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A Proposal of The Abel-Liouville Relationship With the Wronskian For The Treatment of The Barbero-Immirzi Parameter In The Tetrahedral Space-Time

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Abstract: *The paper presents an approach using the Abel-Liouville formula with the wronskian of the area and volume, particularly a tetrahedral geometric structure. This proposal aims to identify a possible connection of these operators with the Barbero-Immirzi parameter. The work is far from the truth for the parameter, but it presents a different and possible view in each regime. It is presented briefly, some approaches referenced in different fields of the parameter and comments of the values estimated by the researchers. At the end, we considered the reasonableness of the methodology and questioning the possible inconsistencies of the BI parameter. We also discuss whether the parameter is fixed or will always have variation due to different regimes, until it is shown otherwise via the experiment.*

Keywords— *Abel-Liouville, BI parameter, Wronskian*

Introduction

Loop Quantum Gravity (LQG) theoretically makes a consistent treatment of the parameter for a particular quantum correction that is the Barbero-Immirzi (BI) parameter. There may be divergences if detected experimentally. The objective of this work is to describe aspects in certain regions or regimes on the parameter in a comparative way with the proposal to be analyzed using the Abel-Liouville formula with the wronskian of the area and volume in a tetrahedral region of space-time.

We will briefly present the studies with their indications and field for the approximations of the Barbero-Immirzi parameter, the proposal of the relationship of the Abel-Liouville formula with the wronskian of the area and volume in a tetrahedral region of space-time and at the end, we will discuss through the graph the possible effects and trends of the estimated parameter.

Studies Involving the Barbero-Immirzi

Studies consider the parameter with value $\gamma \approx 0{,}274$ to have the expected inclination of 1/4 in the relationship between entropy and the area of the event horizon of a black hole. This treatment is given by counting the number of configurations of the spin network by generating a certain area of the isolated horizon[1].

In the treatment of the quantum black hole, in particular Kerr's, in the extreme case, there is no gravity on the surface and there is no Hawking radiation. We have the radius of horizon r_H, angular momentum J, mass M, with $a = J/M$ and $a^2 = r_H^2 = J$, with extreme boundary of the horizon area reduced to

$$A = 4\pi({r_H}^2 + a^2) = 8\pi J \tag{1}$$

Being $J = 1$, $j = 1/2$ (spin) and $\gamma = \sqrt{3}/6$, with the ratio of the quantized area

$$A = 8\pi\gamma {\ell_p}^2 \sum_{i=1}^{4} \sqrt{j(j+1)} \tag{2}$$

This shows the autovalues for the LQG area operator by setting the BI parameter, where it considers horizon drilling on four 1/2 spin links or two perforations per line. In this type of work, it observes estimates for the BI parameter of $0{,}263 < \gamma < 0{,}288$ [2,3,4].

It is provisional, because there are several open questions, by verifying the extreme rotation of the black hole is LQG's Hamiltonian solution and the radiation condition. They are effective approximations, but still require more fundamental treatment.

In Rovelli's [5]works, he finds it more interesting to calculate the quantum spectra of certain geometric quantities. Immirzi considered a certain transformation introduced by Barbero and observed that if we quantize the theory through staggered elementary variables, we obtain different spectra for the same geometric amount.

According to Rovelli, they worked suggesting the fixation of the BI parameter using the entropy of black holes[6]. It was shown that in [7], incorrectly, the derivation of the Entropy of Bekenstein-Hawking with the constant of the LQG, reaching an approximate value for $\gamma \approx 0{,}079577$.

Rovelli believes that indetermination is given by a single parameter and that in general there are two length scales in quantum gravity, the Planck constant and the quantized area of the LQG, being independent of each other, if there is no a priori, some correction factor for the Barbero-Immirzi parameter [5].

In general, the studies that seek to estimate the BI parameter are in the scope of Cosmology, but there are attempts in Particle Physics. The limits presented in the literature are compatible with the ferionic spectrum. In the case of leptons and quarks, they are not compatible with the BI parameter.

Khatsymovsky discovered the area spectrum, considering the Fadeev formulation of gravitation for the parameter at the approximate value of $\gamma = 0{,}39\ldots$. Majhi et al obtained $0{,}159 < \gamma < 0{,}225$, in the treatment of the bekenstein-hawking area law, making the relationship of the parameters that characterized the quantum states of horizons in isolation. In the case of entropy for the arbitrary non-rotating isolated horizon, they obtained the value for $\gamma \approx 0{,}349699$[8].

