m}^2$, $c_{44} = 11.3 \cdot 10^{10} \text{ N/m}^2$, $c_{66} = 10 \cdot 10^{10} \text{ N/m}^2$.

Additional quantities required for the calculation were taken from [5].

Thus, there is a transverse wave polarized perpendicular to the YZ plane, whose velocity $V_3 = \sqrt{\Gamma_{11} / \rho}$ is not affected by the piezoelectric effect. On the contrary, this effect is significant for quasi-longitudinal and quasi-transverse waves polarized in the YZ plane, whose velocities, respectively V_1 and V_2 , are determined by the relation

$$2\rho V_{1,2}^2 = \Gamma_{22} + \Gamma_{33} \pm \sqrt{(\Gamma_{22} - \Gamma_{33})^2 + 4(\Gamma_{23})^2} \quad (10)$$

How significant the influence of the piezoelectric effect in this crystal is seen from Figure.2

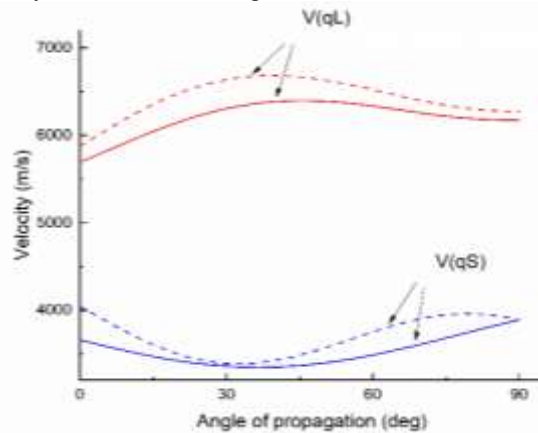


Figure. 2. Velocities of the bulk waves in the YZ plane of LiTaO₃ crystal versus angle of propagation with respect to the Y axis. --- with piezoelectric effect, — without piezoelectric effect.

Figure 3 shows the anisotropy of the effective photoelastic constant of the LiTaO₃ crystal. It can be seen from this figure that when quasi-longitudinal acoustic waves are propagated in a crystal, the values of the effective photoelastic constant of the crystal are much higher than when quasi-transverse acoustic waves are propagated. The piezoelectric effect was taken into account when calculating these values.

As follows from formula (4), the effective photoelastic constant, and hence the components of the photoelastic tensor P_{ijkl} , can be determined from measurements of the relative intensity of light scattering.

To measure the photoelastic constants of crystals, we used the well-known Dixon-Cohen method [11], which consists in comparing the intensity of light diffracted by an

acoustic wave in the sample under study and in the reference sample, the photoelastic constants of which are known. Note that Bragg light scattering is a dynamic method for measuring photoelastic constants and, therefore, allows one to determine only the absolute value of these constants.

However, if measurements are taken for different directions of propagation of elastic waves and for different polarizations of light, then in some cases it is also possible to determine the relative sign of the photoelastic constants.

To estimate the value of p_{eff} in relation (4), in this work, based on the phenomenological theory of the photoelastic effect, we calculated the anisotropy of the effective photoelastic constant p_{eff} in uniaxial crystals for the general case of propagation of transverse acoustic waves at an arbitrary angle to the Z axis in the crystallographic plane YOZ (plane (100)).

The calculation was carried out using a specially developed program in the object-oriented JAVA language. The initial data from Table 1 were used for the calculation. The calculation results for lithium tantalate crystals are shown in Figs. 3. The data obtained were used in setting up acousto-optical experiments with lithium tantalate crystals.

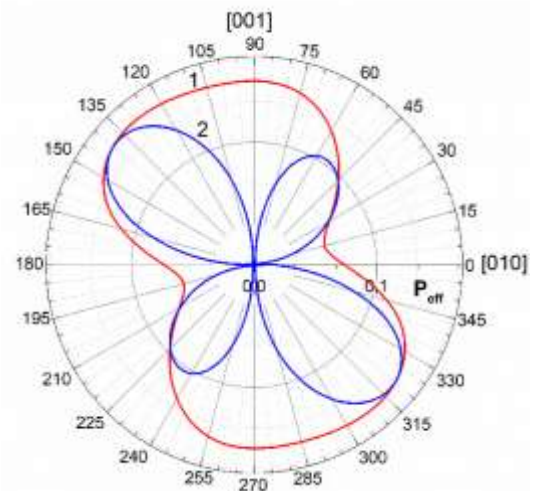


Figure. 3. Anisotropy of the effective photoelastic constant of the LiTaO₃ crystals in symmetry plane (100),

In the calculations, it became clear that the piezoelectric effect in piezoelectric crystals has a significant effect on the speed of the acoustic wave propagating in the crystals.

Using the effective photoelastic constants, expression (5) was used to calculate the relationship between the acousto-optic quality factor and the direction of the wave vector of longitudinal acoustic waves in the (100) plane. According to calculations, the maximum value of the quality factor of the crystal for longitudinal acoustic waves was $M_2 = 1.63 \cdot 10^{-15} \text{ s}^3 \cdot \text{kg}^{-1}$, while for transverse acoustic waves $M_2 = 8 \cdot 10^{-15} \text{ s}^3 \cdot \text{kg}^{-1}$.

CONCLUSION.

A study found that the square of the effective photoelastic constant in lithium tantalate crystals diffraction

of light by longitudinal waves along the [100] axis (without rotation of the polarization plane) is ~ 0.15 , which is a relatively large value. The acousto-optic M_2 -factor for isotropic diffraction ($2.6 \cdot 10^{-15} \text{ s}^3/\text{kg}$) is also quite high for transverse waves along the [001] axis. It is possible to apply the results obtained on the dependence of the acousto-optic properties of LiTaO_3 crystals on the direction of the light wave vector to acousto-optic devices to measure the frequency of acoustic waves based on the phenomenon of acoustic activity. Specifically, it is possible to choose the most optimal geometry for the Bragg diffraction of light by transverse acoustic waves in lithium tantalate crystals. An acoustic wave propagating in a LiTaO_3 crystal at a sufficiently high speed ($V=4300 \text{ m/s}$) indicates the crystal's importance.

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