

Modeling and Theoretical Validations of the Potential Radiation Energy with C-14 Radionuclide under Constant Emissivity with Standard Atmospheric Determinants

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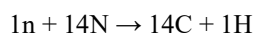
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Abstract: Carbon-14 has long been used as a tracer for the actual estimation of radioactive materials, especially in archeological, biological, biomedical, and environmental samples. Achieving data sets that can be generated from modeling the C-14 tracer and fraction in a biomaterial would actively enhance the quantification of radiation energy that can be emitted with time against the identified parameters. Precise assessment of the radiation exposure from ingested ¹⁴C is an essential component of the computed and theoretical modeling designs. Consequently, this paper illuminates the computational and theoretically validated models of C-14 biomaterial from the initial activity of 100% to the final range of activities, which also relates to the surface area of 5 to 95% of the biomaterial, estimating the decay time as theoretically supported by the Stefan-Boltzmann law under standard conditions of half-life, decay constant, atmospheric emissivity, and atmospheric and radiative temperature. The generated outcomes with the respective potential radiation energies were modeled against time ($y^l = 31.207e^{-1E-04 \cdot xl}$), and the final expected activities ($y^l = 0.3121x$) and with decay time and final expected activities ($y = 32173e^{-0.038x}$). Hence, the models predicts that as the carbon-14 isotope undergoes radioactive decay, it creates radiation (energy) and turns into some other element.

Keywords— Carbon-14, radioactivity, radiation energy, Computational modeling and Stefan's Boltzmann law of radiation

1. INTRODUCTION

Kamen Martin and Ruben Sam discovered carbon-14, sometimes known as radiocarbon, or ¹⁴C, on the 27th of February 1940 [1]. The radiocarbon dating technique is widely used to date archaeological, geological, and hydrogeological samples because its nucleus possesses eight neutrons and six protons, which are present in organic compounds [2]. On Earth, there are three naturally occurring forms of carbon: carbon-12, carbon-13, and carbon-14 which characterize 99%, 1%, and 0.000000001 percent of all carbon respectively [3]. Given that the activity of the current radiocarbon standard is around 14 disintegrations per minute (dpm) per gram of carbon, carbon-14 has a half-life of 5730–40 years and decays into nitrogen-14 by beta-decay with 14.003241 amu [4]. The chemical properties of the various isotopes of carbon do not significantly differ from one another, and this detail is widely used in biochemical investigation in a process known as carbon labeling, where the atoms of carbon-12 of a specified chemical compound are substituted with those of carbon-14 or carbon-13 atoms, to trace them along chemical reactions involving the given compound. The generation of carbon-14 emanates from the thermal absorption of neutrons from nitrogen atoms in the higher stratum of the troposphere and stratosphere. As cosmic rays change as they enter the atmosphere, their transformation results in the production of neutrons (¹n) which take part in the next reaction. [6]:



The largest rates of carbon-14 synthesis occur between 9 and 15 km (30,000 to 50,000 feet) above the earth's surface and at high geomagnetic latitudes, but carbon-14 is easily mixed and dispersed throughout the atmosphere, where it combines with oxygen to generate radioactive carbon dioxide. Additionally dissolving in water, carbon dioxide permeates the oceans [7]. By generating spallation reactions in oxygen, rapid neutrons can also produce carbon-14 in ice. Then, carbon-14 undergoes radioactive beta decay. Carbon-14 with a half-life of 5730 years, decays into the stable, non-radioactive isotope nitrogen-14 by ejecting an electron and an antineutrino [6]. Initially, the 156keV - decay of the nuclide was used to count ¹⁴C; but, since 1977, accelerator mass spectrometry (AMS) has gradually taken its place as the primary method. Accelerator mass spectrometry has taken over as the preferred technique for the majority of measurements of longer-lived radionuclides, of which carbon-14 is the most well-known [9]. Because we can count the atoms directly rather than waiting for decays to be counted, this technique makes it possible to quantify considerably smaller amounts of carbon than were previously conceivable using decay counting. In reality, it is now possible to analyze carbon samples ranging from 0.05 to 0.5 mg. Cross-referencing corals' ¹⁴C and U-Th ages enables calibration to be accomplished [10]. We now have a calibration over the last 50,000 years, which is essentially the