With Dreyer's [9]work, he considers the area of the event horizon because of the large number of sides of the spin network that cross the surface and each side of the network with spin *j*, which contributes to a part, giving a total area of the surface. Each spin *j* increases the dimension by one factor (2*j* + 1), which is the dimension of spin *j* representation.

If you have a large N-sided number with spins *j*, with *i* = 1, ..., *N*, drilling the horizon, entropy is given by the logarithm of this spin representation dimension, in the form

$$S = N.\ln(2j+1) \tag{3}$$

Being $N = \frac{A}{A_j}$, calculated by the area of black hole *A*, so we have

$$N = \frac{A}{8\pi\gamma {\ell_P}^2\sqrt{j(j+1)}} \tag{4}$$

Considered hawking entropy of Equation (3), then it is form $S = A/4{\ell_P}^2$, then, using equations (3) and (4), isolating *N*, we must

$$\frac{A}{4{\ell_P}^2} = N.\ln(2j+1)$$

$$\frac{A}{4{\ell_P}^2} = \frac{A}{8\pi\gamma {\ell_P}^2\sqrt{j(j+1)}}.\ln(2j+1)$$

$$1 = \frac{ln\ (2j\ +\ 1)}{2\pi\gamma\sqrt{j(j+1)}}$$

$$\gamma = \frac{\ln(2j\ +\ 1)}{2\pi\sqrt{j(j+1)}} \tag{5}$$

For spin *j* = 1/2, we must γ, the parameter in the form

$$\gamma = \frac{ln\ 2}{\pi\sqrt{3}} \tag{6}$$

If the spins *1/2* and *1*, we will have for the parameter in Dreyer's proposal for $0{,}127$ and $0{,}123$, respectively.

The relationship of Abel-Liouville with Wronskian in the Tetraheric region for the Barbero-Immirzi parameter

The purpose of this work is to relate the Abel-Liouville formula with the Wronskian of the area and volume in isolation, a tetrahedral space-time region. When considering a homogeneous linear equation of 2nd order variable coefficients, we have the form [10]

$$y'' + p(x)y' + q(x)y = 0 \tag{7}$$

Being p and q defined and continuous in interval I. Imposing the condition for the tetrahedron area as Being A and volume V, according to the W wronskian, we have

$$W = AV' - A'V \tag{8}$$

For W', we also have

$$W' = AV'' - A''V \tag{9}$$

Wronskian solution for area and volume will be 1st order linear solution in the form

$$W' + p(x)W = 0 \tag{10}$$

Consequently, there will be a constant (in this case the BI parameter), such that the wronskian will be given by the Abel-Liouville formula, in the form

$$W = \gamma . e^{-\int p(x)dx} \tag{11}$$

Imposing the condition of x being the length of one of the edges of a regular tetrahedron, then the area of only one face is of the shape $A = x^2\sqrt{3}$ and its volume $V = x^3\sqrt{2}/12$. Using equation (8), we have

$$W = AV' - A'V = \frac{x^2\sqrt{3}x^2\sqrt{2}}{4} - \frac{2x\sqrt{3}x^3\sqrt{2}}{12}$$

$$W = AV' - A'V = \frac{3x^4\sqrt{6}}{12} - \frac{2x^4\sqrt{6}}{12} = \frac{x^4\sqrt{6}}{12} \tag{12}$$

Already for W'' we have the form

$$W'' = AV'' - A''V = \frac{6x^2\sqrt{3}x\sqrt{2}}{12} - \frac{2\sqrt{3}x^3\sqrt{2}}{12} = \frac{x^3\sqrt{6}}{3} \tag{13}$$

When we replace in equation (10), the results of equations (12) and (13), we have

$$\frac{x^3\sqrt{6}}{3} + p(x)\frac{x^4\sqrt{6}}{12} = 0 \tag{14}$$

What we naturally found for $p(x) = -4/x$. reusing equation (11) with the new substitution of $p(x)$, we will have

$$W = \gamma . e^{-\int p(x)dx} = \gamma . e^{-\int -4/x dx} = \gamma . e^{4lnx} \tag{15}$$

