

On Generalized Triple Transformation

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Abstract. In this paper, we introduce a new transformation , namely , **triple g-transformation** with three parameters s, α and β and denote it as T_{3g} . This transformation consider as generalized to some types of triple transformations, also it is considered as composition of three g-transformations with one parameter. We defined it as the following form :

$$T_{3g}(f(x, y, z)) = p_1(s)p_2(\alpha)p_3(\beta) \int_0^\infty \int_0^\infty \int_0^\infty e^{q_1(s)x-q_2(\alpha)y-q_3(\beta)z} f(x, y, z) dx dy dz$$

Keywords: integral transform, triple transformation, g-transformation, double transformation.

I . Introduction:

The integral transformations are one of import methods for solving many of problems. Such that we can convert ordinary differential equation to algebraic equation and then we back by inverse of this integral transformation, but the case partial differential equations we use the integral transformation to convert partial differential equations to ordinary differential equations [5]. By using double integral transformation we can convert partial differential equations of two independent variables to algebraic equations , thus we need to triple transformation to solve partial differential equations with three independent variables. the triple integral transformation is has import applications [1,2]. This transformation is distinguished by the generalities of the most known integral transformations and the possibility of finding new integral transformations from it.

In [4] , H.Jaferi presented general integral transformation and he called it g-transformation also he studied properties of g-transformation and its applications in differential equations. By taking g-transformation with three parameters s, α and β , T_{3g} -transformation is constructed. In this paper , theorems and examples related with T_{3g} -transformation are presented.

2.Triple g-Transformation:

2.1 Definition [4]:

g-transformation $g(f(x))$ for a piecewise function $f(x)$ where $x \in [0, \infty]$ and $|f(x)| \leq M e^{kx}$ is defined by the following integral

$$T_g^s(f(x)) = p(s) \int_0^\infty e^{-q(s)x} f(x) dx = \bar{f}_s, p(s) \neq 0 \quad (1)$$

such that the integral is convergent for some $q(s)$, s is positive constant, and $\|g(f(x))\| \leq \frac{p(s)M}{k-q(s)}$

2.2 Definition [4]:

Double T_{2g} -transformation for a piecewise function $f(x,y)$ where $x \in [0, \infty]$, $y \in [0, \infty]$ and $|f(x,y)| \leq M e^{k(x+y)}$ is defind by the follwoing integral:

$$\begin{aligned} T_{2g}^{s\alpha}(f(x,y)) &= T_g^s \left(T_g^\alpha(f(x,y)) \right) \\ &= P_1(s)P_2(\alpha) \int_0^\infty \int_0^\infty e^{-q_1(s)x-q_2(\alpha)y} f(x,y) dx dy \\ &= p(s) \int_0^\infty \int_0^\infty e^{-q_1(s)x-q_2(\alpha)y} f(x,y) dx dy = \bar{f}_{s\alpha} \end{aligned} \quad (2)$$

Where $p(s) = P_1(s)P_2(s)$
 such that the integral is convergent for some $q_1(s), q_2(\alpha)$ are positive functions, and $\|T_{2g}^{s\alpha}(f(x,y))\| \leq \frac{P_1 P_2 M}{k - q_1 q_2}$ (3)

2.3 Definition:

Let f be a continuous function of three variables then the triple g-transformation of $f(x,y,z)$ is defined as following :

$$\begin{aligned} T_{3g}^{s\alpha\beta}(f(x,y,z)) &= T_g^s \left(T_g^\alpha \left(T_g^\beta(f(x,y,z)) \right) \right) \\ &= P_1(s)P_2(\alpha)P_3(\beta) \int_0^\infty \int_0^\infty \int_0^\infty e^{-q_1(s)x-q_2(\alpha)y-q_3(\beta)z} f(x,y,z) dx dy dz \\ &= p_{(s,\alpha,\beta)} \int_0^\infty \int_0^\infty \int_0^\infty e^{-q_1(s)x-q_2(\alpha)y-q_3(\beta)z} f(x,y,z) dx dy dz \end{aligned}$$

where $p_{(s,\alpha,\beta)} = p_1(s) \cdot p_2(\alpha) \cdot p_3(\beta)$ and $x,y,z > 0$ and s, α and β are positive constants , and $\sup \left| \frac{f(x,y,z)}{e^{ax+by+cz}} \right| < 0$ for some $a, b, c \in R$. The inverse of T_{3g} – transform is defined as following :

$$f(x,y,z) = \frac{1}{2\pi i} \int_{\infty-i\infty}^{\infty+i\infty} \int_{\infty-i\infty}^{\infty+i\infty} \int_{\infty-i\infty}^{\infty+i\infty} P_1(s)P_2(\alpha)P_3(\beta) e^{q_1(s)x-q_2(\alpha)y-q_3(\beta)z} F(s, \alpha, \beta) ds d\alpha d\beta$$

$$= \frac{1}{2} T_{3g}^{s\alpha\beta} [(e^{ax+by+cz}) - \frac{1}{2} T_{3g}^{s\alpha\beta} (e^{-ax-by-cz})]$$

