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Tripolar Intuitionistic Fuzzy RG-ideals of RG-algebra

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***Abstract**: The purpose of this paper is to introduce the concept of tripolar on intuitionistic fuzzy of RG-algebra, as well as to state and prove various theorems and properties. The tripolar intuitionistic fuzzy RG-algebras and tripolar intuitionistic fuzzy RG-ideals are also investigated for their fuzzy relations.*

Keywords— RG-algebra, intuitionistic fuzzy RG-subalgebra, intuitionistic fuzzy RG-ideal, tripolar intuitionistic fuzzy RG-subalgebra, tripolar intuitionistic fuzzy RG-ideal, the homeomorphic of them.

1. Introduction

After the introduction of fuzzy sets by Zadeh [18], there have been a lot of generalizations of this core concept. The notion of intuitionistic fuzzy sets developed by Atanassov [1-4] is one among them.Fuzzy sets give a degree of membership of an element in a given set, while intuitionistic fuzzy sets give both degrees of membership and of non membership.

Both degrees belong to the interval [$\beth$,1], and their sum should not exceed 1. It is known that the class of BCK-algebra is a proper subclass of the class of BCI-algebras. Recently, Senapati and et al. [16-17] introduced intuitionistic fuzzy H-ideals in BCK-algebras. The concept of fuzzy translations in fuzzy subalgebras and ideals in BCK/BCI-algebras has been discussed respectively. R.A.K. Omar, [14] have developed the notion of RG-algebras, RG-ideals, RG-subalgebras and examined the relations among them and P. Patthanangkoor, offered the concept of homomorphism of RG-algebras and investigated certain associated properties,[15]. A.T. Hameed and et al. proposed and investigated new concepts of fuzzy RG-subalgebras and fuzzy RG-ideals of RG-algebra and investigate some of its features, several theorems, properties are stated and proved in [9]. A.T. Hameed and S.M. Abrahem introduced the notion of doubt fuzzy RG-ideals of RG-algebras and studied the homomorphism image and inverse image of doubt fuzzy RG-ideals also prove that the Cartesian product of doubt fuzzy RG-ideals are doubt fuzzy RG-ideals in [10]. A.T. Hameed and et al. introduced and studied new concepts of intuitionistic fuzzy RG-subalgebras and intuitionistic fuzzy RG-ideals of RG-algebra and investigate some of its properties, several theorems, properties are stated and proved in [11]. A.T. Hameed and S.M. Abrahem introduced the notion of tripolar fuzzy RG-ideals of RG-algebras and studied the homomorphism image and inverse image of tripolar fuzzy RG-ideals also prove that the Cartesian product of tripolar fuzzy RG-ideals are tripolar fuzzy RG-ideals in [12].

2. Preliminaries

As a prelude to the following sections, we provide some definitions and preliminary findings here.

Definition 2.1. [14]: An algebra $(X;*, \beth)$ is referred to as RG-algebra if All of the following conditions are met: $\forall j, y, z \in X$,

(i) $j * \beth = j$,

(ii) $j * y = (j * z) * (y * z)$,

(iii) $j * y = y * j = \beth$ imply $j = y$.

Remark 2.2. [14]: In $(X;*, \beth)$ an RG-algebra, As previously stated, there *is a* definition. a binary relation $(\leq)$ by putting $j \leq y \leftrightarrow j*y = \beth$.

Definition 2.3.[9]:

Let $(X;*, \beth)$ *be an* RG-algebra and S be a nonempty subset of X .Then S is known an RG-**subalgebra of X** if $j*y \in S$, for any $j, y \in S$.

Definition 2.4. [14,15]: Let $(X;*, \beth)$ *be an* RG-algebra, a nonempty subset Iof X is called **an RG-ideal of X** if $\forall j, y \in X$

i) $\beth \in I$,

ii) $j * y \in I$ and $\beth * j \in I$ imply $\beth * y \in I$.

Proposition 2.5. [14,15]: In an RG-algebra $(X;*, \beth)$, every RG-ideal *is a* RG-subalgebra of X.

Proposition 2.6. [14]: In any RG-algebra $(X;*, \beth)$, the following hold: $\forall j, y, z \in X$,

i) $j * j = \beth$,

ii) $\beth * (\beth * j) = j$,

iii) $j * (j * y) = y$,

iv) $j * y = \beth \leftrightarrow y * j = \beth$,

v) $j * \beth = \beth$ implies $j = \beth$,

vi) $\beth * (y * j) = j * y$.

Theorem 2.7. [15]:

If h: $(X; *, ב) \to (Y; *`, ב`)$ *is a* homomorphism of an RG-algebras respectively X ,Y, then

1) h(ב) = ב′.

2) h is injective ↔ if ker h ={ ב}.

Definition 2.8. [17]:

For any t∈ [ב,1] and a fuzzy subset μ of a nonempty set **X**, the set

U(μ, t) ={j ∈ X | μ(j) ≥t} is known to as **an upper level cut of μ**, and the set

L(μ, t) ={j∈X | μ(j) ≤t} is known to as **lower level cut of μ.**

Definition 2.9.[9]:

Let (X;*, ב) *be an* RG-algebra; a fuzzy subset μ of X *is called* a **fuzzy RG-subalgebra of X**, if ∀y, j ∈ X, μ (j * y) ≥ min {μ (j), μ (y)} .

Definition 2.10.[9]:

Let (**X**;∗, **0**) **be an** RG-algebra, a fuzzy subset **μ** of **X** is called a **fuzzy RG-ideal of X** if It meets the following requirements.: ∀**y**, **x**∈**X**,

(i) μ (ב) ≥ μ (j) ,

(v) μ (ב ∗ y) ≥ min {μ (j * y), μ (ב ∗ j)} .

Proposition 2.11. [9] :

Every fuzzy RG-ideal of RG-algebra (X; ∗ , ב) is a fuzzy RG-*subalgebra* of X.

Proposition 2.12.[9]:

1- The intersection of any set of fuzzy RG-subalgebras of RG-algebra (X; ∗ , ב) is also fuzzy RG-subalgebra of X.
2- The union of any set of fuzzy RG-subalgebras of RG-algebra is also fuzzy RG-subalgebra, where is chain (Noetherian).
3- The intersection of any set of fuzzy RG-ideals of RG-algebra (X; ∗ , ב) is also fuzzy RG-ideal of X.
4- The union of any set of fuzzy RG-ideals of RG-algebra is also fuzzy RG-ideal, where is chain (Noetherian).

Definition 2.13.[1,2]:

Let h :(J; ∗, ב)→(Y; ∗ ′, ב') be a homeomorphism from *the set* X into the set Y. If μ is a fuzzy subset of X, then *the fuzzy* subset h(μ) in Y defined by:

h
(μ)(q)=

$$\begin{cases} \sup\{\mu(j) : j \in h^{-1}(q)\} & \text{if } h^{-1}(q) = \{j \in X, h(j) = q\} \neq \emptyset \\ ב & \text{otherwies} \end{cases}$$

It's alleged that **the image of μ under f**.

Definition 2.14.[8]:

1) A fuzzy subset μ of algebra (X; ∗ , ב) has inf property if for any subset T of X, there exist $t_0 \in$ T such that μ (t_0) = $\inf_{t\in T} \mu(t)$.

2) A fuzzy subset μ of algebra (X; ∗ , ב) has inf property if for any subset T of X, there exist $t_0\in$ T such that μ(t ב) = sup {μ(t)| t∈T}.

Definition 2.15.[10]:

Let (X; * , ב) *be an* RG-algebra. μ *be a* fuzzy subset of X**,** μ is known *as a* **doubt fuzzy RG-subalgebra of X** if ∀j, y ∈ X μ(j * y) ≤ max{μ(j), μ(y)},

Definition 2.16.[10]:

Let (**X;*, 0**) ***be an*** RG-algebra, a fuzzy subset **μ** of **X** is known as a **doubt fuzzy RG-ideal of X** if ∀**y**, **x∈X**,

1. μ(ב)≤ μ(j).

2.μ(ב ∗ y) ≤ max{μ(j*y), μ(ב*j)}.

Proposition 2.17.[10]:

Every doubt fuzzy RG-ideal of RG-algebra (X; ∗ , ב) is a doubt fuzzy RG-subalgebra of X.

Definition 2.18.[1]:

If $A=\{(j, \mu_A(j), \nu_A(j)) | j\in X\}$ and $B =\{(j, \mu_B(j), \nu_B(j)) | j\in X\}$ are two intuitionistic fuzzy subsets of X, then

1)$A\subseteq B$ if and only if j ∈ X, $\mu_A(j)\leq\mu_B(j)$ and $\nu_A(j)\geq\nu_B(j)$.

2)$A = B$ iff $j \in X, \mu_A(j)=\mu_B(j)$ and $\nu_A(j) = \nu_B(j)$.

3)$A \cap B = \{(j,(\mu_A \cap \mu_B)(j), (\nu_A \cup \nu_B)(j) | j \in X\}$.

4)$A \cup B = \{(j,(\mu_A \cup \mu_B)(j), (\nu_A \cap \nu_B)(j) | j \in X\}$.

Proposition 2.19.[10]:

1- The intersection *of any* set of doubt fuzzy RG-subalgebras of RG-algebra (X; ∗ , ב) i*s also* doubt fuzzy RG-subalgebra of X , where is chain (Arterian).
2- The union of any set of doubt fuzzy RG-subalgebras of RG-algebra is also doubt fuzzy RG-subalgebra**.**
3- The intersection of any set of doubt fuzzy RG-ideals of RG-algebra (X; *, ב) *is also* doubt fuzzy RG-ideal of X, where is chain (Arterian).
4- The union of any set of doubt fuzzy RG-ideals of RG-algebra is also doubt fuzzy RG-ideal.

Definition 2.20[11].

Let $A=\{(j, \mu_A(j), \nu_A(j)) | j\in X\}$ *be an* intuitionistic fuzzy subset of

RG-algebra (X; ∗ , ב). A is called to *be an* **intuitionistic fuzzy RG-subalgebra of X** if

(IFS_1) $\mu_A(j*y)\geq \min\{\mu_A(j), \mu_A(y)\}$,

(IFS_2) $\nu_A(j*y) \leq \max\{\nu_A(j), \nu_A(y)\}$.

That mean μ_A *is a* fuzzy RG-subalgebra and ν_A *is a* doubt fuzzy RG-subalgebra.

Proposition 2.21[11].

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An intuitionistic *fuzzy subset* $A=\{(j,\mu_A(j),\ \nu_A(j))|\ j\in X\}$ *is an* intuitionistic fuzzy RG- subalgebra of RG-algebra (X; * , ב), if any $t\in[$ ב$,1]$, *the set* $U(\mu_A,t)$ and $L(\nu_A,s)$ are RG-subalgebras.

Proposition 2.22[11].

In an intuitionistic fuzzy subalgebra $A=\{(j,\mu_A(j),\nu_A(j))|\ j\in X\}$ if the upper level and lower level of (X; * , ב), are RG-subalgebra, for all $t\in[$ ב$,1]$, then A *is an* intuitionistic fuzzy RG-subalgebra of X.

Definition 2.23[11].

Let $A=\{(j,\mu_A(j),\ \nu_A(j))|\ j\in X\}$ *be an* intuitionistic *fuzzy subset* of RG-algebra (X; * , ב). A *is said* to *be* ***an* intuitionistic fuzzy RG-ideal of X** if, ∀y,j ∈X, then

1) $\mu_A($ ב$)\geq\mu_A(j)$ and $\nu_A($ ב$)\leq\nu_A(j)$,
2) $\mu_A($ ב $*\,y)\geq\min\{\mu_A(j*y),\mu_A($ ב $*\,j)\}$ and $\nu_A($ ב $*\,y)\leq\max\{\nu_A(j*y),\nu_A($ ב$*j)\}$.

That means μ_A is a fuzzy RG-ideal and ν_A *is a* doubt fuzzy RG-ideal.

Proposition 2.24[11].

If an intuitionistic fuzzy subset $A=\{(j,\mu_A(j),\nu_A(j))|\ j\in X\}$ *is an* intuitionistic fuzzy RG- ideal of RG-algebra (X; *, ב), then for any $t\in[$ ב$,1]$, *the set* $U(\mu_A,t)$ and $L(\nu_A,s)$ are RG-ideals of X.

Proposition 2.25[11].

If An intuitionistic *fuzzy subset* $A=\{(j,\mu_A(j),\nu_A(j))|\ j\in X\}$ the sets $U(\mu_A,t)$ and $L(\nu_A,s)$ are RG-ideals of RG-algebra (X; * , ב), for all $t\in[$ ב$,1]$, then A *is an* intuitionistic fuzzy subset is intuitionistic fuzzy RG-ideal of RG-algebra X.

Proposition 2.26[11].

Let $A=\{(j,\mu_A(j),\nu_A(j))|\ j\in X\}$ *be an* intuitionistic fuzzy RG-ideal of RG-algebra (X; * , ב), then A *is an* intuitionistic fuzzy RG-subalgebra of X.

