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Digraphic Topology On Directed Edges

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Abstract: In this paper, we study the digraphic topology $\tau_{IƊ}$ for a directed edges of a digraph. We give some properties of this topology, in particular we prove that $\tau_{IƊ}$ is an Alexandroff topology and when two digraphs are isomorphic, their digraphic topologies will be homeomorphic. We give some properties matching digraphs and homeomorphic topology spaces. Finally, we investigate the connectedness of this topology and some relations between the connectedness of the digraph and the topology $\tau_{IƊ}$.

Keywords— digraph, topology, Alexandroff topology and separation axioms.

1.Introduction:

Topological structures are mathematical models that can be used to analyze data without the concept of distance. Topological structures, in our opinion, are a crucial adjustment for the extraction and processing of knowledge [2]. This publication provides a few topological fundamentals that are pertinent to our study. One of the most crucial structures in discrete mathematics is the graph [1]. Two observations explain their pervasiveness. Graphs are mathematically elegant, to start, from a theoretical standpoint. Although a graph merely has a set of vertices and a relationship between pairs of vertices, it is a simple structure, yet graph theory is a vast and diverse field of study. This is partially because graphs can be thought of as topological spaces, combinatorial objects, and many other mathematical structures in addition to being relational structures [1]. This brings us to our second argument about the significance of graphs: many ideas may be abstractly represented by graphs[3], which makes them very helpful in real-world applications. Several earlier studies on the subject of topological graphs we can see in [4-11]. In this paper we discuss a new method to generate topology $\tau_{IƊ}$ on graph by using new method of taking neighborhood is determining a vertex on the digraph and calculate each vertex and its edges indgree of it and we defined $S_{IƊ} = \{ \overrightarrow{Ẹ_{ṿ}} \mid ṿ \in Ṿ \}$, where $Ẹ_{ṿ}$ is the set of all edges indgree to ṿ, we have $Ẹ = \bigcup_{ṿ\in Ṿ} \overrightarrow{Ẹ_{ṿ}}$. hence $S_{IƊ}$ forms a subbasis for a topology $\tau_{IƊ}$ on Ẹ, called digraphic topology,(briefly digtopology) of Ɗ.

2.PRELIMINARIES:

In this work , some basic notions of graph theory [1-2], and topology [2] are presented. A graph (resp., directed graph or digraph) Ɗ = (Ṿ, Ẹ) consists of a vertex set Ṿ and an edge set Ẹ of unordered (resp., ordered) pairs of elements of Ṿ. To avoid ambiguities, we assume that the vertex and edge sets are disjoint. We say that two vertices ṿ and w of a graph (resp., digraph Ɗ) are adjacent if there is an edge of the form ṿw (rsep., $\overrightarrow{wṿ}$ or $\overrightarrow{ṿw}$) joining them, and the vertices ṿ and w are then incident with such an edge. A subdigraph H of a digraph Ɗ is a digraph, each of whose vertices belong to Ṿ and each of whose edges belong to Ẹ. The degree of a vertex ṿ of Ɗ is the number of edges incident with ṿ, and written deg(ṿ). A vertex of degree zero is an isolated vertex. In digraph, the outdegree, of a vertex ṿ of Ɗ is the number of edges of the form $\overrightarrow{ṿw}$ and denoted by $d^{+}(ṿ)$, similarly, the indegree of a vertex ṿ of Ɗ is the number of edges of the form $\overrightarrow{wṿ}$, and denoted by $d^{-}(ṿ)$,. A vertex of out-degree and in-degree are zero is an isolated vertex . A topology τ on a set X is a combination of subset of X, called open, such that the union of the member of any subset of τ is a member of τ, the intersection of the members of any finite subset of τ is a member of τ, and both empty set and X are in τ and the ordered pair (X, τ) is called topological space. The topology $\tau = P(X)$ on X is called discrete topology while the topology $\tau = \{X, \emptyset\}$ on X is called indiscrete topology. A topology in which arbitrary intersection of open set is open called Alexandroff space.

3.DIGRAPHIC TOPOLOGY.

In this section, we introduce our new subbasis family to generated a topology on the set of edges Ẹ of a digraph Ɗ = (Ṿ,Ẹ).