2.4 Examples :

$$1 - T_{3g}^{s\alpha\beta} (1) = p \int_0^\infty \int_0^\infty \int_0^\infty e^{q_1(s)x - q_2(\alpha)y - q_3(\beta)z} (1) dx dy dz$$

$$= \frac{p}{2} \left[\frac{1}{(q_1-a)(q_2-b)(q_3-c)} - \frac{1}{(q_1+a)(q_2+b)(q_3+c)} \right]$$

$$= \frac{p}{2} \left[\frac{(q_1+a)(q_2+b)(q_3+c) - (q_1-a)(q_2-b)(q_3-c)}{(q_1^2 - a^2)(q_2^2 - b^2)(q_3^2 - c^2)} \right]$$

$$= p \int_0^\infty \int_0^\infty e^{-q_2(\alpha)y - q_3(\beta)z} \left[\frac{1}{-q_1(s)} e^{q_1(s)x} \right]_0^\infty dy dz$$

$$= \frac{p}{2} \left[\frac{2q_1 q_2 c + 2b q_1 q_3 + 2a q_2 q_3 + 2ab c}{(q_1^2 - a^2)(q_2^2 - b^2)(q_3^2 - c^2)} \right]$$

$$= \frac{p}{q_1} \int_0^\infty \int_0^\infty e^{-q_2(\alpha)y - q_3(\beta)z} dy dz$$

$$T_{3g}^{s\alpha\beta} (\sinh(ax + by + cz))$$

$$T_{3g}^{s\alpha\beta} (1) = \frac{p}{q_1 q_2 q_3}$$

$$= \frac{p(q_1 q_2 c + bq_1 q_3 + aq_2 q_3 + ab c)}{(q_1^2 - a^2)(q_2^2 - b^2)(q_3^2 - c^2)}$$

$$2 - T_{3g}^{s\alpha\beta} (e^{ax+by+cz})$$

$$= p \int_0^\infty \int_0^\infty \int_0^\infty e^{q_1 x - q_2 y - q_3 z} e^{ax+by+cz} dx dy dz$$

2.5 Proposition:

$$T_{3g}(c_1 f(x, y, z) \mp c_2 h(x, y, z))$$

$$= c_1 T_{3g}(f(x, y, z)) \mp c_2 T_{3g}(h(x, y, z))$$

Proof:

$$T_{3g}^{s\alpha\beta} (c_1 f(x, y, z) \mp c_1 h(x, y, z))$$

$$= p \int_0^\infty \int_0^\infty \int_0^\infty [c_1 f(x, y, z)]$$

$$\mp c_2 h(x, y, z)] e^{q_1 x - q_2 y - q_3 z} dx dy dz$$

$$= c_1 p \int_0^\infty \int_0^\infty \int_0^\infty f(x, y, z) e^{-q_1 x - q_2 y - q_3 z} dx dy dz$$

$$\mp c_2 p \int_0^\infty \int_0^\infty \int_0^\infty h(x, y, z) e^{-q_1 x - q_2 y - q_3 z} dx dy dz$$

$$= c_1 T_{3g}^{s\alpha\beta} (f(x, y, z)) \mp c_2 T_{3g}^{s\alpha\beta} (h(x, y, z))$$

2.6 $T_{3g}^{s\alpha\beta}$ -Transformation for selected functions.