Definition 2.27. [1]:

A mapping h: (X;*, ב) → (Y;∗, ב) *be a* homeomorphism of BCK-algebra for any

IFSA=$\{(j,\mu_A(j),\nu_A(j))|\ j\in X\}$ in Y, we define

$$\text{IFS } A^h=\left\{\left(j,\mu_A^h(j),\nu_A^h(j)\right)\middle|\, j\in X\right\} \text{ in X by } \mu_A^h(j)=\mu_A(h(j)),$$

$$\nu_A^h(j)=\nu_A(h(j)),\ \forall j\in X.$$

Definition 2.28. [12]:

Let $\Psi=\{(j,\mu_\Psi^N(j),\mu_\Psi^P(j),\tau_\Psi^P(j)):j\in X\}$ *is a* tripolar fuzzy subset of X and $r,t\in[$ ב$,1]$, $k\in[-1,$ ב$]$ the set $\Psi^{k,r,t}=\{j\in X:\mu_\Psi^N(j)\leq k,\mu_\Psi^P(j)\geq r,\tau_\Psi^P(j)\geq t\}$ is said to be **(k,r,t) −tri –cut set of** $\Psi=\left(X:\mu_\Psi^N,\mu_\Psi^P,\tau_\Psi^P\right)$.

Definition 2.29. [12]:

A tripolar fuzzy subset $\Psi=\{(j,\mu_\Psi^N(j),\mu_\Psi^P(j),\tau_\Psi^P(j)):j\in X\}$ of X *is* ***A* tripolar fuzzy subalgebra of X** if it satisfies the following for all $j,y\in X$

(TS₁) $\mu_\Psi^N(j*y)\leq\max\{\mu_\Psi^N(j),\mu_\Psi^N(y)\}$,

(TS₂) $\mu_\Psi^P(j*y)\geq\min\{\mu_\Psi^P(j),\mu_\Psi^P(y)\}$,

(TS₃) $\tau_\Psi^P(j*y)\leq\max\{\tau_\Psi^P(j),\tau_\Psi^P(y)\}$.

Theorem 2.30. [12]:

Let $\Psi=\{(j,\mu_\Psi^N(j),\mu_\Psi^P(j),\tau_\Psi^P(j)):j\in X\}$ *be a* tripolar fuzzy subset of RG-algebra X . If Ψ is a tripolar fuzzy subalgebra of X, then for any $r,t\in[$ ב$,1]$, $k\in[-1,$ ב$]$, $\Psi^{k,r,t}$ *is a* subalgebra of X .

Theorem 2.31. [12]:

Let $\Psi=\{(j,\mu_\Psi^N(j),\mu_\Psi^P(j),\tau_\Psi^P(j)):j\in X\}$ *be a* tripolar fuzzy subset of RG-algebra X .if $\Psi^{k,r,t}$ *is a* subalgebra of X for every $r,t\in[$ ב$,1]$, $k\in[-1,$ ב$]$,then Ψ *is a* tripolar fuzzy subalgebra of X

Definition 2.32. [12]:

A tripolar fuzzy subset $\Psi=\{(j,\mu_\Psi^N(j),\mu_\Psi^P(j),t_\Psi^P(j)):j\in X\}$ of X *is* ***A* tripolar fuzzy RG-ideal of X** if it satisfies the following for all $j,y\in X$

(TI₁) $\mu_\Psi^N($ ב$)\leq\mu_\Psi^N(j)$ and $\mu_\Psi^N($ ב $*\,y)\leq\max\{\mu_\Psi^N(j*y),\mu_\Psi^N($ ב $*\,j)\}$,

(TI₂) $\mu_\Psi^P($ ב$)\geq\mu_\Psi^P(j)$ and $\mu_\Psi^P($ ב $*\,y)\geq\min\{\mu_\Psi^P(j*y),\mu_\Psi^P($ ב $*\,j)\}$,

(TI₃) $\tau_\Psi^P($ ב$)\leq\tau_\Psi^P(j)$ and $\tau_\Psi^P($ ב $*\,y)\leq\max\{\tau_\Psi^P(j*y),\tau_\Psi^P($ ב $*\,j)\}$.

Theorem 2.33. [12]:

Let $\Psi=\{(j,\mu_\Psi^N(j),\mu_\Psi^P(j),\tau_\Psi^P(j)):j\in X\}$ *be a* tripolar fuzzy subset of RG- algebra X. If Ψ is a tripolar fuzzy RG-ideal of X, then for any $r,t\in[$ ב$,1]$, $k\in[-1,$ ב$]$, $\Psi^{k,r,t}$ *is an* RG-ideal of X .

Theorem 2.34. [12]:

Let $\Psi=\{(j,\mu_\Psi^N(j),\mu_\Psi^P(j),\tau_\Psi^P(j)):j\in X\}$ *be a* tripolar fuzzy subset of RG- algebra X.if $\Psi^{k,r,t}$ is an RG-ideal of X for all $r,t\in[$ ב$,1]$, $k\in[-1,$ ב$]$.

Then Ψ *is a* tripolar fuzzy RG-ideal of X .

X	ב	1	2	3
$\mu^N_{\Psi 1}$	-0.7	-0.4	-0.4	-0.4
$\mu^P_{\Psi 1}$	0.4	0.2	0.2	0.2
$\tau^P_{\Psi 1}$	0.1	0.3	0.3	0.3
$v^N_{\Psi 2}$	0.5	0.3	0.3	0.3
$v^P_{\Psi 2}$	-0.6	-0.3	-0.3	-0.3
$\sigma^P_{\Psi 2}$	0.3	0.2	0.2	0.2

Proposition 2.35. [12]:

Every tripolar fuzzy RG-ideal of RG-algebra (X;∗, ב) *is a* tripolar fuzzy subalgebra of X**.**

3. Tripolar Intuitionistic Fuzzy RG-subalgebras of RG-algebra

In this section, we give the topic of a tripolar intuitionistic fuzzy RG-subalgebras of RG-algebra X.

Definition 3.1.

Let $A=\{(j, \mu_A(j), \nu_A(j)) | j\in X\}$ *be an* intuitionistic fuzzy subset of RG-algebra (X; ∗ , ב), a tripolar fuzzy subset $\Psi_1 = \{(j, \mu^N_{\Psi 1}(j), \mu^P_{\Psi 1}(j), \tau^P_{\Psi 1}(j)) : j \in X\}$ of X and a tripolar doubt fuzzy subset $\Psi_2 = \{(j, v^N_{\Psi 2}(j), v^P_{\Psi 2}(j), \sigma^P_{\Psi 2}(j)) : j \in X\}$ of X . Then a fuzzy subset $\Phi = \{(j, \mu^N_{\Psi 1}(j), \mu^P_{\Psi 1}(j), \tau^P_{\Psi 1}(j), v^N_{\Psi 2}(j), v^P_{\Psi 2}(j), \sigma^P_{\Psi 2}(j)) : j \in X\}$ is said to *be a* **tripolar intuitionistic fuzzy RG-subalgebra of X** denoted by $\Phi = (X: \mu^N_{\Psi 1}, \mu^P_{\Psi 1}, \tau^P_{\Psi 1}, v^N_{\Psi 2}, v^P_{\Psi 2}, \sigma^P_{\Psi 2})$ if it satisfies the following for all $y, j \in X$,

(TIFS₁) $\mu^N_{\Psi 1}(j * y) \le \max\{\mu^N_{\Psi 1}(j), \mu^N_{\Psi 1}(y)\}$ and $v^N_{\Psi 2}(j * y) \ge \min\{v^N_{\Psi 2}(j), v^N_{\Psi 2}(y)\}$,

(TIFS₂) $\mu^P_{\Psi 1}(j * y) \ge \min\{\mu^P_{\Psi 1}(j), \mu^P_{\Psi 1}(y)\}$ and $v^P_{\Psi 2}(j * y) \le \max\{v^P_{\Psi 2}(j), v^P_{\Psi 2}(y)\}$,

(TIFS₃) $\tau^P_{\Psi 1}(j * y) \le \max\{\tau^P_{\Psi 1}(j), \tau^P_{\Psi 1}(y)\}$ and $\sigma^P_{\Psi 2}(j * y) \ge \min\{\sigma^P_{\Psi 2}(j), \sigma^P_{\Psi 2}(y)\}$.

Example 3.2.

Let X = { ב, 2, 1, 3} in which ∗ is defined , as seen here,:

*	ב	1	2	3
ב	ב	1	2	3
1	1	ב	3	2
2	2	3	ב	1
3	3	2	1	ב

We determine the tripolar fuzzy subset of $\Phi = (X: \mu^N_{\Psi 1}, \mu^P_{\Psi 1}, \tau^P_{\Psi 1}, v^N_{\Psi 2}, v^P_{\Psi 2}, \sigma^P_{\Psi 2})$ by:

Then $\Phi = (X: \mu^N_{\Psi 1}, \mu^P_{\Psi 1}, \tau^P_{\Psi 1}, v^N_{\Psi 2}, v^P_{\Psi 2}, \sigma^P_{\Psi 2})$ is a tripolar intuitionistic fuzzy RG-subalgebra of X.

Proposition 3.3.

Every tripolar intuitionistic fuzzy RG-subalgebra $\Phi = (X: \mu^N_{\Psi 1}, \mu^P_{\Psi 1}, \tau^P_{\Psi 1}, v^N_{\Psi 2}, v^P_{\Psi 2}, \sigma^P_{\Psi 2})$ of RG-algebra (X; ∗ , ב), satisfies the inequalities

$\mu^N_{\Psi 1}(ב) \le \mu^N_{\Psi 1}(j)$, $\mu^P_{\Psi 1}(ב) \ge \mu^P_{\Psi 1}(j)$, $\tau^P_{\Psi 1}(ב) \le \tau^P_{\Psi 1}(j)$ and

$v^N_{\Psi 2}(ב) \ge v^N_{\Psi 2}(j)$, $v^P_{\Psi 2}(ב) \le v^P_{\Psi 2}(j)$, $\sigma^P_{\Psi 2}(ב) \ge \sigma^P_{\Psi 2}(j)$,

for all j ∈X.

Proof.

For any j ∈ X, we have

$\mu^N_{\Psi 1}(ב) = \mu^N_{\Psi 1}(j * j) \le \max\{\mu^N_{\Psi 1}(j), \mu^N_{\Psi 1}(j)\} = \mu^N_{\Psi 1}(j)$

$\mu^P_{\Psi 1}(ב) = \mu^P_{\Psi 1}(j * j) \ge \min\{\mu^P_{\Psi 1}(j), \mu^P_{\Psi 1}(j)\} = \mu^P_{\Psi 1}(j)$ and

$\tau^P_{\Psi 1}(ב) = \tau^P_{\Psi 1}(j * j) \le \max\{\tau^P_{\Psi 1}(j), \tau^P_{\Psi 1}(j)\} = \tau^P_{\Psi 1}(j)$.

Thus

$v^N_{\Psi 2}(ב) = v^N_{\Psi 2}(j * j) \ge \min\{v^N_{\Psi 2}(j), v^N_{\Psi 2}(j)\} = v^N_{\Psi 2}(j)$

$v^P_{\Psi 2}(ב) = v^P_{\Psi 2}(j * j) \le \max\{v^P_{\Psi 2}(j), v^P_{\Psi 2}(j)\} = v^P_{\Psi 2}(j)$ and

$\sigma^P_{\Psi 2}(ב) = \sigma^P_{\Psi 2}(j * j) \ge \min\{\sigma^P_{\Psi 2}(j), \sigma^P_{\Psi 2}(j)\} = \sigma^P_{\Psi 2}(j)$. ■

Proposition 3.4.

An tripolar intuitionistic fuzzy subset $\Phi = (X: \mu^N_{\Psi 1}, \mu^P_{\Psi 1}, \tau^P_{\Psi 1}, v^N_{\Psi 2}, v^P_{\Psi 2}, \sigma^P_{\Psi 2})$ is an tripolar intuitionistic fuzzy RG-subalgebra of RG-algebra (X; ∗ , ב), if for any t, s ∈ [ב,1], the sets $U(\mu^P_{\Psi 1}, t)$, $U(v^N_{\Psi 2}, t)$, $U(\sigma^P_{\Psi 2}, t)$ are RG-subalgebras of X and $L(\mu^N_{\Psi 1}, s)$, $L(\tau^P_{\Psi 1}, s)$, $L(v^P_{\Psi 2}, s)$ are RG-subalgebras of X.

Proof

Let $\Phi = (X: \mu^N_{\Psi 1}, \mu^P_{\Psi 1}, \tau^P_{\Psi 1}, v^N_{\Psi 2}, v^P_{\Psi 2}, \sigma^P_{\Psi 2})$ *be a* tripolar intuitionistic fuzzy RG-subalgebra of X and sets

$U(\mu^{P}_{\Psi1},t)$, $U(v^{N}_{\Psi2},t)$, $U(\sigma^{P}_{\Psi2},t)$ and $L(\mu^{N}_{\Psi1},s)$, $L(\tau^{P}_{\Psi1},s)$, $L(v^{P}_{\Psi2},s)$ are nonempty sets.

If follows that for $j \in U(\mu^{P}_{\Psi1},t)$, $y \in U(\mu^{P}_{\Psi1},t)$,then $\mu^{P}_{\Psi1}(j) \geq t, \mu^{P}_{\Psi1}(y) \geq t$ which follow $\mu^{P}_{\Psi1}(j*y) \geq \min\{\mu^{P}_{\Psi1}(j),\mu^{P}_{\Psi1}(y)\} \geq t$, So that $j*y \in U(\mu^{P}_{\Psi1},t)$.