entire range of radiocarbon dating, but this includes certain assumptions about the consistency of the marine ¹⁴C record. Although the phrase "half-life" can be used to describe different types of decay, whether exponential or not, it is most frequently used to describe atoms experiencing radioactive decay. Carbon-14 dating is one of the most well-known half-life uses [11]. William Libby formulated the method of carbon-14 dating, with the possibility of carbon-14 being generated continuously in the atmosphere [12]. When an animal or plant dies, carbon-14 is integrated into both through photosynthesis and is then consumed by animals. By measuring the amount of carbon-14 in a sample, one can determine the exact time the plant or animal died. For instance, with a half-life of roughly 5,730 years, it can be used to accurately determine dates up to 50,000 years ago [12]. Three comparable formulas for exponential decay are presented below:

$$(1) \quad N(t) = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$$(2) \quad N(t) = N_0 e^{-\frac{t}{\tau}}$$

$$(3) \quad N(t) = N_0 e^{-\lambda t}$$

Where:

N_0 is the initial quantity

N_t is the remaining quantity after time, t

$t_{1/2}$ is the half-life

τ is the mean lifetime

λ is the decay constant

Production varies as a result of changes in cosmic ray intensity since the atmospheric stock of carbon-14 at equilibrium is approximately 140×10^{15} Bq and the rate of annual production of natural ¹⁴C equals 1.40×10^{15} Bq [14]. The specific activity, measured in Becquerels of ¹⁴C/ kg of the entire carbon, remain unchanged within the mechanisms of the terrestrial environment and is in balance with the specific activity of the atmosphere's CO₂ [14], [15], and [16]. The ¹⁴C-specific activities for the biological compartments of the terrestrial environment, unaffected by nuclear facilities, peaked in the mid-1960s (more than 400 Bq.kg⁻¹ of C), due to the fallout from atmospheric nuclear armament testing, which was at its peak at the time (Figure 1) [17]. With the end of testing and the ongoing rise in CO₂ from fossil fuels, these activities have gradually declined since then (by less than 0.5% per year) (gasoline, coal, gas). When compared to the level in 1950 (226 Bq.kg⁻¹ C), before atmospheric testing, the definite bio-activities of the terrestrial sections are today around 238 Bq ¹⁴C.kg⁻¹ C

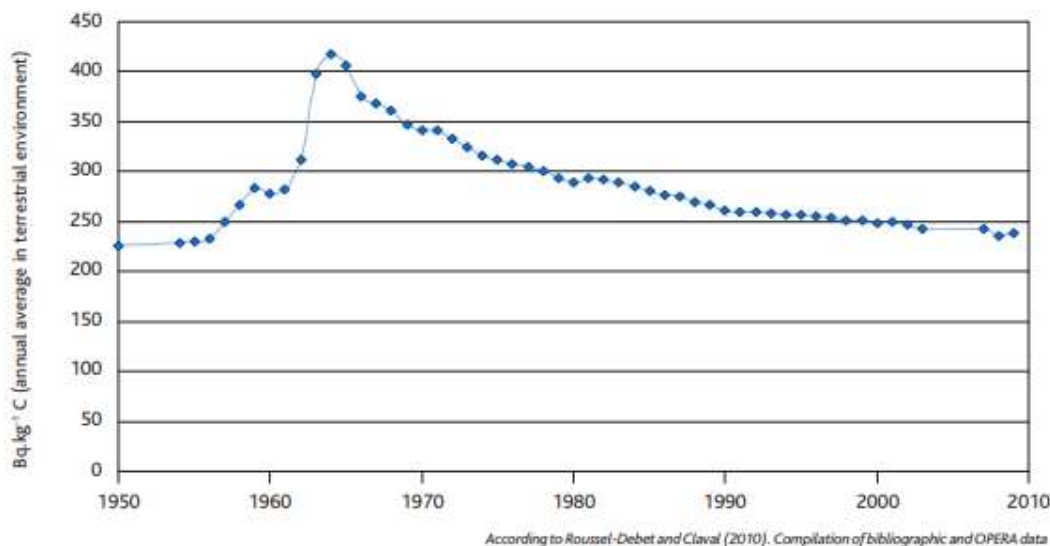


Figure 1. Changes in the net activity of C-14 for biological samples in terrestrial system within the last sixty years