I D	$f(x)$	$T_{3g}^{s\alpha\beta} (f(x, y, z))$ $= p_{(s,\alpha,\beta)} \int_0^\infty \int_0^\infty \int_0^\infty e^{q_1(s)x - q_2(\alpha)y - q_3(\beta)z} f(x, y, z) dx dy dz$
1	$K, K \text{ consta}$	$K \frac{p}{q_1 q_2 q_3}$
2	$\sin(ax + by + cz)$	$p \left[\frac{aq_2 q_3 - abc + bq_1 q_3 + cq_1 q_2}{(q_1^2 + a^2)(q_2^2 + b^2)(q_3^2 + c^2)} \right]$
3	$\cos(ax + by + cz)$	$p \left[\frac{q_1 q_2 q_3 - bcq_1 - abq_3 - acq_2}{(q_1^2 + a^2)(q_2^2 + b^2)(q_3^2 + c^2)} \right]$

4	$x^k y^m z^n$	$\frac{pk! m! n!}{(q_1)^{k+1} (q_2)^{m+1} (q_3)^{n+1}}$
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2.7 Proposition:

Let $f(x,y,z)$, $x,y,z \in [0, \infty]$ be a function such that $|f(x,y,z)| \leq \mu e^{k_1 x + k_2 y + k_3 z}$ and $T_{3g}^{s\alpha\beta}(f(x,y,z)) = F(s,\alpha,\beta)$ exists, then

$$\begin{aligned} T_{3g}^{s\alpha\beta}(f(x+a, y+b, z+c)) &= \\ e^{aq_1+bq_2+cq_3} &\left[T_{3g}(f(u, v, w)) - \right. \\ T_{2g}\left(\int_0^c f(u, v, w) e^{-q_3 w} dw\right) &- \\ T_{2g}\left(\int_0^b f(u, v, w) e^{-q_2 v} dv\right) &+ \\ T_{sg}\left(\int_0^c \int_0^b f(u, v, w) e^{-q_2 v} e^{-q_3 w} dv dw\right) &- \\ T_{2sg}\left(\int_0^a f(u, v, w) e^{-q_1 u} du\right) &+ \\ T_{sg}\left(\int_0^c \int_0^a f(u, v, w) e^{-q_1 u} e^{-q_3 w} dudw\right) &+ \\ T_{sg}\left(\int_0^b \int_0^a f(u, v, w) e^{-q_1 u} e^{-q_2 v} dudv\right) &- \\ p(s) \int_0^c \int_0^b \int_0^a f(u, v, w) e^{-q_1 u} e^{-q_2 v} e^{-q_3 w} du dv dw (f(u, v, w)) \left. \right] \end{aligned}$$

Proof :

$$\begin{aligned} T_{3g}^{s\alpha\beta}(f(x+a, y+b, z+c)) &= \\ p_{(s,\alpha,\beta)} \int_0^\infty \int_0^\infty \int_0^\infty &e^{q_1(s)x-q_2(\alpha)y-q_3(\beta)z} f(x \\ &+ a, y+b, z+c) dx dy dz \end{aligned}$$

Let $u = x+a, v = y+b, w = z+c$

$$\begin{aligned} T_{3g}^{s\alpha\beta}(f(x+a, y+b, z+c)) &= \\ p_{(s,\alpha,\beta)} \int_c^\infty \int_b^\infty \int_a^\infty &e^{q_1(s)(u-a)-q_2(\alpha)(v-b)-q_3(\beta)(w-c)} f(u, v, w) du dv dw \\ &= p_{(s,\alpha,\beta)} e^{aq_1+bq_2+cq_3} \int_c^\infty \int_b^\infty \int_a^\infty f(u, v, w) e^{-q_1 u} e^{-q_2 v} e^{-q_3 w} du dv dw \end{aligned}$$

$$\begin{aligned} &= p e^{aq_1+bq_2+cq_3} \left[\int_c^\infty \int_b^\infty \int_0^\infty f(u, v, w) e^{-q_1 u} e^{-q_2 v} e^{-q_3 w} \right. \\ &\quad \left. - \int_c^\infty \int_b^a f(u, v, w) e^{-q_1 u} e^{-q_2 v} e^{-q_3 w} du dv dw \right] \end{aligned}$$