Hence $U(\mu^{P}_{\Psi1},t)$ is an RG-subalgebra of X. Similarity, $U(v^{N}_{\Psi2},t)$ and $U(\sigma^{P}_{\Psi2},t)$ are an RG-subalgebras of X.

We prove that $L(\mu^{N}_{\Psi1},s)$ is an RG-subalgebra of X.

$j \in L(\mu^{N}_{\Psi1},s)$ and $y \in L(\mu^{N}_{\Psi1},s)$ and $\mu^{N}_{\Psi1}(j) \leq s$ and $\mu^{N}_{\Psi1}(y) \leq s$.

If follows that $\mu^{N}_{\Psi1}(j*y) \leq \max\{\mu^{N}_{\Psi1}(j),\mu^{N}_{\Psi1}(y)\} \leq s$, So that $j*y \in L(\mu^{N}_{\Psi1},s)$.

Hence $L(\mu^{N}_{\Psi1},s)$ is an RG-subalgebra of X. Similarity, $U(\tau^{P}_{\Psi1},s)$ and $U(v^{P}_{\Psi2},s)$ are an RG-subalgebras of X. ■

Proposition 3.5.

In a tripolar intuitionistic fuzzy subset $\Phi = (X: \mu^{N}_{\Psi1}, \mu^{P}_{\Psi1}, \tau^{P}_{\Psi1}, v^{N}_{\Psi2}, v^{P}_{\Psi2}, \sigma^{P}_{\Psi2})$, if the upper level and lower level of (X; * , ℶ), are RG-subalgebras, for all t, s ∈ [ℶ,1], then Φ is a tripolar intuitionistic fuzzy RG-subalgebra of X.

Proof

Assume that for each t, s ∈ [ℶ,1], the sets $U(\mu^{P}_{\Psi1},t)$, $U(v^{N}_{\Psi2},t)$, $U(\sigma^{P}_{\Psi2},t)$ and $L(\mu^{N}_{\Psi1},s)$, $L(\tau^{P}_{\Psi1},s)$ $L(v^{P}_{\Psi2},s)$ are RG-subalgebras of X. If there exist ǰ, ý ∈ X be such that $\mu^{P}_{\Psi1}(ǰ*ý) < \min\{\mu^{P}_{\Psi1}(ǰ),\mu^{P}_{\Psi1}(ý)\}$, then $ť = \frac{1}{2}(\mu^{P}_{\Psi1}(ǰ*ý) + \min\{\mu^{P}_{\Psi1}(ǰ),\mu^{P}_{\Psi1}(ý)\})$

$\mu^{P}_{\Psi1}(ǰ*ý) < ť$, $ǰ*ý \notin U(\mu_A,ť)$ is not RG-subalgebra that mean it is contradiction.

Hence $U(\mu^{P}_{\Psi1},t)$is an RG-subalgebra of X. Similarity, $U(v^{N}_{\Psi2},t)$ and $U(\sigma^{P}_{\Psi2},t)$ are an RG-subalgebra of X.

Now, $\mu^{N}_{\Psi1}(ǰ*ý) > \max\{\mu^{N}_{\Psi1}(ǰ),\mu^{N}_{\Psi1}(ý)\}$, then $ś = \frac{1}{2}(\mu^{N}_{\Psi1}(ǰ*ý) + \max\{\mu^{N}_{\Psi1}(ǰ),\mu^{N}_{\Psi1}(ý)\})$

$\mu^{N}_{\Psi1}(ǰ*ý) < ś$, $ǰ*ý \notin L(\mu^{N}_{\Psi1},ś)$ is not doubt RG-subalgebra that mean it is contradiction

Hence $L(\mu^{N}_{\Psi1},s)$is an RG-subalgebra of X. Similarity, $L(v^{P}_{\Psi2},s)$ and $L(\tau^{P}_{\Psi1},s)$ are an RG-subalgebra of X.

Hence $\Phi = (X: \mu^{N}_{\Psi1}, \mu^{P}_{\Psi1}, \tau^{P}_{\Psi1}, v^{N}_{\Psi2}, v^{P}_{\Psi2}, \sigma^{P}_{\Psi2})$ *be a* tripolar intuitionistic fuzzy RG-subalgebra X. ■

Theorem 3.6.

Let $\Phi = (X: \mu^{N}_{\Psi1}, \mu^{P}_{\Psi1}, \tau^{P}_{\Psi1}, v^{N}_{\Psi2}, v^{P}_{\Psi2}, \sigma^{P}_{\Psi2})$ be a tripolar intuitionistic fuzzy set of an RG-algebra (X; * , ℶ). Φ is a tripolar intuitionistic fuzzy RG-subalgebra of X ↔ the fuzzy sets $\mu^{P}_{\Psi1}$, $v^{N}_{\Psi2}$ and $\sigma^{P}_{\Psi2}$ are fuzzy RG-subalgebras of X and $\mu^{N}_{\Psi1}$, $\tau^{P}_{\Psi1}$ and $v^{P}_{\Psi2}$ are doubt fuzzy RG-subalgebras of X.

Proof

Since $\Phi = (X: \mu^{N}_{\Psi1}, \mu^{P}_{\Psi1}, \tau^{P}_{\Psi1}, v^{N}_{\Psi2}, v^{P}_{\Psi2}, \sigma^{P}_{\Psi2})$ *be a* tripolar intuitionistic fuzzy RG-subalgebra.

Cleary, $\mu^{P}_{\Psi1}$, $v^{N}_{\Psi2}$ and $\sigma^{P}_{\Psi2}$ are fuzzy RG-subalgebras of X.

For all j, y ∈ X, we have

$\mu^{P}_{\Psi1}(j*y) \geq \min\{\mu^{P}_{\Psi1}(j),\mu^{P}_{\Psi1}(y)\}$, $v^{N}_{\Psi2}(j*y) \geq \min\{v^{N}_{\Psi2}(j),v^{N}_{\Psi2}(y)\}$ and

$\sigma^{P}_{\Psi2}(j*y) \geq \min\{\sigma^{P}_{\Psi2}(j),\sigma^{P}_{\Psi2}(y)\}$.

Hence $\mu^{P}_{\Psi1}$, $v^{N}_{\Psi2}$ and $\sigma^{P}_{\Psi2}$ are fuzzy RG-subalgebras of X.

And $\mu^{N}_{\Psi1}(j*y) \leq \max\{\mu^{N}_{\Psi1}(j),\mu^{N}_{\Psi1}(y)\}$, $\tau^{P}_{\Psi1}(j*y) \leq \max\{\tau^{P}_{\Psi1}(j),\tau^{P}_{\Psi1}(y)\}$ and

$v^{P}_{\Psi2}(j*y) \leq \max\{v^{P}_{\Psi2}(j),v^{P}_{\Psi2}(y)\}$.

Hence $\mu^{N}_{\Psi1}$, $\tau^{P}_{\Psi1}$ and $v^{P}_{\Psi2}$ are doubt fuzzy RG-subalgebras of X.

The conversely, assume that the fuzzy sets $\mu^{P}_{\Psi1}$, $v^{N}_{\Psi2}$ and $\sigma^{P}_{\Psi2}$ are fuzzy RG-subalgebras of X and $\mu^{N}_{\Psi1}$, $\tau^{P}_{\Psi1}$ and $v^{P}_{\Psi2}$ are doubt fuzzy RG-subalgebras of X, by Definition (3.1),

Hence $\Phi = (X: \mu^{N}_{\Psi1}, \mu^{P}_{\Psi1}, \tau^{P}_{\Psi1}, v^{N}_{\Psi2}, v^{P}_{\Psi2}, \sigma^{P}_{\Psi2})$ a tripolar intuitionistic fuzzy RG-subalgebra. ■

Theorem 3.7.

Let h: (X;∗, ℶ) → (Y;∗´, ℶ´)*be a* homomorphism of RG-algebras, if $\Phi = (Y: \mu'^{N}_{\Psi1}, \mu'^{P}_{\Psi1}, \tau'^{P}_{\Psi1}, v'^{N}_{\Psi2}, v'^{P}_{\Psi2}, \sigma'^{P}_{\Psi2})$ is a tripolar intuitionistic fuzzy RG-subalgebra of Y with sup and inf properties, then the pre-image $h^{-1}(\Phi) = (X: h^{-1}(\mu'^{N}_{\Psi1}), h^{-1}(\mu'^{P}_{\Psi1}), h^{-1}(\tau'^{P}_{\Psi1}), h^{-1}(v'^{N}_{\Psi2}), h^{-1}(v'^{P}_{\Psi2}), h^{-1}(\sigma'^{P}_{\Psi2}))$ of Φ under h in X is a tripolar intuitionistic fuzzy RG-subalgebra of X.

Proof

1) Let y, j ∈ X, then

$$h^{-1}(\mu'^{P}_{\Psi1})(j*y) = \mu'^{P}_{\Psi1}(h(j*y)) = \mu'^{P}_{\Psi1}(h(j) *́ h(y)) \geq \min\{\mu'^{P}_{\Psi1}(h(j)),\mu'^{P}_{\Psi1}(h(y))\} = \min\{h^{-1}(\mu'^{P}_{\Psi1})(j), h^{-1}(\mu'^{P}_{\Psi1})(y)\}.$$

Hence $h^{-1}(\mu'^{P}_{\Psi1})$ is a fuzzy RG-subalgebra of X.

Similarity, $h^{-1}(v'^{N}_{\Psi2})$ and $h^{-1}(\sigma'^{P}_{\Psi2})$are fuzzy RG-subalgebras of X.

and

2) $$h^{-1}(\mu'^{N}_{\Psi1})(j*y) = \mu'^{N}_{\Psi1}(h(j*y)) = \mu'^{N}_{\Psi1}(h(j) *́ h(y)) \leq \max\{\mu'^{N}_{\Psi1}(h(j)),\mu'^{N}_{\Psi1}(h(y))\} = \max\{h^{-1}(\mu'^{N}_{\Psi1})(j), h^{-1}(\mu'^{N}_{\Psi1})(y)\}.$$

Hence $h^{-1}(\mu'^{N}_{\Psi1})$ is a doubt fuzzy RG-subalgebra of X.

Similarity, $h^{-1}(\tau'^{P}_{\Psi1})$ and $h^{-1}(v'^{P}_{\Psi2})$ are doubt fuzzy RG-subalgebras of X.

Hence $h^{-1}(\Phi)$ of Φ under h in X is a tripolar intuitionistic fuzzy RG-subalgebra of X. ■

Theorem 3.8.

Let $h: (X;*, \beth) \to (Y;*', \beth')$ *be an* epimorphism of RG-algebras, if $\Phi = (X: \mu^N_{\Psi_1}, \mu^P_{\Psi_1}, \tau^P_{\Psi_1}, v^N_{\Psi_2}, v^P_{\Psi_2}, \sigma^P_{\Psi_2})$ is a tripolar intuitionistic fuzzy RG-subalgebra of X with sup and inf properties, then the image $\Phi^h =$ $(X: (\mu^N_{\Psi_1})^h, (\mu^P_{\Psi_1})^h, (\tau^P_{\Psi_1})^h, (v^N_{\Psi_2})^h, (v^P_{\Psi_2})^h, (\sigma^P_{\Psi_2})^h)$ is a tripolar intuitionistic fuzzy RG-subalgebra of Y.

<u>Proof</u>

Let $a, b \in X$, then $j, y \in Y$ such that h (a)=j, h (b)=y, and $h(a*b) = j*'y$, $h(a) = j$, and $h(b) = y$.

$(\mu^P_{\Psi_1})^h(j*y) = \mu^P_{\Psi_1}(h(a*b))$, $(\mu^P_{\Psi_1})^h(j) = \mu^P_{\Psi_1}(h(a))$, and $(\mu^P_{\Psi_1})^h(y) = \mu^P_{\Psi_1}(h(b))$,

$(\mu^P_{\Psi_1})^h(j*'y) = \mu^P_{\Psi_1}(h(a)*'h(b)) = \mu^P_{\Psi_1}(h(a*b))$
$\geq \min\{\mu^P_{\Psi_1}(h(a)), \mu^P_{\Psi_1}(h(b))\}$
$= \min\{(\mu^P_{\Psi_1})^h(j), (\mu^P_{\Psi_1})^h(y)\}$.

Hence $(\mu^P_{\Psi_1})^h$ is a fuzzy RG-subalgebra of Y.

*	ב	a	b	c
ב	ב	c	b	a
a	a	b	c	ב
b	b	a	ב	c
c	c	ב	a	b

Similarity, $(v^N_{\Psi_2})^h$ and $(\sigma^P_{\Psi_2})^h$ are fuzzy RG-subalgebras of Y.

and

$(\mu^N_{\Psi_1})^h(j*y) = \mu^N_{\Psi_1}(h(a*b))$, $(\mu^N_{\Psi_1})^h(j) = \mu^N_{\Psi_1}(h(a))$, and $(\mu^N_{\Psi_1})^h(y) = \mu^N_{\Psi_1}(h(b))$,

$(\mu^N_{\Psi_1})^h(j*'y) = \mu^N_{\Psi_1}(h(a)*'h(b)) = \mu^N_{\Psi_1}(h(a*b))$
$\leq \max\{\mu^N_{\Psi_1}(h(a)), \mu^N_{\Psi_1}(h(b))\}$
$= \max\{(\mu^N_{\Psi_1})^h(j), (\mu^N_{\Psi_1})^h(y)\}$.