Two destructive methods, such as activity measurement or atom counting, can be used to determine the amount of carbon-14 present in an environmental sample [19]. The test portion's carbon is converted to carbon dioxide, from

which a sample is generated for measurement by liquid scintillation, according to the activity measurement principle [20]. The ability of a surface to emit energy as thermal radiation or electromagnetic radiation with

wavelengths that depend on the temperature of radioactivity released from radioisotopes, radionuclides, or radioactive nuclides, with species of a similar chemical element with different atomic masses whose nuclei are unstable and spontaneously emit radiation in the form of alpha, beta, or gamma radiation, is referred to as the surface's emissivity. In its most basic form, radioactivity releases energy out of the nuclei of specific atoms and isotopes [24]. The term "radioactive nuclei" refers to unstable nuclei that decay by releasing energetic particles such as alpha particles, photons, neutrinos, electrons, protons, or neutrons. Ionizing particles are some of these substances. These are particles powerful enough to dislodge electrons from molecules or atoms. The proportion of unstable nuclei and the rate at which they decay affect how radioactive an area is [22]. The kind and energy of the particles released during nuclear decay also affect radioactivity's effects. Radioactivity is used by scientists and engineers as a heat source for satellites, individualized cancer therapies, medical imaging, radiometric dating, and for investigations into the fundamental principles of nature and the creation of matter [23]. Meanwhile, the amount of energy released is influenced by the surface's temperature as well as how well it can emit radiation (emissivity). Surfaces with emissivity less than 0.85 are often only found in deserts and semi-arid regions [24], [25]. In the thermal infrared

spectrum, vegetation, water, and ice have high emissivity over 0.95 [25]. Therefore, the online computational and theoretical frameworks of applicable carbon-14 biomaterial under typical atmospheric conditions are evaluated in this paper.

2. METHODOLOGY

Real-time online radioactive resources [26] were adopted in modeling the decay profile of C-14 and [27] was for the potential energy of radiation that could be generated from possible carbon-14 radioisotope progressively over time.

2.1 MODELING

The periods of C-14 radio nucleotide decay were estimated with the respective final activities of 5 to 95%, which are also the function of the available surface area were considered in determining the potential radiation energies that could be released at standard atmospheric conditions of 0.8 emissivity for C-14 and 288K of radiative equilibrium atmospheric temperature [28]. The modeling was further validated theoretically in ascertaining the logical precision

3. RESULTS AND DISCUSSION

Table 1. Computational modeling of radioactive C 14 decayed period and the potential energy under standard conditions.

Initial activity (%)	Decayed activity (%)	Final activity (%)	Decay time/ Age (t) (Years)	Potential radiation Energy (KW)	Half life (Years)	Mean lifetime (Years)	Decay constant	Atmospheric emissivity constant	Atmospheric radiative temperature (Kelvin)
100	0	0	0	0	5,730	8,266.64	1.2097×10^{-4}	0.8	288
	95	5	24,764.65	1.56					
	90	10	19,034.65	3.12					
	85	15	15,682.81	4.68					
	80	20	13,304.65	6.24					
	75	25	11,460.00	7.80					
	70	30	9,952.81	9.36					
	65	35	8,678.50	10.92					
	60	40	7,574.65	12.48					
	55	45	6,600.98	14.04					
	50	50	5,730.00	15.60					
	45	55	4,942.11	17.16					
	40	60	4,222.81	18.72					
	35	65	3,561.13	20.28					
	30	70	2,948.50	21.84					
	25	75	2,378.17	23.41					
	20	80	1,844.65	24.97					
15	85	1,343.49	26.53						
10	90	870.98	28.09						
5	95	424.02	29.65						