$$\begin{aligned} &= p e^{aq_1+bq_2+cq_3} \left[\int_c^\infty \int_0^\infty \int_0^\infty f(u, v, w) e^{-q_1 u} e^{-q_2 v} e^{-q_3 w} \right. \\ &\quad - \int_c^\infty \int_0^b \int_0^\infty f(u, v, w) e^{-q_1 u} e^{-q_2 v} e^{-q_3 w} \\ &\quad - \int_c^\infty \int_0^\infty \int_0^a f(u, v, w) e^{-q_1 u} e^{-q_2 v} e^{-q_3 w} du dv dw \\ &\quad \left. + \int_c^\infty \int_0^b \int_0^a f(u, v, w) e^{-q_1 u} e^{-q_2 v} e^{-q_3 w} du dv dw \right] \\ T_{3g}^{s\alpha\beta}(f(x+a, y+b, z+c)) &= \\ e^{aq_1+bq_2+cq_3} &\left[T_{3g}(f(u, v, w)) \right. \\ &\quad - T_{2g}^{s\alpha}\left(p_3 \int_0^c f(u, v, w) e^{-q_3 w} dw\right) \\ &\quad - T_{2g}^{s\beta}\left(p_2 \int_0^b f(u, v, w) e^{-q_2 v} dv\right) \\ &\quad \left. + T_g^s \left(p_2 p_3 \int_0^c \int_0^b f(u, v, w) e^{-q_2 v} e^{-q_3 w} dv dw \right) \right. \\ &\quad - T_{2g}^{\alpha\beta}\left(p_1 \int_0^a f(u, v, w) e^{-q_1 u} du\right) \\ &\quad + T_g^\alpha\left(p_1 p_3 \int_0^c \int_0^a f(u, v, w) e^{-q_1 u} e^{-q_3 w} dudw\right) \\ &\quad + T_g^\beta\left(p_1 p_2 \int_0^b \int_0^a f(u, v, w) e^{-q_1 u} e^{-q_2 v} dudv\right) \\ &\quad \left. - p(s) \int_0^c \int_0^b \int_0^a f(u, v, w) e^{-q_1 u} e^{-q_2 v} e^{-q_3 w} du dv dw (f(u, v, w)) \right] \end{aligned}$$

3.The Convolution of $T_{3g}^{s\alpha\beta}$ -transformation

3.1 Definition [3,6,7]:

Let $f(x, y, z), h(x, y, z)$ be a function defined on R^+ , then the convolution of the functions $f(x, y, z)$ and $h(x, y, z)$ is given as following :

$$(f \ast \ast h)(x, y, z) = \int_0^x \int_0^y \int_0^z f(x-u, y-v, z-w) h(u, v, w) du dv dw$$

3.2 Theorem :

Let $f(x,y,z), h(x,y,z)$ be a function which defined on R^+ then $T_{3g}^{s\alpha\beta}$ -transformation of the convolution $(f \ast\ast h)$ is given as following:

$$T_{3g}^{s\alpha\beta}(f \ast\ast h)(x, y, z) = \frac{1}{p} T_{3g}^{s\alpha\beta}(f(x, y, z)) \cdot T_{3g}^{s\alpha\beta}(h(x, y, z))$$

Proof:

$$T_{3g}^{s\alpha\beta}(f(x, y, z)) \cdot T_{3g}^{s\alpha\beta}(h(x, y, z)) =$$

$$[p \int_0^\infty \int_0^\infty \int_0^\infty f(u, v, w) e^{-q_1 u} e^{-q_2 v} e^{-q_3 w} du dv dw] \cdot [p \int_0^\infty \int_0^\infty \int_0^\infty h(t, r, o) e^{-q_1 t} e^{-q_2 r} e^{-q_3 o} dt dr do]$$

=

$$(p)^2 \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty e^{-q_1(u+t)-q_2(v+r)-q_3(w+o)} f(u, v, w) h(t, r, o) du dv dw dt dr do$$

$$x=u+t \rightarrow u=x-t, y=v+r \rightarrow v=y-r, z=w+o \rightarrow w=z-o$$

$$= (p)^2 \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty e^{-q_1 x - q_2 y - q_3 z} f(x-t, y-r, z-o) h(t, r, o) dx dy dz dt dr do$$

$$= (p)^2 \int_0^\infty \int_0^\infty \int_0^\infty e^{-q_1 x - q_2 y - q_3 z} [f \ast\ast h](x, y, z) dx dy dz$$

Thus

$$T_{3g}^{s\alpha\beta}(f \ast\ast h)(x, y, z) = \frac{1}{p} T_{3g}^{s\alpha\beta}(f(x, y, z)) \cdot T_{3g}^{s\alpha\beta}(h(x, y, z))$$

3.3 Example :

Let $p_1 = s, p_2 = p_3 = 1$

$$q_1 = s, q_2 = \alpha, q_3 = \beta$$

Find

$$T_{3g}^{s\alpha\beta} \left(\frac{s^2}{\alpha \beta (s^2 - 1)(s - 1)(\alpha - 1)(\beta - 1)} \right)$$