Hence $(\mu^N_{\Psi_1})^h$ is a doubt fuzzy RG-subalgebra of Y.

X	ב	1	2	3
$\mu^N_{\Psi_1}$	-0.9	-0.4	-0.4	-0.4
$\mu^P_{\Psi_1}$	0.5	0.4	0.4	0.4
$\tau^P_{\Psi_1}$	0.2	0.3	0.3	0.3
$v^N_{\Psi_2}$	0.6	0.2	0.2	0.2
$v^P_{\Psi_2}$	-0.8	-0.5	-0.5	-0.5
$\sigma^P_{\Psi_2}$	0.7	0.2	0.2	0.2

Similarity, $(\tau^P_{\Psi_1})^h$ and $(v^P_{\Psi_2})^h$ are doubt fuzzy RG-subalgebras of Y.

Hence $\Phi^h =$ $(X: (\mu^N_{\Psi_1})^h, (\mu^P_{\Psi_1})^h, (\tau^P_{\Psi_1})^h, (v^N_{\Psi_2})^h, (v^P_{\Psi_2})^h, (\sigma^P_{\Psi_2})^h)$ is a tripolar intuitionistic fuzzy RG-ideal of Y. ■

<u>4. Tripolar Intuitionistic Fuzzy RG-ideals of RG-algebra</u>

In this section, we give the concept of a tripolar intuitionistic fuzzy RG-ideals of RG-algebra X.

<u>Definition 4.1.</u>

Let $A = \{(j, \mu_A(j), \nu_A(j)) \mid j \in X\}$ *be an* intuitionistic fuzzy subset of RG-algebra (X; * , ב), a tripolar fuzzy subset $\Psi_1 = \{(j, \mu^N_{\Psi_1}(j), \mu^P_{\Psi_1}(j), \tau^P_{\Psi_1}(j)) : j \in X\}$ of X and a tripolar doubt fuzzy subset $\Psi_2 = \{(j, v^N_{\Psi_2}(j), v^P_{\Psi_2}(j), \sigma^P_{\Psi_2}(j)) : j \in X\}$ of X . Then a fuzzy subset $\Phi = \{(j, \mu^N_{\Psi_1}(j), \mu^P_{\Psi_1}(j), \tau^P_{\Psi_1}(j), v^N_{\Psi_2}(j), v^P_{\Psi_2}(j), \sigma^P_{\Psi_2}(j)) : j \in X\}$ is said to *be a* **tripolar intuitionistic fuzzy RG-ideal of X** $(\Phi = (X: \mu^N_{\Psi_1}, \mu^P_{\Psi_1}, \tau^P_{\Psi_1}, v^N_{\Psi_2}, v^P_{\Psi_2}, \sigma^P_{\Psi_2}))$, if it satisfies the following for all $j, y \in X$

(**TIFS₁**) $\mu^N_{\Psi_1}(\beth) \leq \mu^N_{\Psi_1}(j)$, $\mu^P_{\Psi_1}(\beth) \geq \mu^P_{\Psi_1}(j)$, $\tau^P_{\Psi_1}(\beth) \leq \tau^P_{\Psi_1}(j)$ and
$v^N_{\Psi_2}(\beth) \geq v^N_{\Psi_2}(j)$, $v^P_{\Psi_2}(\beth) \leq v^P_{\Psi_2}(j)$, $\sigma^P_{\Psi_2}(\beth) \geq \sigma^P_{\Psi_2}(j)$,

(**TIFS₂**) $\mu^N_{\Psi_1}(\beth * y) \leq \max\{\mu^N_{\Psi_1}(j*y), \mu^N_{\Psi_1}(y)\}$ and
$v^N_{\Psi_2}(\beth * y) \geq \min\{v^N_{\Psi_2}(j*y), v^N_{\Psi_2}(\beth * j)\}$,

(**TIFS₃**) $\mu^P_{\Psi_1}(\beth * y) \geq \min\{\mu^P_{\Psi_1}(j*y), \mu^P_{\Psi_1}(\beth * j)\}$ and
$v^P_{\Psi_2}(\beth * y) \leq \max\{v^P_{\Psi_2}(j*y), v^P_{\Psi_2}(\beth * j)\}$,

(**TIFS₄**) $\tau^P_{\Psi_1}(\beth * y) \leq \max\{\tau^P_{\Psi_1}(j*y), \tau^P_{\Psi_1}(\beth * j)\}$ and
$\sigma^P_{\Psi_2}(\beth * y) \geq \min\{\sigma^P_{\Psi_2}(j*y), \sigma^P_{\Psi_2}(\beth * j)\}$.

<u>Example 4.2.</u>

Let X ={a,b,c}with $*$ and constand (ב) is defined by :

We determine the tripolar fuzzy subset of $\Phi = (X: \mu^N_{\Psi_1}, \mu^P_{\Psi_1}, \tau^P_{\Psi_1}, v^N_{\Psi_2}, v^P_{\Psi_2}, \sigma^P_{\Psi_2})$ by:

Then $\Phi = (X: \mu^N_{\Psi_1}, \mu^P_{\Psi_1}, \tau^P_{\Psi_1}, v^N_{\Psi_2}, v^P_{\Psi_2}, \sigma^P_{\Psi_2})$ *is a* tripolar intuitionistic fuzzy RG-ideal of X.

<u>Proposition 4.3.</u>

An tripolar intuitionistic fuzzy subset $\Phi =$ $(X: \mu^N_{\Psi_1}, \mu^P_{\Psi_1}, \tau^P_{\Psi_1}, v^N_{\Psi_2}, v^P_{\Psi_2}, \sigma^P_{\Psi_2})$ is an tripolar intuitionistic fuzzy RG-ideal of RG-algebra (X; * ,0), if for

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any t, s ∈ [ב,1], the sets $U(\mu^{P}_{\Psi_1},t)$, $U(v^{N}_{\Psi_2},t)$, $U(\sigma^{P}_{\Psi_2},t)$ are RG-ideals of X and $L(\mu^{N}_{\Psi_1},s)$, $L(\tau^{P}_{\Psi_1},s)$, $L(v^{P}_{\Psi_2},s)$ are RG-ideals of X.

Proof

Let $\Phi = (X: \mu^{N}_{\Psi_1}, \mu^{P}_{\Psi_1}, \tau^{P}_{\Psi_1}, v^{N}_{\Psi_2}, v^{P}_{\Psi_2}, \sigma^{P}_{\Psi_2})$ *be a* tripolar intuitionistic fuzzy RG-ideal of X and sets $U(\mu^{P}_{\Psi_1},t)$, $U(v^{N}_{\Psi_2},t)$, $U(\sigma^{P}_{\Psi_2},t)$ and $L(\mu^{N}_{\Psi_1},s)$, $L(\tau^{P}_{\Psi_1},s)$, $L(v^{P}_{\Psi_2},s)$ are nonempty sets.

Since $\mu^{N}_{\Psi_1}(ב) \leq \mu^{N}_{\Psi_1}(j)$, $\mu^{P}_{\Psi_1}(ב) \geq \mu^{P}_{\Psi_1}(j)$, $\tau^{P}_{\Psi_1}(ב) \leq \tau^{P}_{\Psi_1}(j)$ and

$v^{N}_{\Psi_2}(ב) \geq v^{N}_{\Psi_2}(j)$, $v^{P}_{\Psi_2}(ב) \leq v^{P}_{\Psi_2}(j)$, $\sigma^{P}_{\Psi_2}(ב) \geq \sigma^{P}_{\Psi_2}(j)$,

for all j ∈X.

If follows that for $j * y \in U(\mu^{P}_{\Psi_1},t)$, $ב * j \in U(\mu^{P}_{\Psi_1},t)$,then $\mu^{P}_{\Psi_1}(j*y) \geq t$, $\mu^{P}_{\Psi_1}(ב * j) \geq t$ which follow $\mu^{P}_{\Psi_1}(ב * y) \geq \min\{\mu^{P}_{\Psi_1}(j*y), \mu^{P}_{\Psi_1}(ב * j)\} \geq t$, So that $ב * y \in U(\mu^{P}_{\Psi_1},t)$.

Hence $U(\mu^{P}_{\Psi_1},t)$ is an RG-ideal of X.

Similarity, $U(v^{N}_{\Psi_2},t)$ and $U(\sigma^{P}_{\Psi_2},t)$ are an RG-ideals of X.

We prove that $L(\mu^{N}_{\Psi_1},s)$ is an RG-ideal of X.

$j * y \in L(\mu^{N}_{\Psi_1},s)$ and $ב * j \in L(\mu^{N}_{\Psi_1},s)$ and $\mu^{N}_{\Psi_1}(j*y) \leq s$ and $\mu^{N}_{\Psi_1}(ב * j) \leq s$.

If follows that $\mu^{N}_{\Psi_1}(ב * y) \leq \max\{\mu^{N}_{\Psi_1}(j*y), \mu^{N}_{\Psi_1}(ב * j)\} \leq s$, So that $ב * y \in L(\mu^{N}_{\Psi_1},s)$.

Hence $L(\mu^{N}_{\Psi_1},s)$ is an RG-ideal of X.

Similarity, $U(\tau^{P}_{\Psi_1},s)$ and $U(v^{P}_{\Psi_2},s)$ are an RG-ideals of X.

■

Proposition 4.4.

In a tripolar intuitionistic fuzzy subset $\Phi = (X: \mu^{N}_{\Psi_1}, \mu^{P}_{\Psi_1}, \tau^{P}_{\Psi_1}, v^{N}_{\Psi_2}, v^{P}_{\Psi_2}, \sigma^{P}_{\Psi_2})$, if the upper level and lower level of (X; * , ב), are RG-ideals, for all t, s ∈ [ב,1], then Φ is a tripolar intuitionistic fuzzy RG-ideal of X.

Proof

Assume that for each t, s ∈ [ב,1], the sets $U(\mu^{P}_{\Psi_1},t)$, $U(v^{N}_{\Psi_2},t)$, $U(\sigma^{P}_{\Psi_2},t)$ and $L(\mu^{N}_{\Psi_1},s)$, $L(\tau^{P}_{\Psi_1},s)$ $L(v^{P}_{\Psi_2},s)$ are RG-ideals of X.

If there exist ј́ ∈ X *be such that* $\mu^{P}_{\Psi_1}(ב́) < \mu^{P}_{\Psi_1}(ј́)$, then $ť = \frac{1}{2}(\mu^{P}_{\Psi_1}(ב́) + \mu^{P}_{\Psi_1}(ј́))$

$\mu^{P}_{\Psi_1}(ב́) < ť$, $ב́ \notin U(\mu_A, ť)$ is not RG-ideal that mean it is contradiction.

Then $\mu^{P}_{\Psi_1}(ב) \geq \mu^{P}_{\Psi_1}(j)$, for all j ∈X.

If there exist ј́, ý ∈ X be such that $\mu^{P}_{\Psi_1}(ב́ * ý) < \min\{\mu^{P}_{\Psi_1}(ј́ * ý), \mu^{P}_{\Psi_1}(ב́ * ј́)\}$, then $ť = \frac{1}{2}(\mu^{P}_{\Psi_1}(ב́ * ý) + \min\{\mu^{P}_{\Psi_1}(ј́ * ý), \mu^{P}_{\Psi_1}(ב́ * ј́)\})$

$\mu^{P}_{\Psi_1}(ב́ * ý) < ť$, $ב́ * ý \notin U(\mu_A, ť)$ is not RG-ideal that mean it is contradiction.

Then $\mu^{P}_{\Psi_1}(ב́ * ý) \geq \min\{\mu^{P}_{\Psi_1}(ј́ * ý), \mu^{P}_{\Psi_1}(ב́ * ј́)\}$.

Hence $U(\mu^{P}_{\Psi_1},t)$is an RG-ideal of X. Similarity, $U(v^{N}_{\Psi_2},t)$ and $U(\sigma^{P}_{\Psi_2},t)$ are an RG-ideals of X.

If there exist ј́ ∈ X be such that $\mu^{N}_{\Psi_1}(ב́) > \mu^{N}_{\Psi_1}(ј́)$, then $ť = \frac{1}{2}(\mu^{N}_{\Psi_1}(ב́) + \mu^{N}_{\Psi_1}(ј́))$

$\mu^{N}_{\Psi_1}(ב́) > ť$, $ב́ \notin U(\mu^{N}_{\Psi_1}, ť)$ is not RG-ideal that mean it is contradiction.

Then $\mu^{N}_{\Psi_1}(ב) \leq \mu^{N}_{\Psi_1}(j)$, for all j ∈X.

If there exist ј́, ý ∈ X be such that $\mu^{N}_{\Psi_1}(ב́ * ý) > \max\{\mu^{N}_{\Psi_1}(ј́ * ý), \mu^{N}_{\Psi_1}(ב́ * ј́)\}$, then $ť = \frac{1}{2}(\mu^{N}_{\Psi_1}(ב́ * ý) + \max\{\mu^{N}_{\Psi_1}(ј́ * ý), \mu^{N}_{\Psi_1}(ב́ * ј́)\})$

$\mu^{N}_{\Psi_1}(ב́ * ý) > ť$, $ב́ * ý \notin U(\mu^{N}_{\Psi_1}, ť)$ is not RG-ideal that mean it is contradiction.