As $W = \frac{x^4\sqrt{6}}{12}$, replacing in equation (15), we have

$$W = \frac{x^4\sqrt{6}}{12} = \gamma . e^{4lnx}$$

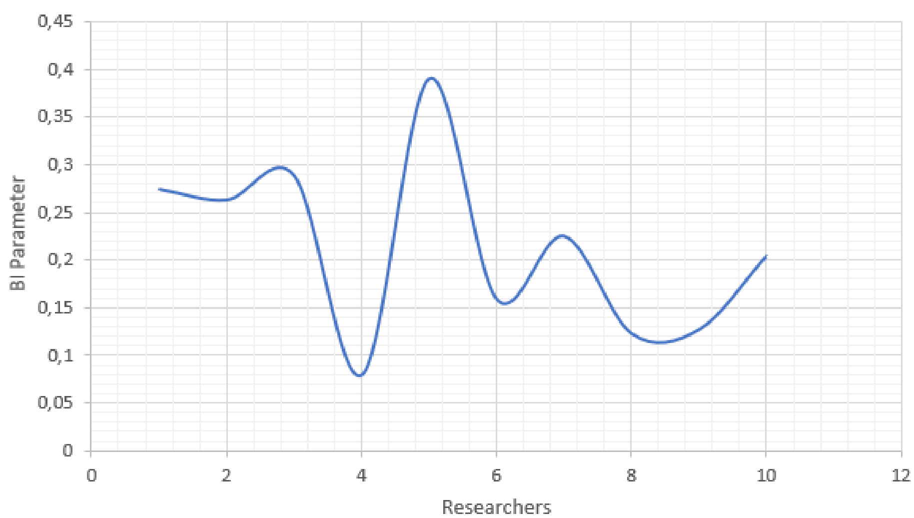
$$\gamma = \frac{x^4\sqrt{6}}{12. e^{4lnx}} \tag{16}$$

Where the equation (16) shows the proposal of this work. When considering x being the planck length $\ell_P = 1{,}6.10^{-35}\, m$, we will have for the Barbero-Immirzi parameter the approximate value $\gamma = 0{,}204124$ .... When we put several values for the length of the tetrahedron, we will find the same parameter, changing only the seventh decimal place, except zero to the value *of x*, which would make it impossible not to have a geometric figure of the time space according to the LQG. Looking at the table below, we will see the estimated parameters cited in this work with the proposal:

Researcher	Identification number	Estimated value of the BI parameter
Pigozzo et al	1	$\gamma = 0{,}274$
Carneiro, Frolov, Zelnikov, Rovelli e Smollin	2 e 3	$\gamma = 0{,}263\ a\ 0{,}288$
Corichi e Krasnov	4	$\gamma = 0{,}079577$
Khatsymovsky	5	$\gamma = 0{,}39$
Majhi	6 e 7	$\gamma = 0{,}159\ a\ 0{,}225$
Dreyer	8 e 9	$\gamma = 0{,}123\ e\ 0{,}127$
Jizreel	10	$\gamma = 0{,}204124$

Table 1. Relationship of researchers with the estimated BI parameter.

Through the table with the identification number, we observed the chart below, the estimated works for the BI parameter



Graph 1. Table 1 researchers estimate for the Barbero-Immirzi parameter.

Results and discussion

Obviously, the discussion will be in the references mentioned in this article, because there are certainly several articles dealing with the BI parameter. When observing graph 1, we noticed the natural oscillation, due to several methods used by the researchers, considering several factors in their methodologies.

The lowest peak of the graph was represented by the parameter of Corichi and Krasnov's works and which, according to Rovelli's notes, considered the incorrect calculation for Bekenstein-Hawking entropy, so a value well below the "expected". The highest peak of the chart was for Khatsymovsky, where he considered Fadeev's formulation for gravitation.

The proposal of this work for the value of the BI parameter is reasonable, but with limitations, because there are still questions such as the non-compatibility for other geometric structures of the time space, despite being fixed for a given geometric figure. Why is there incompatibility of the BI parameter for quarks and léptons? What will be the physics behind this incompatibility? Is the BI parameter unique to Black Hole entropy in LQG or can we work with Particle Physics? These are questions that still need studies, but it is a fact that their existence has an effect on the quantum regime.

For this study, the proposal used is precisely the consideration of the space-time tetrahedral geometric structure, precisely emphasized by LQG. Can the constant that appears in Abel-Liouville's relationship with the Wronskian that relates the area and volume serve as a parameter?

Given the cited works and the proposal presented, the BI parameter will hardly be fixed, as it will involve several aspects such as: object of study analyzed, particle type, dimensionality aspects, treatment with spins, loaded fields, type of universe region, among other factors that contribute to the high complexity in the calculations of approximation of the BI parameter.

Final considerations

The work did not aim to be the truth of the BI parameter, but a proposal, a possibility to look differently to relate what we already know within the LQG theory, which is the quantized area and volume factor, considering the polyhedric structure of space-time. Until the parameter is detected, we will have motivations in various fields of physics to analyze the coupling that the parameter exerts. Each researcher will search in various points of view, work to connect this coupling and identify the possible effects and its causes.

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