Solution:

$$T_{3g}^{s\alpha\beta} \left(\frac{s^2}{\alpha \beta (s^2 - 1)(s - 1)(\alpha - 1)(\beta - 1)} \right)$$

$$= T_{3g}^{s\alpha\beta} \left(\frac{1}{s} \frac{s^3}{\alpha \beta (s^2 - 1)(s - 1)(\alpha - 1)(\beta - 1)} \right)$$

$$= T_{3g}^{s\alpha\beta} \left(\frac{1}{s} \frac{s(s\alpha\beta)}{(s^2 - 1)\alpha^2\beta^2} \cdot \frac{s}{(s - 1)(\alpha - 1)(\beta - 1)} \right)$$

Since

$$T_{3g}^{s\alpha\beta}(\cosh(x)) = \frac{s(s\alpha\beta)}{(s^2 - 1)\alpha^2\beta^2}$$

And

$$T_{3g}^{s\alpha\beta}(e^{x+y+z}) = \frac{s}{(s - 1)(\alpha - 1)(\beta - 1)}$$

Thus

$$T_{3g}^{s\alpha\beta} \left(\frac{s^2}{\alpha \beta (s^2 - 1)(s - 1)(\alpha - 1)(\beta - 1)} \right)$$

4. Applications of T_{3g} -transformation

4.1 Proposition

$$1 - T_{3g}^{s\alpha\beta}(u_x) = q_1 \overline{\overline{u}} - \overline{\overline{u}_{\alpha\beta}(0, y, z)}$$

$$2 - T_{3g}^{s\alpha\beta}(u_{xx}) = q_1^2 \overline{\overline{u}} - q_1 \overline{\overline{u}_{\alpha\beta}(0, y, z)} - \overline{\overline{(u_x)_{\alpha\beta}(0, y, z)}}$$

$$3 - T_{3g}^{s\alpha\beta}(u_x^{(n)}) = q_1^n \overline{\overline{u}} - \sum_{i=0}^{n-1} q^{n-i-1} \overline{\overline{(u_x)_{\alpha\beta}(0, y, z)}}$$

$$4 - T_{3g}^{s\alpha\beta}(u_y) = q_2 \overline{\overline{u}_y} - \overline{\overline{u}_{s\beta}(x, 0, z)}$$

$$5 - T_{3g}^{s\alpha\beta}(u_y^{(m)}) = q_2^m \overline{\overline{u}_y} - \sum_{i=0}^{m-1} q_2^{k-i-1} \overline{\overline{(u_y^{(i)})_{s\beta}(x, 0, z)}}$$

$$6 - T_{3g}^{s\alpha\beta}(u_z) = q_3 \overline{\overline{u}_z} - \overline{\overline{u}_{s\alpha}(x, y, 0)}$$

$$7 - T_{3g}^{s\alpha\beta}(u_z^{(r)}) = q_3^r \overline{\overline{u}_z} - \sum_{i=0}^{r-1} q_3^{k-i-1} \overline{\overline{(u_z^{(i)})_{s\alpha}(x, y, 0)}}$$

$$8 - T_{3g}^{s\alpha\beta}(u_{xyz}) = q_1 q_2 q_3 \overline{\overline{u}} - q_1 q_2 \overline{\overline{(u)_{s\alpha}(x, y, 0)}} - q_1 \overline{\overline{(u_z)_{s\beta}(x, 0, z)}} - \overline{\overline{(u_{yz})_{\alpha\beta}(0, y, z)}}$$

Proof:

$$1 - T_{3g}^{s\alpha\beta}(u_x) = p \int_0^\infty \int_0^\infty \int_0^\infty e^{-q_1 x - q_2 y - q_3 z} u_x dx dy dz$$

$$W = e^{-q_1 x} \rightarrow dw = -q_1 e^{-q_1 x} dx$$

$$dv = u_x dx \rightarrow v = u$$

$$T_{3g}^{s\alpha\beta}(u_x) = p \int_0^\infty \int_0^\infty e^{-q_2 y - q_3 z} [u e^{-q_1 x}]_0^\infty + q_1 \int_0^\infty u e^{-q_1 x} dx dy dz$$

$$= q_1 p \int_0^\infty \int_0^\infty \int_0^\infty e^{-q_1 x - q_2 y - q_3 z} u dx dy dz - p(s) \int_0^\infty$$