Then $\mu^{N}_{\Psi_1}(ב́ * ý) \leq \max\{\mu^{N}_{\Psi_1}(ј́ * ý), \mu^{N}_{\Psi_1}(ב́ * ј́)\}$.

Hence $U(\mu^{N}_{\Psi_1},t)$is an RG-ideal of X. Similarity, $U(v^{P}_{\Psi_2},t)$ and $U(\tau^{P}_{\Psi_1},t)$ are an RG-ideals of X.

Hence $\Phi = (X: \mu^{N}_{\Psi_1}, \mu^{P}_{\Psi_1}, \tau^{P}_{\Psi_1}, v^{N}_{\Psi_2}, v^{P}_{\Psi_2}, \sigma^{P}_{\Psi_2})$ *be a* tripolar intuitionistic fuzzy RG-ideal X. ■

Theorem 4.5.

Let $\Phi = (X: \mu^{N}_{\Psi_1}, \mu^{P}_{\Psi_1}, \tau^{P}_{\Psi_1}, v^{N}_{\Psi_2}, v^{P}_{\Psi_2}, \sigma^{P}_{\Psi_2})$ *be a* tripolar intuitionistic fuzzy set of an RG-algebra (X; * , ב). Φ is a tripolar intuitionistic fuzzy RG-ideal of X ↔ the fuzzy sets $\mu^{P}_{\Psi_1}$, $v^{N}_{\Psi_2}$ and $\sigma^{P}_{\Psi_2}$ are fuzzy RG-ideals of X and $\mu^{N}_{\Psi_1}$, $\tau^{P}_{\Psi_1}$ and $v^{P}_{\Psi_2}$ are doubt fuzzy RG-ideals of X.

Proof

Let $\Phi = (X: \mu^{N}_{\Psi_1}, \mu^{P}_{\Psi_1}, \tau^{P}_{\Psi_1}, v^{N}_{\Psi_2}, v^{P}_{\Psi_2}, \sigma^{P}_{\Psi_2})$ *be a* tripolar intuitionistic fuzzy RG-ideal of X and sets $\mu^{N}_{\Psi_1}, \mu^{P}_{\Psi_1}, \tau^{P}_{\Psi_1}, v^{N}_{\Psi_2}, v^{P}_{\Psi_2}, \sigma^{P}_{\Psi_2}$ are nonempty sets.

And $\mu^{N}_{\Psi_1}(ב) \leq \mu^{N}_{\Psi_1}(j)$, $\mu^{P}_{\Psi_1}(ב) \geq \mu^{P}_{\Psi_1}(j)$, $\tau^{P}_{\Psi_1}(ב) \leq \tau^{P}_{\Psi_1}(j)$ and

$v^{N}_{\Psi_2}(ב) \geq v^{N}_{\Psi_2}(j)$, $v^{P}_{\Psi_2}(ב) \leq v^{P}_{\Psi_2}(j)$, $\sigma^{P}_{\Psi_2}(ב) \geq \sigma^{P}_{\Psi_2}(j)$,

for all j ∈X.

For all j, y ∈ X, we have

$\mu^{P}_{\Psi_1}(ב * y) \geq \min\{\mu^{P}_{\Psi_1}(j*y), \mu^{P}_{\Psi_1}(ב * j)\}$, $v^{N}_{\Psi_2}(ב * y) \geq \min\{v^{N}_{\Psi_2}(j*y), v^{N}_{\Psi_2}(ב * j)\}$ and $\sigma^{P}_{\Psi_2}(ב * y) \geq \min\{\sigma^{P}_{\Psi_2}(j*y), \sigma^{P}_{\Psi_2}(ב * j)\}$.

Hence $\mu^{P}_{\Psi_1}$, $v^{N}_{\Psi_2}$ and $\sigma^{P}_{\Psi_2}$ are fuzzy RG-ideals of X.

And $\mu^{N}_{\Psi_1}(ב * y) \leq \max\{\mu^{N}_{\Psi_1}(j*y), \mu^{N}_{\Psi_1}(ב * j)\}$, $\tau^{P}_{\Psi_1}(ב * y) \leq \max\{\tau^{P}_{\Psi_1}(j*y), \tau^{P}_{\Psi_1}(ב * j)\}$ and $v^{P}_{\Psi_2}(ב * y) \leq \max\{v^{P}_{\Psi_2}(j*y), v^{P}_{\Psi_2}(ב * j)\}$.

Hence $\mu^{N}_{\Psi_1}$, $\tau^{P}_{\Psi_1}$ and $v^{P}_{\Psi_2}$ are doubt fuzzy RG-ideals of X.

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Conversely, assume that the fuzzy sets $\mu^{P}_{\Psi 1}$, $v^{N}_{\Psi 2}$ and $\sigma^{P}_{\Psi 2}$ are fuzzy RG-ideals of X and $\mu^{N}_{\Psi 1}$, $\tau^{P}_{\Psi 1}$ and $v^{P}_{\Psi 2}$ are doubt fuzzy RG-ideals of X, by Definition (3.1),
Hence $\Phi = (X: \mu^{N}_{\Psi 1}, \mu^{P}_{\Psi 1}, \tau^{P}_{\Psi 1}, v^{N}_{\Psi 2}, v^{P}_{\Psi 2}, \sigma^{P}_{\Psi 2})$ a tripolar intuitionistic fuzzy RG-ideal . ■

Proposition 4.6.

If $\Phi = (X: \mu^{N}_{\Psi 1}, \mu^{P}_{\Psi 1}, \tau^{P}_{\Psi 1}, v^{N}_{\Psi 2}, v^{P}_{\Psi 2}, \sigma^{P}_{\Psi 2})$ *is a* tripolar intuitionistic fuzzy RG-ideal of an RG-algebra (X; * , ב), then Φ is a tripolar intuitionistic fuzzy RG-subalgebra of X.

Proof

By Proposition (4.3) , Proposition (2.11) and Proposition (3.5) . ■

Remark 4.7.

The converse of Proposition (4.5) is not true as the following example:

Example 4.8.

Let X = { ב, 2, 1, 3} in which $*$ is defined by the following table :

*	ב	1	2	3
ב	ב	1	2	3
1	1	ב	3	2
2	2	3	ב	1
3	3	2	1	ב

Then $A = \{(j, \mu_A(j), \nu_A(j)) \mid j \in X\}$ *be an* intuitionistic fuzzy RG-subalgebra of X

when μ(j) is fuzzy RG-ideal $\mu(j) = \begin{cases} 0.8 & j = ב \\ 0.3 & j \in \{1,2,3\} \end{cases}$

and ν(j) is a doubt fuzzy RG-subalgebra of X $\nu(j) = \begin{cases} 0.3 & j \in \{ב, 3\} \\ 0.8 & j = 1 \\ 0.9 & j = 2 \end{cases}$.

But ν(j) is not a doubt fuzzy RG-ideal since Let j=1, y=2 then $\nu(ב * 2) = 0.9 \nleq \max\{\nu(1 * 2), \nu(ב * 1)\} = 0.8$.
$A = \{(j, \mu_A(j), \nu_A(j)) \mid j \in X\}$ is not intuitionistic fuzzy RG-ideal of RG-algebra

Theorem 4.9.

Let h: (X;∗, ב) → (Y;∗´, ב´) *be a* homomorphism of RG-algebras, if $\Phi = (Y: \mu'^{N}_{\Psi 1}, \mu'^{P}_{\Psi 1}, \tau'^{P}_{\Psi 1}, v'^{N}_{\Psi 2}, v'^{P}_{\Psi 2}, \sigma'^{P}_{\Psi 2})$ is a tripolar intuitionistic fuzzy RG-ideal of Y with sup and inf properties, then the pre-image $h^{-1}(\Phi) = (X: h^{-1}(\mu'^{N}_{\Psi 1}), h^{-1}(\mu'^{P}_{\Psi 1}), h^{-1}(\tau'^{P}_{\Psi 1}), h^{-1}(v'^{N}_{\Psi 2}), h^{-1}(v'^{P}_{\Psi 2}), h^{-1}(\sigma'^{P}_{\Psi 2}))$ of Φ under h in X is a tripolar intuitionistic fuzzy RG-ideal of X.

Proof

Let $\Phi = (Y: \mu'^{N}_{\Psi 1}, \mu'^{P}_{\Psi 1}, \tau'^{P}_{\Psi 1}, v'^{N}_{\Psi 2}, v'^{P}_{\Psi 2}, \sigma'^{P}_{\Psi 2})$ *be a* tripolar intuitionistic fuzzy RG-ideal of X and sets $\mu'^{N}_{\Psi 1}, \mu'^{P}_{\Psi 1}, \tau'^{P}_{\Psi 1}, v'^{N}_{\Psi 2}, v'^{P}_{\Psi 2}, \sigma'^{P}_{\Psi 2}$ are nonempty sets.
And $\mu'^{N}_{\Psi 1}(ב') \leq \mu'^{N}_{\Psi 1}(j')$, $\mu'^{P}_{\Psi 1}(ב') \geq \mu'^{P}_{\Psi 1}(j')$, $\tau'^{P}_{\Psi 1}(ב') \leq \tau'^{P}_{\Psi 1}(j')$ and $v'^{N}_{\Psi 2}(ב') \geq v'^{N}_{\Psi 2}(j')$, $v'^{P}_{\Psi 2}(ב') \leq v'^{P}_{\Psi 2}(j')$, $\sigma'^{P}_{\Psi 2}(ב') \geq \sigma'^{P}_{\Psi 2}(j')$, for all $j' \in Y$.

Then $h^{-1}(\mu'^{N}_{\Psi 1}(ב')) \leq h^{-1}(\mu'^{N}_{\Psi 1}(j'))$, $h^{-1}(\mu'^{P}_{\Psi 1}(ב')) \geq h^{-1}(\mu'^{P}_{\Psi 1}(j'))$, $h^{-1}(\tau'^{P}_{\Psi 1}(ב) \leq h^{-1}(\tau'^{P}_{\Psi 1}(j'))$ and $h^{-1}(v'^{N}_{\Psi 2}(ב')) \geq h^{-1}(v'^{N}_{\Psi 2}(j'))$, $h^{-1}(v'^{P}_{\Psi 2}(ב')) \leq h^{-1}(v'^{P}_{\Psi 2}(j'))$, $h^{-1}(\sigma'^{P}_{\Psi 2}(ב')) \geq h^{-1}(\sigma'^{P}_{\Psi 2}(j'))$,

Let $j', y' \in Y$ such that $h(j) = j'$ and $h(y) = y'$, $j, y \in X$, then

$h^{-1}(\mu'^{P}_{\Psi 1})(ב' *' y') = \mu'^{P}_{\Psi 1}(h(ב) *' h(y)) = \mu'^{P}_{\Psi 1}(h(ב * y))$

$\geq \min\{\mu'^{P}_{\Psi 1}(h(j) *' h(y)), \mu'^{P}_{\Psi 1}(h(ב) *' h(j))\}$

$= \min\{\mu'^{P}_{\Psi 1}(h(j * y)), \mu'^{P}_{\Psi 1}(h(ב * j))\}$

$= \min\{h^{-1}(\mu'^{P}_{\Psi 1})(j' *' y'), h^{-1}(\mu'^{P}_{\Psi 1})(ב' *' j')\}$.

Hence $h^{-1}(\mu'^{P}_{\Psi 1})$ is a fuzzy RG-ideal of X.
Similarity, $h^{-1}(v'^{N}_{\Psi 2})$ and $h^{-1}(\sigma'^{P}_{\Psi 2})$ are fuzzy RG-ideals of X.

and

$h^{-1}(\mu'^{N}_{\Psi 1})(ב' *' y') = \mu'^{N}_{\Psi 1}(h(ב) *' h(y)) = \mu'^{N}_{\Psi 1}(h(ב * y))$

$\leq \max\{\mu'^{N}_{\Psi 1}(h(j * y)), \mu'^{N}_{\Psi 1}(h(ב * j))\}$

$= \max\{\mu'^{N}_{\Psi 1}(h(j) *' h(y)), \mu'^{N}_{\Psi 1}(h(ב) *' h(j))\}$

$= \max\{h^{-1}(\mu'^{N}_{\Psi 1})(j' *' y'), h^{-1}(\mu'^{N}_{\Psi 1})(ב' *' j')\}$.

Hence $h^{-1}(\mu'^{N}_{\Psi 1})$ is a doubt fuzzy RG-ideal of X.
Similarity, $h^{-1}(\tau'^{P}_{\Psi 1})$ and $h^{-1}(v'^{P}_{\Psi 2})$ are doubt fuzzy RG-ideals of X.

Hence $h^{-1}(\Phi)$ of Φ under h in X is a tripolar intuitionistic fuzzy RG-ideal of X. ■

Theorem 4.10.

Let h: (X;*, ℶ) → (Y;*´, ℶ´) *be an* epimorphism of RG-algebras, if $\Phi = \left(X: \mu^N_{\Psi1}, \mu^P_{\Psi1}, \tau^P_{\Psi1}, v^N_{\Psi2}, v^P_{\Psi2}, \sigma^P_{\Psi2}\right)$ is a tripolar intuitionistic fuzzy RG-ideal of X with sup and inf properties, then the image $\Phi^h =$ $\left(X: (\mu^N_{\Psi1})^h, (\mu^P_{\Psi1})^h, (\tau^P_{\Psi1})^h, (v^N_{\Psi2})^h, (v^P_{\Psi2})^h, (\sigma^P_{\Psi2})^h\right)$ is a tripolar intuitionistic fuzzy RG-ideal of Y.