Definition 3.1: Let Ɗ = (Ṿ,Ẹ) be a digraph, we defined $S_{IƊ} = \{ \overrightarrow{Ẹ_{ṿ}} \mid ṿ \in Ṿ \}$, where $Ẹ_{ṿ}$ is the set of all edges indgree to ṿ, we have $Ẹ = \bigcup_{ṿ\in Ṿ} \overrightarrow{Ẹ_{ṿ}}$. hence $S_{IƊ}$ forms a subbasis for a topology $\tau_{IƊ}$ on Ẹ, called digraphic topology,(briefly digtopology) of Ɗ.

Theorem 3.2: Let Ɗ = (Ṿ,Ẹ) be digraph then (Ẹ, $\tau_{IƊ}$) is topological space .

prove : we will prove that $\tau_{IƊ}$ is topological graph ,

1) Since $Ẹ = \bigcup_{i\in I} W_i$ where $W_i \in \beta_{IƊ}$ such that $\beta_{IƊ}$ is a basis for a topological graph $\tau_{IƊ}$, then $W_i = \bigcap_{i=1}^{n} S_i$ where $S_i \in S_{IƊ}$ and $S_i = \overrightarrow{Ẹ_{ṿ_i}}$, $ṿ_i \in Ṿ$ .Then $Ẹ = \bigcup_{i\in I}(\overrightarrow{Ẹ_{ṿ_i}})$ and so $Ẹ \in \tau_{IƊ}$. Also $\emptyset \in \tau_{IƊ}$ by complement of Ẹ.

2) Let $U_i \in \tau_{IƊ}$, $U_i = \bigcup_{i\in I} W_i$ where $W_i \in \beta_{IƊ}$, $W_i = \bigcap_{i=1}^{n} S_i$ where $S_i \in S_{IƊ}$, $S_i = \overrightarrow{Ẹ_{ṿ_i}}$, $ṿ_i \in Ṿ$, then $U_i =$

$\bigcup_{i\in I}(\bigcap_{j=1}^{n}\overrightarrow{E_{v_i}})$ so $U_i=\bigcup_{i\in I}(\bigcap_{j=1}^{n}\overrightarrow{E_{v_i}})\in\tau_{ID}$. Therefore $\bigcup_{i\in I}U_i\in\tau_{ID}$.

3) Let $A_i, B_i\in\tau_{ID}$, $A_i=\bigcup_{i\in I}W_i$ where $W_i\in\beta_{ID}$, $W_i=\bigcap_{i=1}^{n}S_i$ where $S_i\in S_{ID}$, $S_i=\overrightarrow{E_{v_i}}$, $v_i\in V$ then $A_i=\bigcup_{i\in I}(\bigcap_{j=1}^{n}\overrightarrow{E_{v_i}})$, $B_i=\bigcup_{i\in I}U_i$ where $U_i\in\beta_{ID}$, $U_i=\bigcap_{i=1}^{n}S_i$ where $S_i\in S_{ID}$ $S_i=\overrightarrow{E_{v_i}}$, $v_i\in V$then $B_i=\bigcup_{i\in I}(\bigcap_{j=1}^{n}\overrightarrow{E_{v_i}})$ then:

i) If there are no element in intersection i.e., $A_i\cap B_i=\emptyset$ since $\emptyset\in\tau_{ID}$ then $A_i\cap B_i\in\tau_{ID}$.

ii) If there exist element in intersection $U_i\cap W_i$ then we denote it $\overrightarrow{E_{v_i}}$, $v\in V$ since $A_i=\bigcup_{i\in I}(\bigcap_{j=1}^{n}\overrightarrow{E_{v_i}})$ and$B_i=\bigcup_{k\in I}(\bigcap_{j=1}^{n}\overrightarrow{E_{v_i}})$ then $\overrightarrow{E_v}$ one of these Posts. Therefore $A_i\cap B_i\in\tau_{ID}$.

Example 3.3: Let $D=(V,E)$ be digraph as in Figure (1), such that $V=\{v_1,v_2,v_3,v_4,v_5,v_6\}$, $E=\{e_1,e_2,e_3,e_4,e_5,e_6\}$.