$$T_{3g}^{s\alpha\beta}(u_x) = q_1 \bar{\bar{\bar{u}}} - \overline{(u_x)_{\alpha\beta}(0, y, z)}$$

$$2 - T_{3g}^{s\alpha\beta}(u_{xx}) = p \int_0^\infty \int_0^\infty \int_0^\infty e^{-q_1x - q_2y - q_3z} u_{xx} dx dy dz$$

$$W = e^{-q_1x} \rightarrow dw = -q_1 e^{-q_1x} dx$$

$$dv = u_{xx} dx \rightarrow v = u_x$$

$$T_{3g}^{s\alpha\beta}(u_{xx}) = p \int_0^\infty \int_0^\infty e^{-q_2y - q_3z} [u_x e^{-q_1x}]_0^\infty + q_1 \int_0^\infty u_x e^{-q_1x} dy dz$$

$$= q_1 p(s) \int_0^\infty \int_0^\infty \int_0^\infty e^{-q_1x - q_2y - q_3z} u_x dx dy dz - p(s) \int_0^\infty \int_0^\infty e^{-q_2y - q_3z} u_x(0, y, z) dy dz$$

since

$$T_{3g}^{s\alpha\beta}(u_x) = q_1 \bar{\bar{\bar{u}}} - \overline{(u_x)_{\alpha\beta}(0, y, z)}$$

thus

$$T_{3g}^{s\alpha\beta}(u_{xx}) = q_1 [q_1 \bar{\bar{\bar{u}}} - \overline{(u_x)_{s23}(0, y, z)}] - \overline{(u_x)_{s23}(0, y, z)}$$

$$T_{3g}^{s\alpha\beta}(u_{xx}) = q_1^2 \bar{\bar{\bar{u}}} - q_1 \overline{(u_x)_{\alpha\beta}(0, y, z)} - \overline{(u_x)_{\alpha\beta}(0, y, z)}$$

$$3 - T_{3g}^{s\alpha\beta}(u_x^{(n)}) = q_1^n \bar{\bar{\bar{u}}} - \sum_{i=0}^{n-1} q^{n-i-1} \overline{(u_x)_{\alpha\beta}(0, y, z)}$$

We will proof by using mathematical induction

I) when n=1 we get that (3) is true by (1)

II) suppose that (3) is true when n=k

That is :

$$T_{3g}^{s\alpha\beta}(u_x^{(k)}) = q_1^k \bar{\bar{\bar{u}}} - \sum_{i=0}^{k-1} q^{k-i-1} \overline{(u_x^{(i)})_{\alpha\beta}(0, y, z)}$$

III) let n=k+1 , and let w=u_x^{(k)}, w_x=u_x^{(k+1)}

$$T_{3g}^{s\alpha\beta}(u_x^{(k+1)}) = T_{3g}^{s\alpha\beta}(w_x) = q_1 \bar{\bar{\bar{w}}} - \overline{w_{\alpha\beta}(0, y, z)}$$

$$= q_1 [q_1^k \bar{\bar{\bar{u}}} - \sum_{i=0}^{k-1} q_1^{k-i-1} \overline{(u_x^{(i)})_{\alpha\beta}(0, y, z)}] - \overline{(u_x)_{\alpha\beta}(0, y, z)}$$

$$= q_1^{k+1} \bar{\bar{\bar{u}}} - p \sum_{i=0}^{k+1-1} q_1^{k+1-i-1} \overline{(u_x^{(i)})_{\alpha\beta}(0, y, z)}$$

There of the fact is true for all n $\in \mathbb{Z}^+$

There of the fact is true for all n $\in \mathbb{Z}^+$

$$4 - T_{3g}^{s\alpha\beta}(u_y) = p \int_0^\infty \int_0^\infty \int_0^\infty e^{-q_1x - q_2y - q_3z} u_y dx dy dz$$

$$W = e^{-q_2y} \rightarrow dw = -q_2 e^{-q_2y} dy$$

$$dv = u_y dy \rightarrow v = u$$

$$T_{3g}^{s\alpha\beta}(u_y) = p \int_0^\infty \int_0^\infty e^{-q_1x - q_3z} [u e^{-q_2y}]_0^\infty + q_2 \int_0^\infty u e^{-q_2y} dy dx dz$$

$$= q_2 p \int_0^\infty \int_0^\infty e^{-q_1x - q_3z} u(x, 0, z) dy dz$$

$$T_{3g}^{s\alpha\beta}(u_y) = q_2 T_{3g}^{s\alpha\beta}(u) - \overline{u_{s\beta}(x, 0, z)}$$

$$T_{3g}^{s\alpha\beta}(u_y) = q_2 \bar{\bar{\bar{u}}} - \overline{u_{s\beta}(x, 0, z)}$$