Proof

Let $\Phi = \left(X: \mu^N_{\Psi1}, \mu^P_{\Psi1}, \tau^P_{\Psi1}, v^N_{\Psi2}, v^P_{\Psi2}, \sigma^P_{\Psi2}\right)$ *be a* tripolar intuitionistic fuzzy RG-ideal of X and sets $\mu^N_{\Psi1}, \mu^P_{\Psi1}, \tau^P_{\Psi1}, v^N_{\Psi2}, v^P_{\Psi2}, \sigma^P_{\Psi2}$ are nonempty sets.

And $\mu^N_{\Psi1}(ℶ) \le \mu^N_{\Psi1}(j)$, $\mu^P_{\Psi1}(ℶ) \ge \mu^P_{\Psi1}(j), \tau^P_{\Psi1}(ℶ) \le \tau^P_{\Psi1}(j)$ and

$v^N_{\Psi2}(ℶ) \ge v^N_{\Psi2}(j)$, $v^P_{\Psi2}(ℶ) \le v^P_{\Psi2}(j), \sigma^P_{\Psi2}(ℶ) \ge \sigma^P_{\Psi2}(j)$, for all $j \in X$.

Then $(\mu^N_{\Psi1})^h(ℶ) \le (\mu^N_{\Psi1})^h(j)$, $(\mu^P_{\Psi1})^h(ℶ) \ge (\mu^P_{\Psi1})^h(j), (\tau^P_{\Psi1})^h(ℶ) \le (\tau^P_{\Psi1})^h(j)$

And $(v^N_{\Psi2})^h(ℶ) \ge (v^N_{\Psi2})^h(j), (v^P_{\Psi2})^h(ℶ) \le (v^P_{\Psi2})^h(j)$, $(\sigma^P_{\Psi2})^h(ℶ) \ge (\sigma^P_{\Psi2})^h(j)$.

Let $a, b \in X$, then $j, y \in Y$ such that h (a)=j, h (b)=y, and $h(a * b) = j *' y$, $h(a) = j$, and $h(b) = y$.

$(\mu^P_{\Psi1})^h(j * y) = \mu^P_{\Psi1}(h(a * b)), (\mu^P_{\Psi1})^h(ℶ * j) = \mu^P_{\Psi1}(h(ℶ * a))$, and $(\mu^P_{\Psi1})^h(ℶ * y) = \mu^P_{\Psi1}(h(ℶ * b))$,

$$\begin{aligned}(\mu^P_{\Psi1})^h(ℶ *' y) &= \mu^P_{\Psi1}(h(ℶ) *' h(b)) = \mu^P_{\Psi1}(h(ℶ * b))\\ &\ge \min\{\mu^P_{\Psi1}(h(a) *' h(b)), \mu^P_{\Psi1}(h(ℶ) *' h(a))\}\\ &= \min\{\mu^P_{\Psi1}(h(a * b)), \mu^P_{\Psi1}(h(ℶ * a))\}\\ &= \min\{(\mu^P_{\Psi1})^h(j * y), (\mu^P_{\Psi1})^h(ℶ * j)\}\end{aligned}$$

Hence $(\mu^P_{\Psi1})^h$ is a fuzzy RG-ideal of Y.

Similarity, $(v^N_{\Psi2})^h$ and $(\sigma^P_{\Psi2})^h$ are fuzzy RG-ideals of Y.

and

$(\mu^N_{\Psi1})^h(ℶ * y) = \mu^N_{\Psi1}(h(ℶ * b)), (\mu^N_{\Psi1})^h(ℶ * j) = \mu^N_{\Psi1}\left((h(ℶ) *' h(a))\right)$, and $(\mu^N_{\Psi1})^h(ℶ * y) = \mu^N_{\Psi1}\left((h(ℶ) *' h(b))\right)$,

$$\begin{aligned}(\mu^N_{\Psi1})^h(ℶ *' y) &= \mu^N_{\Psi1}(h(ℶ) *' h(b)) = \mu^N_{\Psi1}(h(ℶ * b))\\ &\le \max\{\mu^N_{\Psi1}(h(a) *' h(b)), \mu^N_{\Psi1}(h(ℶ) *' h(b))\}\\ &= \max\{\mu^N_{\Psi1}(h(a * b)), \mu^N_{\Psi1}(h(ℶ * a))\}\\ &= \max\{(\mu^N_{\Psi1})^h(j * y), (\mu^N_{\Psi1})^h(ℶ * j)\}\end{aligned}$$

Hence $(\mu^N_{\Psi1})^h$ is a doubt fuzzy RG-ideal of Y.

Similarity, $(\tau^P_{\Psi1})^h$ and $(v^P_{\Psi2})^h$ are doubt fuzzy RG-ideals of Y.

Hence $\Phi^h =$ $\left(X: (\mu^N_{\Psi1})^h, (\mu^P_{\Psi1})^h, (\tau^P_{\Psi1})^h, (v^N_{\Psi2})^h, (v^P_{\Psi2})^h, (\sigma^P_{\Psi2})^h\right)$ is a tripolar intuitionistic fuzzy RG-ideal of Y. ■

5. Tripolar Intuitionistic Sub-implicative Fuzzy Ideal of RG-algebra

Here, we explain what the notion is all about. of a tripolar intuitionistic subimplicative fuzzy ideals of RG-algebra.

Definition 5.1.

Let $A = \{(j, \mu_A(j), \nu_A(j)) \mid j \in X\}$ *be an* intuitionistic fuzzy subset of RG-algebra (X; * , ℶ), a tripolar fuzzy subset $\Psi_1 = \{(j, \mu^N_{\Psi1}(j), \mu^P_{\Psi1}(j), \tau^P_{\Psi1}(j)) : j \in X\}$ of X and a tripolar doubt fuzzy subset $\Psi_2 = \{(j, v^N_{\Psi2}(j), v^P_{\Psi2}(j), \sigma^P_{\Psi2}(j)) : j \in X\}$ of X . Then a fuzzy subset $\Phi =$ $\{(j, \mu^N_{\Psi1}(j), \mu^P_{\Psi1}(j), \tau^P_{\Psi1}(j), v^N_{\Psi2}(j), v^P_{\Psi2}(j), \sigma^P_{\Psi2}(j)) : j \in X\}$

is said *to be a* **tripolar intuitionistic subimplicative fuzzy ideal of X** ($\Phi = \left(X: \mu^N_{\Psi1}, \mu^P_{\Psi1}, \tau^P_{\Psi1}, v^N_{\Psi2}, v^P_{\Psi2}, \sigma^P_{\Psi2}\right)$), if it satisfies the following for all $j, y \in X$

(TIFS₁) $\mu^N_{\Psi1}(ℶ) \le \mu^N_{\Psi1}(j)$, $\mu^P_{\Psi1}(ℶ) \ge \mu^P_{\Psi1}(j), \tau^P_{\Psi1}(ℶ) \le \tau^P_{\Psi1}(j)$ and

$v^N_{\Psi2}(ℶ) \ge v^N_{\Psi2}(j)$, $v^P_{\Psi2}(ℶ) \le v^P_{\Psi2}(j), \sigma^P_{\Psi2}(ℶ) \ge \sigma^P_{\Psi2}(j)$,

(TIFS₂) $\mu^N_{\Psi1}(j * y^2) \le \max\{\mu^N_{\Psi1}(z * ((j * y) * (y * j^2))), \mu^N_{\Psi1}(z)\}$ and

$v^N_{\Psi2}(j * y^2) \ge \min\{v^N_{\Psi2}(z * ((j * y) * (y * j^2))), v^N_{\Psi2}(z)\}$,

(TIFS₃) $\mu^P_{\Psi1}(j * y^2) \ge \min\{\mu^P_{\Psi1}(z * ((j * y) * (y * j^2))), \mu^P_{\Psi1}(z)\}$ and

$v^P_{\Psi2}(j * y^2) \le \max\{v^P_{\Psi2}(z * ((j * y) * (y * j^2))), v^P_{\Psi2}(z)\}$,

(TIFS₄) $\tau^P_{\Psi1}(j * y^2) \le \max\{\tau^P_{\Psi1}(z * ((j * y) * (y * j^2))), \tau^P_{\Psi1}(z)\}$ and

$\sigma^P_{\Psi2}(j * y^2) \ge \min\{\sigma^P_{\Psi2}(z * ((j * y) * (y * j^2))), \sigma^P_{\Psi2}(z)\}$.

Proposition 5.2.

An tripolar intuitionistic fuzzy subset $\Phi =$ $\left(X: \mu^N_{\Psi1}, \mu^P_{\Psi1}, \tau^P_{\Psi1}, v^N_{\Psi2}, v^P_{\Psi2}, \sigma^P_{\Psi2}\right)$ is an tripolar intuitionistic subimplicative fuzzy ideal of RG-algebra (X; *

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, ב), if for any t, s ∈ [ב,1], the sets $U(\mu^P_{\Psi_1}, t)$, $U(v^N_{\Psi_2}, t)$, $U(\sigma^P_{\Psi_2}, t)$ are subimplicative ideals of X and $L(\mu^N_{\Psi_1}, s)$, $L(\tau^P_{\Psi_1}, s)$, $L(v^P_{\Psi_2}, s)$ are subimplicative ideals of X.

Proof

Let $\Phi = (X: \mu^N_{\Psi_1}, \mu^P_{\Psi_1}, \tau^P_{\Psi_1}, v^N_{\Psi_2}, v^P_{\Psi_2}, \sigma^P_{\Psi_2})$ *be a* tripolar intuitionistic subimplicative fuzzy ideal of X and sets $U(\mu^P_{\Psi_1}, t)$, $U(v^N_{\Psi_2}, t)$, $U(\sigma^P_{\Psi_2}, t)$ and $L(\mu^N_{\Psi_1}, s)$, $L(\tau^P_{\Psi_1}, s)$, $L(v^P_{\Psi_2}, s)$ are nonempty sets.

Since $\mu^N_{\Psi_1}(ב) \le \mu^N_{\Psi_1}(j)$, $\mu^P_{\Psi_1}(ב) \ge \mu^P_{\Psi_1}(j)$, $\tau^P_{\Psi_1}(ב) \le \tau^P_{\Psi_1}(j)$ and

$v^N_{\Psi_2}(ב) \ge v^N_{\Psi_2}(j)$, $v^P_{\Psi_2}(ב) \le v^P_{\Psi_2}(j)$, $\sigma^P_{\Psi_2}(ב) \ge \sigma^P_{\Psi_2}(j)$,

for all j ∈X.

If follows that for $(z * ((j * y) * (y * j^2))) \in U(\mu^P_{\Psi_1}, t)$, $z \in U(\mu^P_{\Psi_1}, t)$,then $\mu^P_{\Psi_1}(z * ((j * y) * (y * j^2))) \ge t$, $\mu^P_{\Psi_1}(z) \ge t$ which follow $\mu^P_{\Psi_1}(j * y^2) \ge \min\{\mu^P_{\Psi_1}(z * ((j * y) * (y * j^2))), \mu^P_{\Psi_1}(z)\} \ge t$, So that $j * y^2 \in U(\mu^P_{\Psi_1}, t)$.

Hence $U(\mu^P_{\Psi_1}, t)$ is an subimplicative ideal of X.

Similarity, $U(v^N_{\Psi_2}, t)$ and $U(\sigma^P_{\Psi_2}, t)$ are subimplicative ideals of X.

We prove that $L(\mu^N_{\Psi_1}, s)$ is a subimplicative ideal of X.

$(z * ((j * y) * (y * j^2))) \in L(\mu^N_{\Psi_1}, s)$ and $z \in L(\mu^N_{\Psi_1}, s)$ and $\mu^N_{\Psi_1}(z * ((j * y) * (y * j^2))) \le s$ and $\mu^N_{\Psi_1}(z) \le s$.

If follows that $\mu^N_{\Psi_1}(j * y^2) \le \max\{\mu^N_{\Psi_1}(z * ((j * y) * (y * j^2))), \mu^N_{\Psi_1}(z)\} \le s$, So that $j * y^2 \in L(\mu^N_{\Psi_1}, s)$.

Hence $L(\mu^N_{\Psi_1}, s)$ is subimplicative ideal of X.

Similarity, $U(\tau^P_{\Psi_1}, s)$ and $U(v^P_{\Psi_2}, s)$ are subimplicative ideals of X. ■

Proposition 5.3.

In a tripolar intuitionistic fuzzy subset $\Phi = (X: \mu^N_{\Psi_1}, \mu^P_{\Psi_1}, \tau^P_{\Psi_1}, v^N_{\Psi_2}, v^P_{\Psi_2}, \sigma^P_{\Psi_2})$, if the upper level and lower level of (X; ∗ , ב), are subimplicative ideals, for all t, s ∈ [ב,1], then Φ is a tripolar intuitionistic subimplicative fuzzy ideal of X.