We have,$\overrightarrow{E_{v_1}}=\emptyset$, $\overrightarrow{E_{v_2}}=\{e_1\}$, $\overrightarrow{E_{v_3}}=\{e_2\}$,$\overrightarrow{E_{v_4}}=\{e_3\}$, $\overrightarrow{E_{v_5}}=\{e_4,e_6\}$,$\overrightarrow{E_{v_6}}=\{e_5\}$.and $S_{ID}=\{\emptyset,\{e_1\},\{e_2\},\{e_3\},\{e_5\},\{e_4,e_6\}\}$.

By taking finitely intersection the basis obtained is : $\{\emptyset,\{e_1\},\{e_2\},\{e_3\},\{e_5\},\{e_4,e_6\}\}$. Then by taking all unions the topology can be written as:

$\tau_{ID}=\{E(D),\emptyset,\{e_1\},\{e_2\},\{e_3\},\{e_5\},\{e_4,e_6\},\{e_1,e_2\},\{e_1,e_3\},\{e_1,e_5\},\{e_2,e_3\},\{e_2,e_5\},\{e_3,e_5\},\{e_1,e_2,e_3\},\{e_1,e_2,e_5\},\{e_1,e_3,e_5\},\{e_2,e_3,e_5\},\{e_1,e_4,e_6\},\{e_2,e_4,e_6\},\{e_3,e_4,e_6\},\{e_5,e_4,e_6\},\{e_1,e_2,e_3,e_5\},\{e_1,e_2,e_4,e_6\},\{e_1,e_3,e_4,e_5\},\{e_1,e_5,e_4,e_5\},\{e_2,e_3,e_4,e_5\},\{e_2,e_5,e_4,e_5\},\{e_3,e_5,e_4,e_5\},\{e_1,e_2,e_3,e_4,e_6\},\{e_1,e_2,e_5,e_4,e_6\},\{e_1,e_3,e_5,e_4,e_6\},\{e_2,e_3,e_5,e_4,e_6\}\}$.

Then τ_{ID} is topology is called digtopology τ_{ID} .

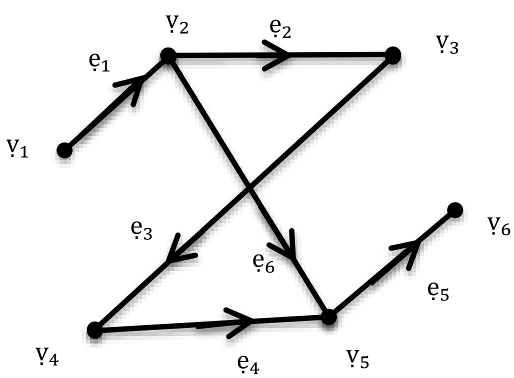


Fig.(1)

Remark 3.4: Let C_n be cyclic digraph if every edges are in the same directed then we get the digtopology τ_{ID} on C_n is discrete, and if the edges are not all in the same direction we get the digtopology τ_{ID} on C_n is not discrete .

This Remark illustrates in the next two Examples.

Example 3.5: Let C_5 be cyclic digraph such that every edges are in the same direction, show in Figure (2).

We have, $\overrightarrow{E_{v_1}}=\{e_2\}$, $\overrightarrow{E_{v_2}}=\{e_3\}$, $\overrightarrow{E_{v_3}}=\{e_4\}$, $\overrightarrow{E_{v_4}}=\{e_5\}$, $\overrightarrow{E_{v_5}}=\{e_6\}$ $\overrightarrow{E_{v_6}}=\{e_1\}$.

And $S_{ID}=\{\{e_1\},\{e_2\},\{e_3\},\{e_4\},\{e_5\},\{e_6\}\}$.