5- We will proof by using mathematical induction

I) when m=1 we get that (5) is true by (4)

II) suppose that (5) is true when m=k

That is :

$$T_{3g}^{s\alpha\beta}(u_y^{(k)}) = q_2^k \bar{\bar{\bar{u}}} - \sum_{i=0}^{k-1} q_2^{k-i-1} \overline{(u_y^{(i)})_{s\beta}(x, 0, z)}$$

III) When m=k+1 and let w=u_y^{(k)}, w_y=u_y^{(k+1)}

$$T_{3g}^{s\alpha\beta}(u_y^{(k+1)}) = T_{3g}^{s\alpha\beta}(w_y) = q_2 \bar{\bar{\bar{u}}} - \overline{w_{s\beta}(x, 0, z)}$$

$$= q_2 [q_2^k \bar{\bar{\bar{u}}} - \sum_{i=0}^{k-1} q_2^{k-i-1} \overline{(u_y^{(i)})_{s\beta}(x, 0, z)}] - \overline{(u_y)_{s\beta}(x, 0, z)}$$

$$= q_2^{k+1} \bar{\bar{\bar{u}}} - \sum_{i=0}^{k+1-1} q_2^{k+1-i-1} \overline{(u_y^{(i)})_{s\beta}(x, 0, z)}$$

Therefore (5) is true for all m $\in \mathbb{Z}^+$

$$6 - T_{3g}^{s\alpha\beta}(u_z) = p \int_0^\infty \int_0^\infty e^{-q_1x - q_2y} [\int_0^\infty e^{-q_3z} u_z dz] dx dy$$

$$W = e^{-q_3z} \rightarrow dw = -q_3 e^{-q_3z} dz$$

$$dv = u_z dz \rightarrow v = u$$

$$T_{3g}^{s\alpha\beta}(u_z) = p \int_0^\infty \int_0^\infty e^{-q_1x - q_2y} [u e^{-q_3z}]_0^\infty + q_3 \int_0^\infty u e^{-q_3z} dz dx dy$$

$$= q_3 p \int_0^\infty \int_0^\infty e^{-q_1x - q_2y - q_3z} u dx dy dz - p(s) \int_0^\infty \int_0^\infty e^{-q_1x - q_2y} u(x, y, 0) dx dy$$

$$T_{3g}^{s\alpha\beta}(u_z) = q_3 T_{3g}^{s\alpha\beta}(u) - \overline{u_{s\alpha}(x, y, 0)}$$

$$T_{3g}^{s\alpha\beta}(u_z) = q_3 \overline{\overline{\overline{u}_z}} - \overline{\overline{u}_{s\alpha}(x, y, 0)}$$

7- We will proof by using mathematical induction

I) when r=1 we get that (7) is true by

II) suppose that (7) is true when r=k

That is :

$$T_{3g}^{s\alpha\beta}(u_z^{(k)}) = q_3^k \overline{\overline{\overline{u}_z}} - \sum_{i=0}^{k-1} q_3^{k-i-1} \overline{(u_z^{(i)})_{s\alpha}(x, y, 0)}$$

III) let r=k+1 , and let w=u_z^{(k)} , w_z=u_z^{(k+1)}

$$\begin{aligned} T_{3g}^{s\alpha\beta}(u_z^{(k+1)}) &= T_{3g}^{s\alpha\beta}(w_z) = q_3 \overline{\overline{\overline{u}_z}} - \overline{\overline{w}_{s\alpha}(x, y, 0)} \\ &= q_3 \left[q_3^k \overline{\overline{\overline{u}_z}} - \sum_{i=0}^{k-1} q_3^{k-i+1} \overline{(u_z^{(i)})_{s\alpha}(x, y, 0)} \right] - \\ &\quad \overline{(u_z^{(k)})_{s\alpha}(x, y, 0)} \\ &= q_3^{k+1} \overline{\overline{\overline{u}_z}} - \sum_{i=0}^{k+1-1} q_3^{k+1-i+1} \overline{(u_z^{(i)})_{s\alpha}(x, y, 0)} \end{aligned}$$