Table 2. Theoretical validation of the computational modeling

Source of C-14	Initial activity (IA) of C-14 (%)	Final activity (FA) of C-14 (%)	Decay period relationship (years)		Potential radiation energy (PRE = εAσT ⁴)					
			$t = \frac{5730 \ln\left(\frac{FA}{100}\right)}{-0.693}$		ε	A (m ²)	σ (× 10 ⁻⁸) (W/m ² ·K ⁴)	T (K)	T ⁴ (K ⁴)	PRE(× 10 ⁻⁸) (KW)
Fossil biomaterial	100	0								
		5	-17,165.55	24,769.91	0.8	5	5.67	288	6879707136	1.56032E+11
		10	-13,193.81	19,038.69	0.8	10	5.67	288	6879707136	3.12064E+11
		15	-10,870.50	15,686.14	0.8	15	5.67	288	6879707136	4.68095E+11
		20	-92,22.08	13,307.47	0.8	20	5.67	288	6879707136	6.24127E+11
		25	-7,943.47	11,462.43	0.8	25	5.67	288	6879707136	7.80159E+11
		30	-6,898.76	9,954.93	0.8	30	5.67	288	6879707136	9.36191E+11
		35	-6,015.48	8,680.35	0.8	35	5.67	288	6879707136	1.09222E+12
		40	-5,250.35	7,576.26	0.8	40	5.67	288	6879707136	1.24825E+12
		45	-4,575.45	6,602.38	0.8	45	5.67	288	6879707136	1.40429E+12
		50	-3,971.73	5,731.22	0.8	50	5.67	288	6879707136	1.56032E+12
		55	-3,425.61	4,943.15	0.8	55	5.67	288	6879707136	1.71635E+12
		60	-2,927.03	4,223.71	0.8	60	5.67	288	6879707136	1.87238E+12
		65	-2,468.39	3,561.89	0.8	65	5.67	288	6879707136	2.02841E+12
		70	-2,043.75	2,949.13	0.8	70	5.67	288	6879707136	2.18444E+12
		75	-1,648.42	2,378.67	0.8	75	5.67	288	6879707136	2.34048E+12
		80	-1,278.61	1,845.04	0.8	80	5.67	288	6879707136	2.49651E+12
85	-931.23	1,343.77	0.8	85	5.67	288	6879707136	2.65254E+12		
90	-603.72	871.16	0.8	90	5.67	288	6879707136	2.80857E+12		
95	-293.91	424.11	0.8	95	5.67	288	6879707136	2.9646E+12		

The Stefan-Boltzmann Law, which states that all things with temperatures greater than absolute zero emit energy, is used to compute the radiation energy of a subject surface [29]. The Stefan-Boltzmann formula is as follows:

$$P = \epsilon A \sigma T^4$$

Where:

P: Radiation Energy

ε: Emissivity of the Surface. Check emissivity table below of common materials.

A: Surface Area, in m².

σ: Stefan-Boltzmann Constant, $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$

T: Absolute Temperature

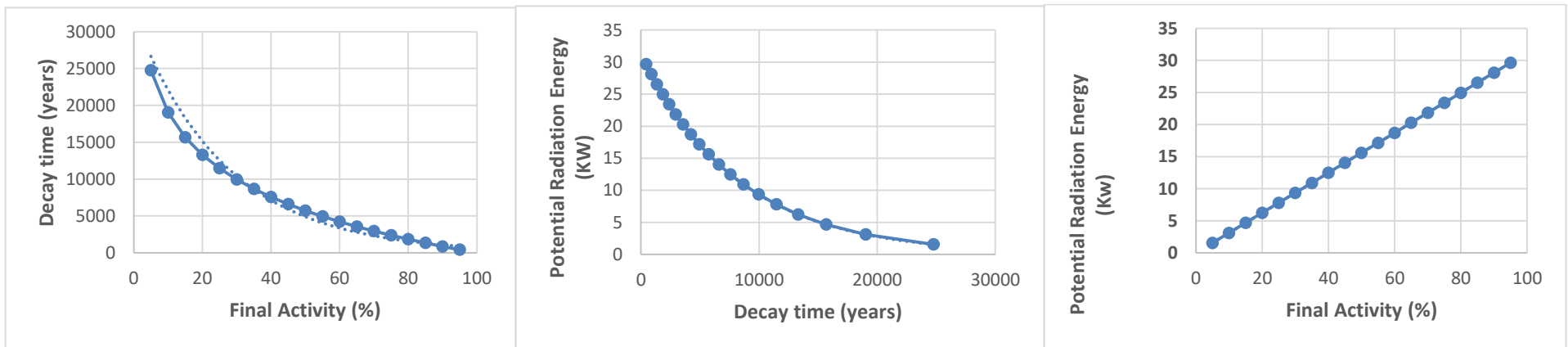


Figure 1. Decay time against the final activity

Figure 2. Potential radiation energy against the decay time

Figure 3. Potential radiation energy against the final activity

Table 3. Generated Models

Interaction	Model	Correlation (R ²)
Decay time (y) and Final activity (x)	$y = 32173e^{-0.038x}$	0.9520
Potential Radiation Energy (y ¹) and Decay time (x ¹)	$y^1 = 31.207e^{-1E-04 x^1}$	1
Potential Radiation Energy(y ¹) and Final activity (x)	$y^1 = 0.3121x$	1