$\tau_{ID}=\{E(D),\emptyset,\{e_1\},\{e_2\},\{e_3\},\{e_4\},\{e_5\},\{e_6\},\{e_1,e_2\},\{e_1,e_3\},\{e_1,e_4\},\{e_1,e_5\},\{e_1,e_6\},\{e_2,e_3\},\{e_2,e_4\},\{e_2,e_5\},\{e_2,e_6\},\{e_3,e_4\},\{e_3,e_5\},\{e_3,e_6\},\{e_4,e_5\},\{e_4,e_6\},\{e_1,e_2,e_3\},\{e_1,e_2,e_4\},\{e_1,e_2,e_5\},\{e_1,e_2,e_6\},\{e_1,e_3,e_4\},\{e_1,e_3,e_5\},\{e_1,e_3,e_6\},\{e_1,e_4,e_5\},\{e_1,e_4,e_6\},\{e_2,e_3,e_4\},\{e_2,e_3,e_5\},\{e_2,e_3,e_6\},\{e_2,e_4,e_5\},\{e_2,e_4,e_6\},\{e_3,e_4,e_5\},\{e_3,e_4,e_6\},\{e_1,e_2,e_3,e_4\},\{e_1,e_2,e_3,e_5\},\{e_1,e_2,e_3,e_6\}\{e_1,e_2,e_4,e_5\},\{e_1,e_2,e_4,e_6\},\{e_1,e_3,e_4,e_5\},\{e_1,e_3,e_4,e_6\},\{e_2,e_3,e_4,e_5\},\{e_1,e_3,e_4,e_6\},\{e_1,e_2,e_3,e_4,e_5\},\{e_1,e_2,e_3,e_4,e_6\},\{e_1,e_2,e_3,e_5,e_6\}\{e_1,e_3,e_4,e_5,e_6\},\{e_2,e_3,e_4,e_5,e_6\}\}$.Then we get the digtopology τ_{ID} of C_6 is discrete topology.

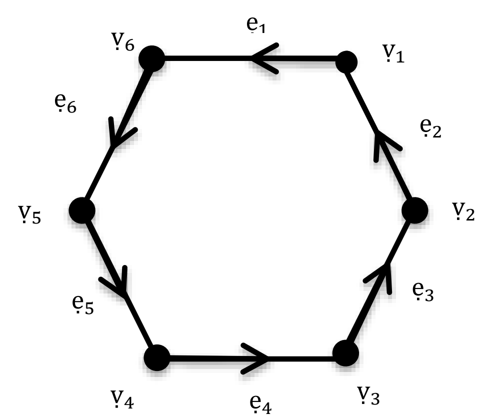
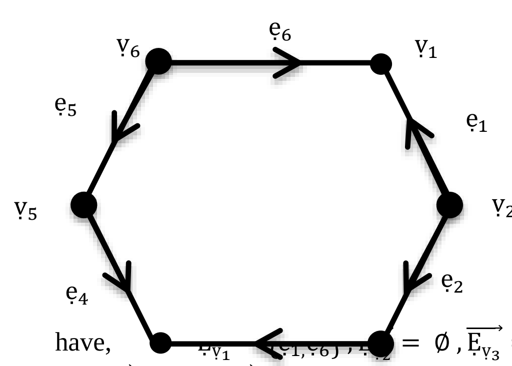


Fig.

Example 3.6: Let C_5 be cyclic digraph such that the edges are not all in the same direction, show in Figure (3).



We have, $\overrightarrow{E_{v_1}}=\{e_1,e_6\}$, $\overrightarrow{E_{v_2}}=\emptyset$, $\overrightarrow{E_{v_3}}=\{e_2\}$, $\overrightarrow{E_{v_4}}=\{e_3,e_4\}$, $\overrightarrow{E_{v_5}}=\{e_5\}$,$\overrightarrow{E_{v_6}}=\emptyset$.

Fig. (3)

And $S_{ID} = \{\emptyset, \{e_1, e_6\}, \{e_3, e_4\}, \{e_2\}, \{e_5\}\}$.

$\tau_{ID} = \{E(D), \emptyset, \{e_1, e_6\}, \{e_3, e_4\}, \{e_2\}, \{e_5\}, e_2, e_5\}, \{e_1, e_6, e_2\}, \{e_1, e_6, e_5\}\{e_3, e_4, e_2\}, \{e_3, e_4, e_5\}, \{e_1, e_6, e_3, e_4\}, \{e_1, e_6, e_2, e_5\}\{e_1, e_6, e_5, e_2\}\{e_3, e_4, e_2, e_5\}, \{e_3, e_4, e_5, e_2\}, \{e_1, e_6, e_3, e_4, e_2\}, \{e_1, e_6, e_3, e_4, e_5\}\}$. Then we get the digtopology τ_{ID} of C_6 is not discrete topology .

Remark 3.7: Let P_n be a path digraph if every edges are in the same directed then we get the digtopology τ_{ID} on P_n is discrete, and if the edges are not all in the same direction we get the digtopology τ_{ID} on P_n is not necessary discrete .
This Remark illustrates in the next Examples.