Therefore (7) is true for all r $\in \mathbf{Z}^+$

$$\begin{aligned} 8- T_{3g}^{s\alpha\beta}(u_{xyz}) &= p \int_0^\infty \int_0^\infty \int_0^\infty u_{xyz} e^{-q_1 x - q_2 y - q_3 z} dx dy dz \\ &= p \int_0^\infty \int_0^\infty e^{-q_2 y - q_3 z} [[u_{yz} e^{-q_1 x}]_0^\infty + q_1 \int_0^\infty u_{yz} e^{-q_1 x} dx] dy dz \\ &= q_1 p \int_0^\infty \int_0^\infty \int_0^\infty u_{yz} e^{-q_1 x - q_2 y - q_3 z} dx dy dz - \overline{(u_{yz})_{\alpha\beta}(0, y, z)} \\ &= q_1 p \left[\int_0^\infty \int_0^\infty e^{-q_1 x - q_3 z} [[u_z e^{-q_2 y}]_0^\infty \right. \\ &\quad \left. + q_2 \int_0^\infty u_z e^{-q_2 y} dy] dx dz \right] \\ &\quad - \overline{(u_{yz})_{\alpha\beta}(0, y, z)} \\ &= q_1 q_2 p \int_0^\infty \int_0^\infty \int_0^\infty u_z e^{-q_1 x - q_2 y - q_3 z} dx dy dz - q_1 \overline{(u_z)_{s\beta}(x, 0, z)} \\ &\quad - \overline{(u_{yz})_{\alpha\beta}(0, y, z)} \end{aligned}$$

$$\begin{aligned} &= q_1 q_2 p(s) \left[\int_0^\infty \int_0^\infty e^{-q_1 x - q_2 y} [[u e^{-q_3 z}]_0^\infty \right. \\ &\quad \left. + q_3 \int_0^\infty u e^{-q_3 z} dz] dx dy \right] - q_1 \overline{(u_z)_{s\beta}(x, 0, z)} \\ &\quad - \overline{(u_{yz})_{\alpha\beta}(0, y, z)} \end{aligned}$$

$$\begin{aligned} T_{3g}^{s\alpha\beta}(u_{xyz}) &= q_1 q_2 q_3 \overline{\overline{\overline{u}}} - q_1 q_2 \overline{(u)_{s\alpha}(x, y, 0)} \\ &\quad - q_1 \overline{(u_z)_{s\beta}(x, 0, z)} - \overline{(u_{yz})_{\alpha\beta}(0, y, z)} \end{aligned}$$

4.2 Example :

Solve the following equation

$$u_y = 1 , u(x, 0, z) = 1$$

Solution:

By taking $T_{3g}^{s\alpha\beta}$ - transformation for the equation we get

$$T_{3g}^{s\alpha\beta}(u_x) = T_{3g}^{s\alpha\beta}(1)$$

$$q_2 \overline{\overline{\overline{u}}} - \overline{u_{s\beta}(x, 0, z)} = \frac{p}{q_1 q_2 q_3}$$

We note that :

$$\overline{u_{s\beta}(x, 0, z)} = p \int_0^\infty \int_0^\infty u(x, 0, z) e^{-q_1 y - q_3 z} dy dz = \frac{p}{q_1 q_3}$$

Thus

$$q_2 \overline{\overline{\overline{u}}} - \frac{p}{q_1 q_3} = \frac{p}{q_1 q_2 q_3}$$

$$q_2 \overline{\overline{\overline{u}}} = \frac{q_2 p + p}{q_1 q_2 q_3} \rightarrow \overline{\overline{\overline{u}}} = \frac{p(q_2 + 1)}{q_2^2 q_1 q_3}$$

We take $T_{3g}^{s\alpha\beta^{-1}}$ - transformation for both sides , we get :

$$\begin{aligned} u(x, y, z) &= T_{3g}^{s\alpha\beta^{-1}} \left(\frac{p(q_2 + 1)}{q_2^2 q_1 q_3} \right) = T_{3g}^{s\alpha\beta^{-1}} \left(\frac{p}{q_2^2 q_1 q_3} + \frac{p}{q_1 q_2 q_3} \right) \\ &= T_{3g}^{s\alpha\beta^{-1}} \left(\frac{p}{q_2^2 q_1 q_3} \right) + T_{3g}^{s\alpha\beta^{-1}} \left(\frac{p}{q_1 q_2 q_3} \right) \\ &= x^0 y^0 z^0 + x^1 y^0 z^0 = 1 + x \end{aligned}$$

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