Proof

Assume that for each t, s ∈ [ב,1], the sets $U(\mu^P_{\Psi_1}, t)$, $U(v^N_{\Psi_2}, t)$, $U(\sigma^P_{\Psi_2}, t)$ and $L(\mu^N_{\Psi_1}, s)$, $L(\tau^P_{\Psi_1}, s)$ $L(v^P_{\Psi_2}, s)$ are subimplicative ideals of X.

If there exist ȷ́ ∈ X *be* such that $\mu^P_{\Psi_1}(\acute{ב}) < \mu^P_{\Psi_1}(\acute{j})$, then $\acute{t} = \frac{1}{2}(\mu^P_{\Psi_1}(\acute{ב}) + \mu^P_{\Psi_1}(\acute{j}))$

$\mu^P_{\Psi_1}(\acute{ב}) < \acute{t}$, $\acute{ב} \notin U(\mu_A, \acute{t})$ is not subimplicative ideal that mean it is contradiction.

Then $\mu^P_{\Psi_1}(ב) \ge \mu^P_{\Psi_1}(j)$, for all j ∈X.

If there exist j, y ∈ X *be* such that $\mu^P_{\Psi_1}(j * y^2) < \min\{\mu^P_{\Psi_1}(z * ((j * y) * (y * j^2))), \mu^P_{\Psi_1}(z)\}$, then $\acute{t} = \frac{1}{2}(\mu^P_{\Psi_1}(j * y^2) + \min\{\mu^P_{\Psi_1}(z * ((j * y) * (y * j^2))), \mu^P_{\Psi_1}(z)\})$

$\mu^P_{\Psi_1}(j * y^2) < \acute{t}$, $j * y^2 \notin U(\mu_A, \acute{t})$ is not subimplicative ideal that mean it is contradiction.

Then $\mu^P_{\Psi_1}(j * y^2) \ge \min\{\mu^P_{\Psi_1}(z * ((j * y) * (y * j^2))), \mu^P_{\Psi_1}(z)\}$.

Hence $U(\mu^P_{\Psi_1}, t)$is a subimplicative ideal of X. Similarity, $U(v^N_{\Psi_2}, t)$ and $U(\sigma^P_{\Psi_2}, t)$ are an subimplicative ideals of X.

If there exist ȷ́ ∈ X *be* such that $\mu^N_{\Psi_1}(\acute{ב}) > \mu^N_{\Psi_1}(\acute{j})$, then $\acute{t} = \frac{1}{2}(\mu^N_{\Psi_1}(\acute{ב}) + \mu^N_{\Psi_1}(\acute{j}))$

$\mu^N_{\Psi_1}(\acute{ב}) > \acute{t}$, $\acute{ב} \notin U(\mu^N_{\Psi_1}, \acute{t})$ is not subimplicative ideal that mean it is contradiction.

Then $\mu^N_{\Psi_1}(ב) \le \mu^N_{\Psi_1}(j)$, for all j ∈X.

If there exist j, y ∈ X be such that $\mu^N_{\Psi_1}(j * y^2) > \max\{\mu^N_{\Psi_1}(z * ((j * y) * (y * j^2))), \mu^N_{\Psi_1}(z)\}$, then $\acute{t} = \frac{1}{2}(\mu^N_{\Psi_1}(j * y^2) + \max\{\mu^N_{\Psi_1}(z * ((j * y) * (y * j^2))), \mu^N_{\Psi_1}(z)\})$

$\mu^N_{\Psi_1}(j * y^2) > \acute{t}$, $j * y^2 \notin U(\mu^N_{\Psi_1}, \acute{t})$ is not subimplicative ideal that mean it is contradiction.

Then $\mu^N_{\Psi_1}(j * y^2) \le \max\{\mu^N_{\Psi_1}(z * ((j * y) * (y * j^2))), \mu^N_{\Psi_1}(z)\}$.

Hence $U(\mu^N_{\Psi_1}, t)$is a subimplicative ideal of X. Similarity, $U(v^P_{\Psi_2}, t)$ and $U(\tau^P_{\Psi_1}, t)$ are an subimplicative ideals of X.

Hence $\Phi = (X: \mu^N_{\Psi_1}, \mu^P_{\Psi_1}, \tau^P_{\Psi_1}, v^N_{\Psi_2}, v^P_{\Psi_2}, \sigma^P_{\Psi_2})$ *be a* tripolar intuitionistic subimplicative fuzzy ideal of X. ■

Theorem 5.4.

Let $\Phi = (X: \mu^N_{\Psi_1}, \mu^P_{\Psi_1}, \tau^P_{\Psi_1}, v^N_{\Psi_2}, v^P_{\Psi_2}, \sigma^P_{\Psi_2})$ *be a* tripolar intuitionistic fuzzy set of an RG-algebra (X; ∗ , ב). Φ is a tripolar intuitionistic subimplicative fuzzy ideal of X ↔ the fuzzy sets $\mu^P_{\Psi_1}$, $v^N_{\Psi_2}$ and $\sigma^P_{\Psi_2}$ are subimplicative fuzzy ideals of X and $\mu^N_{\Psi_1}$, $\tau^P_{\Psi_1}$ and $v^P_{\Psi_2}$ are doubt subimplicative fuzzy ideals of X.

Proof

Let $\Phi = (X: \mu^N_{\Psi_1}, \mu^P_{\Psi_1}, \tau^P_{\Psi_1}, v^N_{\Psi_2}, v^P_{\Psi_2}, \sigma^P_{\Psi_2})$ *be a* tripolar intuitionistic subimplicative fuzzy ideal of X and sets $\mu^N_{\Psi_1}, \mu^P_{\Psi_1}, \tau^P_{\Psi_1}, v^N_{\Psi_2}, v^P_{\Psi_2}, \sigma^P_{\Psi_2}$ are nonempty sets.

And $\mu^N_{\Psi_1}(ב) \le \mu^N_{\Psi_1}(j)$, $\mu^P_{\Psi_1}(ב) \ge \mu^P_{\Psi_1}(j)$, $\tau^P_{\Psi_1}(ב) \le \tau^P_{\Psi_1}(j)$ and

$v^N_{\Psi_2}(ב) \ge v^N_{\Psi_2}(j)$, $v^P_{\Psi_2}(ב) \le v^P_{\Psi_2}(j)$, $\sigma^P_{\Psi_2}(ב) \ge \sigma^P_{\Psi_2}(j)$,

for all j ∈X.

For all j, y ∈ X, we have

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$\mu^{P}_{\Psi 1}(j * y^2) \geq \min\{\mu^{P}_{\Psi 1}(z * ((j * y) * (y * j^2))), \mu^{P}_{\Psi 1}(z)\}$,
$v^{N}_{\Psi 2}(j * y^2) \geq \min\{v^{N}_{\Psi 2}(z * ((j * y) * (y * j^2))), v^{N}_{\Psi 2}(z)\}$ and
$\sigma^{P}_{\Psi 2}(j * y^2) \geq \min\{\sigma^{P}_{\Psi 2}(z * ((j * y) * (y * j^2))), \sigma^{P}_{\Psi 2}(z)\}$.
Hence $\mu^{P}_{\Psi 1}$, $v^{N}_{\Psi 2}$ and $\sigma^{P}_{\Psi 2}$ are subimplicative fuzzy ideals of X.
And $\mu^{N}_{\Psi 1}(j * y^2) \leq \max\{\mu^{N}_{\Psi 1}(z * ((j * y) * (y * j^2))), \mu^{N}_{\Psi 1}(z)\}$, $\tau^{P}_{\Psi 1}(j * y^2) \leq \max\{\tau^{P}_{\Psi 1}(z * ((j * y) * (y * j^2))), \tau^{P}_{\Psi 1}(z)\}$ and $v^{P}_{\Psi 2}(j * y^2) \leq \max\{v^{P}_{\Psi 2}(z * ((j * y) * (y * j^2))), v^{P}_{\Psi 2}(z)\}$.
Hence $\mu^{N}_{\Psi 1}$, $\tau^{P}_{\Psi 1}$ and $v^{P}_{\Psi 2}$ are doubt subimplicative fuzzy ideals of X.

Conversely, assume that the fuzzy sets $\mu^{P}_{\Psi 1}$, $v^{N}_{\Psi 2}$ and $\sigma^{P}_{\Psi 2}$ are subimplicative fuzzy ideals of X and $\mu^{N}_{\Psi 1}$, $\tau^{P}_{\Psi 1}$ and $v^{P}_{\Psi 2}$ are doubt subimplicative fuzzy ideals of X, by Definition (5.1), hence $\Phi = (X: \mu^{N}_{\Psi 1}, \mu^{P}_{\Psi 1}, \tau^{P}_{\Psi 1}, v^{N}_{\Psi 2}, v^{P}_{\Psi 2}, \sigma^{P}_{\Psi 2})$ a tripolar intuitionistic subimplicative fuzzy ideal. ■

Proposition 5.5.

If $\Phi = (X: \mu^{N}_{\Psi 1}, \mu^{P}_{\Psi 1}, \tau^{P}_{\Psi 1}, v^{N}_{\Psi 2}, v^{P}_{\Psi 2}, \sigma^{P}_{\Psi 2})$ is a tripolar intuitionistic subimplicative fuzzy ideal of an RG-algebra (X; * , ב), then Φ is a tripolar intuitionistic fuzzy RG-ideal of X.

X	ב	1	2
$\mu^{N}_{\Psi 1}$	-0.2	-0.1	-0.1
$\mu^{P}_{\Psi 1}$	0.5	0.3	0.3
$\tau^{P}_{\Psi 1}$	0.2	0.3	0.3
$v^{N}_{\Psi 2}$	0.5	0.3	0.3
$v^{P}_{\Psi 2}$	-0.2	-0.1	-0.1
$\sigma^{P}_{\Psi 2}$	0.2	0.3	0.3

Proof:

Let Ψ = $(X: \mu^{N}_{\Psi}, \mu^{P}_{\Psi}, \tau^{P}_{\Psi})$is a tripolar subimplicative fuzzy ideal of X , then

$\mu^{P}_{\Psi 1}(j * y^2) \geq \min\{\mu^{P}_{\Psi 1}(z * ((j * y) * (j * y^2))), \mu^{P}_{\Psi 1}(z)\}$.

Take j = y , then

$\mu^{P}_{\Psi 1}(j * j^2) \geq \min\{\mu^{P}_{\Psi 1}(z * ((j * j) * (j * j^2))), \mu^{P}_{\Psi 1}(z)\}$, i.e.,

$\mu^{P}_{\Psi 1}(ב * j) \geq \min\{\mu^{P}_{\Psi 1}(z * j), \mu^{P}_{\Psi 1}(ב * z)\}$.

Hence $\mu^{P}_{\Psi 1}$ is a fuzzy RG-ideal of X.
Similarity, $v^{N}_{\Psi 2}$ and $\sigma^{P}_{\Psi 2}$ are fuzzy RG-ideals of X.
And

$\mu^{N}_{\Psi 1}(j * y^2) \leq \max\{\mu^{N}_{\Psi 1}(z * ((j * y) * (j * y^2))), \mu^{N}_{\Psi 1}(z)\}$.

Take j = y, then

$\mu^{N}_{\Psi 1}(j * j^2) \leq \max\{\mu^{N}_{\Psi 1}(z * ((j * j) * (j * j^2))), \mu^{N}_{\Psi 1}(z)\}$,i.e.,

$\mu^{N}_{\Psi 1}(ב * j) \leq \max\{\mu^{N}_{\Psi 1}(z * j), \mu^{N}_{\Psi 1}(ב * z)\}$.

Hence $\mu^{N}_{\Psi 1}$ is a fuzzy RG-ideal of X.

Similarity, $\tau^{P}_{\Psi 1}$ and $v^{P}_{\Psi 2}$ are fuzzy RG-ideals of X.
Then $\Phi = (X: \mu^{N}_{\Psi 1}, \mu^{P}_{\Psi 1}, \tau^{P}_{\Psi 1}, v^{N}_{\Psi 2}, v^{P}_{\Psi 2}, \sigma^{P}_{\Psi 2})$ is a tripolar intuitionistic fuzzy RG-ideal of X. ■

Remark 5.6.

The next example shows that the converse of Proposition (5.5)is not true in general.

Example 5.7.

Let X = { ב, 1,2} in which * is define by the following table:

*	ב	1	2
ב	ב	2	1
1	1	ב	2
2	2	1	ב

It is easy to show that (X;∗ , ב) is an RG-algebra.$\Phi = (X: \mu^{N}_{\Psi 1}, \mu^{P}_{\Psi 1}, \tau^{P}_{\Psi 1}, v^{N}_{\Psi 2}, v^{P}_{\Psi 2}, \sigma^{P}_{\Psi 2})$is a tripolar intuitionistic fuzzy RG-ideal of X as this table

But not a tripolar intuitionistic subimplicative fuzzy ideal of RG-algebra X,because if

z = ב , j = 1 , y = ב we get

$\mu^{N}_{\Psi 1}((1 * ב) * ב) = \mu^{N}_{\Psi 1}(1) = -0.1 \leq \max\{\mu^{N}_{\Psi 1}(ב * ((1 * ב) * ((ב * 1) * 1), \mu^{N}_{\Psi 1}(ב)\} = \mu^{N}_{\Psi 1}(ב) = -0.2$.

Proposition 5.8.