Example 3.8: Let P_5 be a path digraph, show in Figure (4).

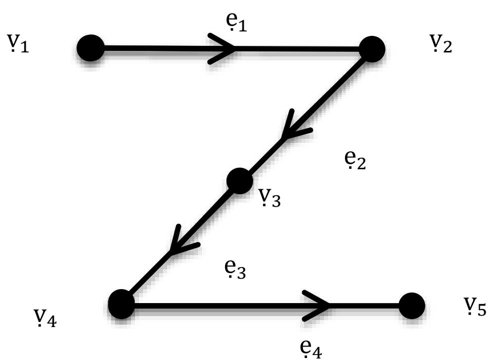


Fig.(4)

We have $\overrightarrow{E_{v_1}} = \emptyset$, $\overrightarrow{E_{v_2}} = \{e_1\}$, $\overrightarrow{E_{v_3}} = \{e_2\}$, $\overrightarrow{E_{v_4}} = \{e_3\}$, $\overrightarrow{E_{v_5}} = \{e_4\}$.And $S_{ID} = \{\emptyset, \{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}\}$.

$\tau_{ID} = \{E(D), \emptyset, \{e_1\}\{e_2\}, \{e_3\}, \{e_4\}, \{e_1, e_2\}, \{e_1, e_3\}, \{e_1, e_4\}, \{e_2, e_3\}\}, \{e_2, e_4\}, \{e_3, e_4\}, \{e_1, e_2, v_3\}, \{e_1, e_2, e_4\}, \{e_2, e_3, e_4\}\}$.

We note the ine.to.digsp. τ_{ID} is discret topology.

Example 3.9: Let P_5 be a path digraph , show in Figure (5).

We have, $\overrightarrow{E_{v_1}} = \{e_1\}$, $\overrightarrow{E_{v_2}} = \emptyset$, $\overrightarrow{E_{v_3}} = \{e_2\}$, $\overrightarrow{E_{v_4}} = \{e_3\}$, $\overrightarrow{E_{v_5}} = \{e_4\}$.

And $S_{ID} = \{\emptyset, \{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}\}$. $T_{ID} = \{E(D), \emptyset, \{e_1\}\{e_2\}, \{e_3\}, \{e_4\}, \{e_1, e_2\}, \{e_1, e_3\}, \{e_1, e_4\}, \{e_2, e_3\}, \{e_2, e_4\}, \{e_3, e_4\}, \{e_1, e_2, v_3\}, \{e_1, e_2, e_4\}, \{e_2, e_3, e_4\},$

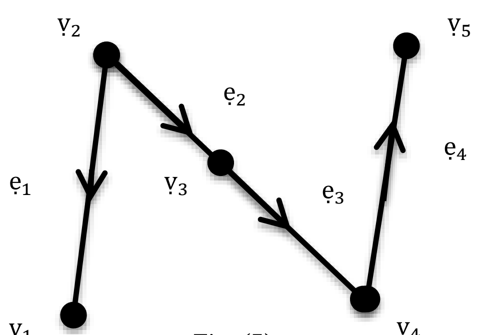


Fig. (5).

$\{e_1, e_2, e_3, e_4\}\}$.We note the digtopology τ_{ID} is discret topology.

Example 3.10: Let P_5 be a path digraph, show in Figure (6) then we get the digtopology τ_{ID} of P_5 is not discrete .

We have, $\overrightarrow{E_{v_1}} = \{e_1\}$, $\overrightarrow{E_{v_2}} = \emptyset$, $\overrightarrow{E_{v_3}} = \{e_2, e_3\}$, $\overrightarrow{E_{v_4}} = \emptyset$, $\overrightarrow{E_{v_5}} = \{e_4\}$.And $S_{ID} = \{\emptyset, \{e_1\}, \{e_2, e_3\}, \{e_4\}\}$.

$\tau_{ID} = \{E(D), \emptyset, \{e_1\}, \{e_2, e_3\}, \{e_4\}, \{e_1, e_4\}, \{e_1, e_2, e_3\}\}, \{e_2, e_3, e_4\}, \{e_1, e_2, e_3, e_4\}\}$.We note the ine.to.digsp. τ_{ID} is not discret topology.