Technically, the outcomes, Table 1 were based on the constant values for the atmospheric emissivity [25], the carbon-14 material decay factor [7], the carbon-14 mean lifespan [7], and the atmospheric radioactive temperature in Kelvin [28].

As a result of the decay effects that simultaneously disperse the relevant radiation energy and the decay period, the initial radioactive activity of 100% was arbitrarily designed to decay down to 95 to 5% final activity. With increased C-14 activity at constant temperature, the link between the decay time and the potential radiation energy rises. A proper presentation of the computational modeling used for the aforementioned theoretical validation has been demonstrated using graphics. This model describes the atmosphere as a single homogenous layer of gases in thermal equilibrium at temperature T_a , behaving as a grey body with an emissivity and an absorptivity determined by. This demonstrates that some of the energy generated by the heated surface gets absorbed by the atmosphere. Atmospheric absorption controls how much energy is absorbed with the earth's surface typically has a temperature of 288 K [25].

This temperature would be equivalent to an atmospheric emissivity of roughly 0.8 for the single-layer atmosphere model. Thus, the model achieves its primary goal of showing how an atmosphere can cause a planet's surface to be warmer than it would be in the absence of an atmosphere [25]. An atmosphere can absorb some radiation from a planet's surface and then re-emit part of it.

The geometry of the closely formulated peat of ^{14}C dates is matched with the shape of the ^{14}C calibration curve using the non-linear connection between calendar and C-14 ages [25]. Similarly, a mathematical model to WMD (wiggle match dating) was introduced by Blaauw et al. (2004), making it possible to calculate the accuracy of WMD chronologies where the investigations of the Holocene's rapid climatic transformation alongside their potential causes with high-precision sequence [30]. Also, the dendrochronological calibration curve, which was matched with the natural ^{14}C fluctuations (wiggles), was found in the Holocene AMS dating with elevated bog deposits from Engbertsdijkswenen, the Netherlands. The ^{14}C record and other conventionally dated peat cores exhibit an unexpected and unreported reservoir impact when compared [14]. As a result, carbon-14 dating represents a fundamental leap in our understanding of the past of our planet. In actuality, it is helping to "reconstruct world history." Researchers can learn about ancient civilizations, planetary changes, and climate changes using this dating technique. Different civilizations and faiths use various dating techniques.

The age of the absolute dating of any substance prior to the present, is a particularly useful feature of carbon-14 dating. This implies that it can be applied and evaluated against dates everywhere in the world. However, it remains the primary method for dating the last 50,000 years and is regarded as the "most important development in absolute dating in archaeology." Scientists seek to use this instrument to solve the puzzles surrounding the origin of mankind, the duration

of his existence, his travels, and the construction of a timeline of human history.

4. CONCLUSION

Data on carbon-14 and the models describing this radionuclide's fate in terrestrial environments are based on understanding of the equilibrium carbon cycle. Due to the existence of both inorganic and organic carbon, whether it be in the form of solid, liquid, or gas, the carbon cycle which includes carbon-14, is exceedingly complex. The level of C-14 in plants at the time wood is laid down or in animals at the time of death is equivalent to the level of C-14 in the atmosphere at that time because biomaterials, such as plants, fix atmospheric carbon during photosynthesis. The date of death or fixation can be concluded from the subsequent decrease in the amount due to radioactive decay. The initial C-14 level can be calculated or directly compared with known year-by-year data from tree-ring data (dendrochronology) to 10,000 years ago or from cave deposits (speleothems) to around 45,000 years of age. The age of the bio sample from formation and the quantity of energy released into the atmosphere have increased dramatically over the previous several periods, according to a precise theoretical calculation or direct comparison models like those with carbon-14.

5. ACKNOWLEDGEMENT

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