If $\Phi = (X: \mu^{N}_{\Psi 1}, \mu^{P}_{\Psi 1}, \tau^{P}_{\Psi 1}, v^{N}_{\Psi 2}, v^{P}_{\Psi 2}, \sigma^{P}_{\Psi 2})$ is a tripolar intuitionistic subimplicative fuzzy ideal of an RG-algebra (X; * , ב), then Φ is a tripolar intuitionistic fuzzy RG-subalgebra of X.

Proof

By Proposition (5.5) and Proposition (4.6) .■

Remark 5.9.

The next example shows that the converse of Proposition (5.8) is not true in general by Example (5.8).

Theorem 5.10.

Let h: (X;∗, ב) → (Y;∗´, ב´) *be a* homomorphism of RG-algebras, if $\Phi = (Y: \mu'^{N}_{\Psi 1}, \mu'^{P}_{\Psi 1}, \tau'^{P}_{\Psi 1}, v'^{N}_{\Psi 2}, v'^{P}_{\Psi 2}, \sigma'^{P}_{\Psi 2})$ is a tripolar intuitionistic subimplicative fuzzy ideal of Y with sup and inf properties, then the pre-image $h^{-1}(\Phi) =$

$\left(\begin{matrix} X: h^{-1}(\mu'^{N}_{\Psi_1}), h^{-1}(\mu'^{P}_{\Psi_1}), h^{-1}(\tau'^{P}_{\Psi_1}), h^{-1}(v'^{N}_{\Psi_2}), \\ h^{-1}(v'^{P}_{\Psi_2}), h^{-1}(\sigma'^{P}_{\Psi_2}) \end{matrix}\right)$ of Φ under h in X is a tripolar intuitionistic subimplicative fuzzy ideal of X.

Proof

Let $\Phi = \left(Y: \mu'^{N}_{\Psi_1}, \mu'^{P}_{\Psi_1}, \tau'^{P}_{\Psi_1}, v'^{N}_{\Psi_2}, v'^{P}_{\Psi_2}, \sigma'^{P}_{\Psi_2}\right)$ *be a* tripolar intuitionistic subimplicative fuzzy ideal of X and sets $\mu'^{N}_{\Psi_1}, \mu'^{P}_{\Psi_1}, \tau'^{P}_{\Psi_1}, v'^{N}_{\Psi_2}, v'^{P}_{\Psi_2}, \sigma'^{P}_{\Psi_2}$ are nonempty sets.

And $\mu'^{N}_{\Psi_1}(\text{ב}') \le \mu'^{N}_{\Psi_1}(j')$, $\mu'^{P}_{\Psi_1}(\text{ב}') \ge \mu'^{P}_{\Psi_1}(j')$, $\tau'^{P}_{\Psi_1}(\text{ב}') \le \tau'^{P}_{\Psi_1}(j')$ and

$v'^{N}_{\Psi_2}(\text{ב}') \ge v'^{N}_{\Psi_2}(j')$, $v'^{P}_{\Psi_2}(\text{ב}') \le v'^{P}_{\Psi_2}(j')$, $\sigma'^{P}_{\Psi_2}(\text{ב}') \ge \sigma'^{P}_{\Psi_2}(j')$, for all $j' \in Y$.

Then $h^{-1}(\mu'^{N}_{\Psi_1}(\text{ב}')) \le h^{-1}(\mu'^{N}_{\Psi_1}(j'))$,

$h^{-1}(\mu'^{P}_{\Psi_1}(\text{ב}')) \ge h^{-1}(\mu'^{P}_{\Psi_1}(j'))$, $h^{-1}(\tau'^{P}_{\Psi_1}(\text{ב}) \le h^{-1}(\tau'^{P}_{\Psi_1}(j'))$ and

$h^{-1}(v'^{N}_{\Psi_2}(\text{ב}')) \ge h^{-1}(v'^{N}_{\Psi_2}(j'))$, $h^{-1}(v'^{P}_{\Psi_2}(\text{ב}')) \le h^{-1}(v'^{P}_{\Psi_2}(j'))$, $h^{-1}(\sigma'^{P}_{\Psi_2}(\text{ב}')) \ge h^{-1}(\sigma'^{P}_{\Psi_2}(j'))$,

Let $j', y', z' \in Y$ such that $h(j) = j'$, $h(y) = y'$ and $h(z) = z'$, $j, y, z \in X$, then

$h^{-1}(\mu'^{P}_{\Psi_1})(j' \ast' y'^2) = \mu'^{P}_{\Psi_1}(h(j) \ast h(y^2)) = \mu'^{P}_{\Psi_1}(h(j \ast y^2))$

$\ge \min\{\mu'^{P}_{\Psi_1}(h(z \ast ((j \ast y) \ast (y \ast j^2)))), \mu'^{P}_{\Psi_1}(h(\text{ב}))\}$

$= \min\{\mu'^{P}_{\Psi_1}(h(z) \ast h((h(j) \ast h(y) \ast' (h(y) \ast h(j^2)), \mu'^{P}_{\Psi_1}(h(z))\}$

$= \min\{\mu'^{P}_{\Psi_1}(h(z) \ast h((h(j) \ast h(y) \ast' (h(y) \ast h(j^2)), \mu'^{P}_{\Psi_1}(h(z))\}$

$= \min\{h^{-1}(\mu'^{P}_{\Psi_1})(z' \ast'((j' \ast' y') \ast'(y' \ast j'^2))), h^{-1}(\mu'^{P}_{\Psi_1})(z')\}$.

Hence $h^{-1}(\mu'^{P}_{\Psi_1})$ is a subimplicative fuzzy ideal of X.

Similarity, $h^{-1}(v'^{N}_{\Psi_2})$ and $h^{-1}(\sigma'^{P}_{\Psi_2})$ are subimplicative fuzzy ideals of X.

and

$h^{-1}(\mu'^{N}_{\Psi_1})(j' \ast' y'^2) = \mu'^{N}_{\Psi_1}(h(j) \ast h(y^2)) = \mu'^{N}_{\Psi_1}(h(j \ast y^2))$

$\le \max\{\mu'^{N}_{\Psi_1}(h(z \ast ((j \ast y) \ast (y \ast j^2)))), \mu'^{N}_{\Psi_1}(h(\text{ב}))\}$

$= \max\{\mu'^{N}_{\Psi_1}(h(z) \ast h((h(j) \ast h(y) \ast' (h(y) \ast h(j^2)), \mu'^{N}_{\Psi_1}(h(z))\}$

$= \max\{\mu'^{N}_{\Psi_1}(h(z) \ast h((h(j) \ast h(y) \ast' (h(y) \ast h(j^2)), \mu'^{N}_{\Psi_1}(h(z))\}$

$= \max\{h^{-1}(\mu'^{N}_{\Psi_1})(z' \ast'((j' \ast' y') \ast'(y' \ast j'^2))), h^{-1}(\mu'^{N}_{\Psi_1})(z')\}$.

Hence $h^{-1}(\mu'^{N}_{\Psi_1})$ is a doubt subimplicative fuzzy ideal of X.

Similarity, $h^{-1}(\tau'^{P}_{\Psi_1})$ and $h^{-1}(v'^{P}_{\Psi_2})$ are doubt subimplicative fuzzy ideals of X.

Hence $h^{-1}(\Phi)$ of Φ under h in X is a tripolar intuitionistic subimplicative fuzzy ideal of X. ■

Theorem 5.11.

Let $h: (X; \ast, \text{ב}) \to (Y; \ast', \text{ב}')$ *be an* epimorphism of RG-algebras, if $\Phi = \left(X: \mu^{N}_{\Psi_1}, \mu^{P}_{\Psi_1}, \tau^{P}_{\Psi_1}, v^{N}_{\Psi_2}, v^{P}_{\Psi_2}, \sigma^{P}_{\Psi_2}\right)$ is a tripolar intuitionistic subimplicative fuzzy ideal of X with sup and inf properties, then the image $\Phi^h = \left(X: (\mu^{N}_{\Psi_1})^h, (\mu^{P}_{\Psi_1})^h, (\tau^{P}_{\Psi_1})^h, (v^{N}_{\Psi_2})^h, (v^{P}_{\Psi_2})^h, (\sigma^{P}_{\Psi_2})^h\right)$ is a tripolar intuitionistic subimplicative fuzzy ideal of Y.

Proof

Let $\Phi = \left(X: \mu^{N}_{\Psi_1}, \mu^{P}_{\Psi_1}, \tau^{P}_{\Psi_1}, v^{N}_{\Psi_2}, v^{P}_{\Psi_2}, \sigma^{P}_{\Psi_2}\right)$ *be a* tripolar intuitionistic subimplicative fuzzy ideal of X and sets $\mu^{N}_{\Psi_1}, \mu^{P}_{\Psi_1}, \tau^{P}_{\Psi_1}, v^{N}_{\Psi_2}, v^{P}_{\Psi_2}, \sigma^{P}_{\Psi_2}$ are nonempty sets.

And $\mu^{N}_{\Psi_1}(\text{ב}) \le \mu^{N}_{\Psi_1}(j)$, $\mu^{P}_{\Psi_1}(\text{ב}) \ge \mu^{P}_{\Psi_1}(j)$, $\tau^{P}_{\Psi_1}(\text{ב}) \le \tau^{P}_{\Psi_1}(j)$ and

$v^{N}_{\Psi_2}(\text{ב}) \ge v^{N}_{\Psi_2}(j)$, $v^{P}_{\Psi_2}(\text{ב}) \le v^{P}_{\Psi_2}(j)$, $\sigma^{P}_{\Psi_2}(\text{ב}) \ge \sigma^{P}_{\Psi_2}(j)$, for all $j \in X$.

Then $(\mu^{N}_{\Psi_1})^h(\text{ב}) \le (\mu^{N}_{\Psi_1})^h(j)$, $(\mu^{P}_{\Psi_1})^h(\text{ב}) \ge (\mu^{P}_{\Psi_1})^h(j)$, $(\tau^{P}_{\Psi_1})^h(\text{ב}) \le (\tau^{P}_{\Psi_1})^h(j)$

And $(v^{N}_{\Psi_2})^h(\text{ב}) \ge (v^{N}_{\Psi_2})^h(j)$, $(v^{P}_{\Psi_2})^h(\text{ב}) \le (v^{P}_{\Psi_2})^h(j)$, $(\sigma^{P}_{\Psi_2})^h(\text{ב}) \ge (\sigma^{P}_{\Psi_2})^h(j)$.

Let $a, b \in X$, then $j, y \in Y$ such that h (a)=j, h (b)=y, h (c)=z and $h(a \ast b^2) = j \ast' y^2$ andf $(c \ast ((a \ast b) \ast (b \ast a^2))) = (z \ast'((j \ast' y) \ast'(y \ast' j^2)))$.

$(\mu^{P}_{\Psi_1})^h(j \ast' y^2) = \mu^{P}_{\Psi_1}(h(a) \ast' h(b^2)) = \mu^{P}_{\Psi_1}(h(a \ast b^2))$

$\ge \min\{\mu^{P}_{\Psi_1}(h(c) \ast'((h(a) \ast' h(b)) \ast (h(b) \ast' h(a^2)))), \mu^{P}_{\Psi_1}(h(c))\}$

$= \min\{\mu^{P}_{\Psi_1}(h(c \ast ((a \ast b) \ast (b \ast a^2)))), \mu^{P}_{\Psi_1}(h(c))\}$

$= \min\{(\mu^{P}_{\Psi_1})^h(z * ((j * y) * (y * j^2))), (\mu^{P}_{\Psi_1})^h(z)\}$

.

Hence $(\mu^{P}_{\Psi_1})^h$ is a subimplicative fuzzy ideal of Y.
Similarity, $(v^{N}_{\Psi_2})^h$ and $(\sigma^{P}_{\Psi_2})^h$ are subimplicative fuzzy ideals of Y.

and

$(\mu^{N}_{\Psi_1})^h(j *' y^2) = \mu^{N}_{\Psi_1}(h(a) *' h(b^2)) = \mu^{N}_{\Psi_1}(h(a * b^2))$

$\leq \max\{\mu^{N}_{\Psi_1}(h(c) *' ((h(a) *' h(b)) * (h(b) *' h(a^2)))), \mu^{N}_{\Psi_1}(h(c))\}$

$= \max\{\mu^{N}_{\Psi_1}(h(c * ((a * b) * (b * a^2)))), \mu^{N}_{\Psi_1}(h(c))\}$

$= \max\{(\mu^{N}_{\Psi_1})^h(z * ((j * y) * (y * j^2))), (\mu^{N}_{\Psi_1})^h(z)\}$

.

Hence $(\mu^{N}_{\Psi_1})^h$ is a doubt subimplicative fuzzy ideal of Y.
Similarity, $(\tau^{P}_{\Psi_1})^h$ and $(v^{P}_{\Psi_2})^h$ are doubt subimplicative fuzzy ideals of Y.

Hence $\Phi^h = \left(X: (\mu^{N}_{\Psi_1})^h, (\mu^{P}_{\Psi_1})^h, (\tau^{P}_{\Psi_1})^h, (v^{N}_{\Psi_2})^h, (v^{P}_{\Psi_2})^h, (\sigma^{P}_{\Psi_2})^h\right)$ *is a* tripolar intuitionistic subimplicative fuzzy ideals of Y. ■

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