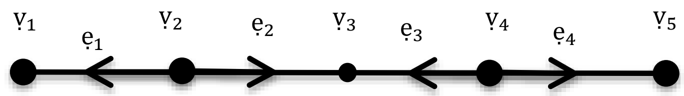


Fig.(6)

Remark 3.11: Let S_n be a star digraph then :

(i) If every edges are direction to center vertex then the digtopology τ_{ID} is indiscreet topology.

(ii) If every edges are not direction to center vertex then the digtopology τ_{ID} is discreet topology.

(iii) If most edge is direction to center vertex then the digtopology τ_{ID} is not discrete topology.This Remark illustrates in the next Examples.

Example 3.12: Let S_5 be a star digraph ,show in Figure(7).
In Figure(7)(A),every edges are directed to center vertex.
We have $\overrightarrow{E_{v_1}} = \{e_1, e_2, e_3, e_4, e_5\}$, $\overrightarrow{E_{v_2}} = \emptyset$, $\overrightarrow{E_{v_3}} = \emptyset$, $\overrightarrow{E_{v_4}} = \emptyset$, $\overrightarrow{E_{v_5}} = \emptyset$, $\overrightarrow{E_{v_6}} = \emptyset$ .And $S_{ID} = \{\emptyset, E(D)\}$

$\tau_{ID} = \{\emptyset, E(D)\}$ then we get the digtopology τ_{ID} is indiscrete topology.But in Figure (7)(B) every the edges not direction to center vertixe We note, $\overrightarrow{E_{v_1}} = \emptyset$, $\overrightarrow{E_{v_2}} = \{e_1\}$, $\overrightarrow{E_{v_3}} = \{e_2\}$, $\overrightarrow{E_{v_4}} = \{e_3\}$, $\overrightarrow{E_{v_5}} = \{e_4\}$, $\overrightarrow{E_{v_6}} = \{e_5\}$.

And $S_{ID} = \{\emptyset, \{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_5\}\}$

$\tau_{ID} = \{E(D), \emptyset, \{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_5\}, \{e_1, e_2\}, \{e_1, e_3\}, \{e_1, e_4\}, \{e_1, e_5\}, \{e_2, e_3\}, \{e_2, e_4\}, \{e_2, e_5\}, \{e_3, e_4\}, \{e_3, e_5\}, \{e_4, e_5\}, \{e_1, e_2, e_3\}, \{e_1, e_2, e_4\}, \{e_1, e_2, e_5\}, \{e_1, e_3, e_4\}, \{e_1, e_3, e_5\}, \{e_1, e_4, e_5\}, \{e_2, e_3, e_4\}, \{e_2, e_3, e_5\}, \{e_2, e_4, e_5\}, \{e_3, e_4, e_5\}, \{e_1, e_2, e_3, e_4\}, \{e_1, e_2, e_3, e_5\}, \{e_1, e_2, e_4, e_5\},$

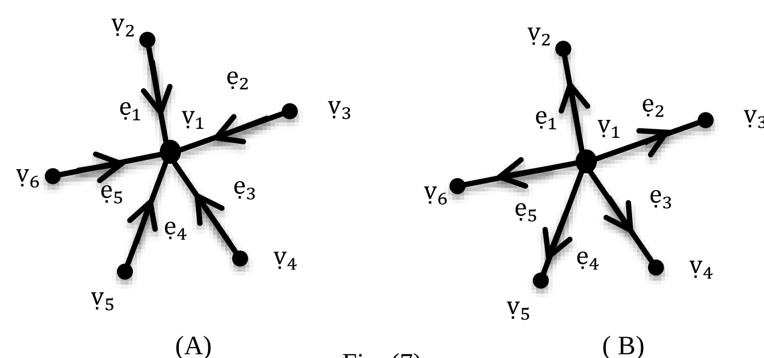


Fig. (7)

$\{e_1, e_3, e_4, e_5\}, \{e_2, e_3, e_4, e_5\}\}$.we get the digtopology τ_{ID} of S_5 is discrete topology.

Example 3.13: Let S_4 be a star digraph ,show in Figure (8).

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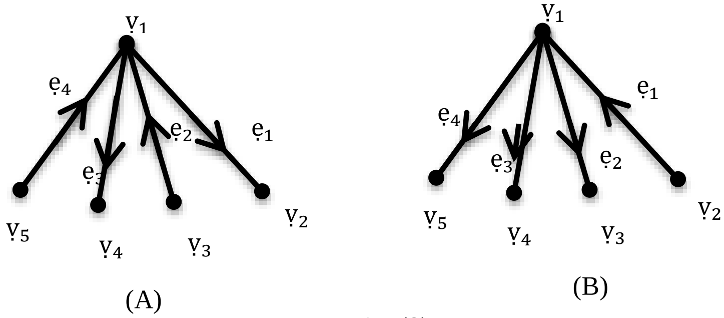

Fig. (8)

We note, in Figure (8)(A), is most edges is not same directed, $\overrightarrow{E_{v_1}} = \{e_2, e_4\}$, $\overrightarrow{E_{v_2}} = \{e_1\}$, $\overrightarrow{E_{v_3}} = \emptyset$, $\overrightarrow{E_{v_4}} = \{e_3\}$, $\overrightarrow{E_{v_5}} = \emptyset$. And $S_{ID} = \{\emptyset, \{e_1\}, \{e_3\}, \{e_2, e_4\}\}$.

$\tau_{ID} = \{E(D), \emptyset, \{e_1\}, \{e_3\}, \{e_2, e_3\}, \{e_1, e_3\}, \{e_1, e_2, e_4\}\}, \{e_3, e_2, e_4\}\}$

We get the digtopology τ_{ID} is not discret topology .

But in Figure (8)(B), we note only one edges is directed to center vertixe , hence $\overrightarrow{E_{v_1}} = \{e_1\}$, $\overrightarrow{E_{v_2}} = \emptyset$, $\overrightarrow{E_{v_3}} = \{e_2\}$, $\overrightarrow{E_{v_4}} = \{e_3\}$, $\overrightarrow{E_{v_5}} = \{e_4\}$ .And $S_{ID} = \{\emptyset, \{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}\}$.

$\tau_{ID} = \{E(D), \emptyset, \{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_1, e_2\}, \{e_1, e_3\}, \{e_1, e_4\}, \{e_2, e_3\}, \{e_2, e_4\}, \{e_3, e_4\}, \{e_1, e_2, e_3\}, \{e_1, e_2, e_4\}, \{e_1, e_3, e_4\}, \{e_2, e_3, e_4\}\}$. We get the digtopology τ_{ID} of S_5 is discrete topology.

Proposition 3.14: Suppose that τ_{ID} is the digtopology of the adigraph $D = (V, E)$, then $\{e\} \in \tau_{ID}$ if $\overrightarrow{I_e}^{v} \neq \overrightarrow{I_{é}}^{v}$ for all $é \in E$

Prove: Let $e \in E$ then $\overrightarrow{I_e}^{v} = \{v\}$ for some $v \in V$ and byhypothesis $\overrightarrow{I_e}^{v} \neq \overrightarrow{I_{é}}^{v}$ for all $é \in E$ then we get e is only edge is directed to v and hence then $\overrightarrow{E_v} = \{e\}$ and by definition digtopology τ_{ID} we get $\{e\} \in \tau_{ID}$.

Remark 3.15: Let $D = (V, E)$ be a digraph, then the digtopology τ_{ID} is not necessary to be discrete topology in general .The Example 3.3, illustrate Remark 3.15 **.**

Corollary 3.16: Let $D = (V, E)$ be a digraph if $\overrightarrow{I_e}^{v} \neq \overrightarrow{I_{é}}^{v}$ for every distance pair of edge $e, é \in E$, then digtopology τ_{ID} is discrete topology.

Prove: Since $\overrightarrow{I_e}^{v} \neq \overrightarrow{I_{é}}^{v}$ for every distance pair of edge in digraph D then by Proposition 3.14, $\{e\} \in \tau_{ID}$ for all $e \in E$, hence we get the digtopology τ_{ID} is discrete topology.

Example 3.17: According to Example 3.5,we note that $\overrightarrow{I_{e_1}}^{v} = \{v_5\}$, $\overrightarrow{I_{e_2}}^{v} = \{v_1\}$, $\overrightarrow{I_{e_3}}^{v} = \{v_2\}$, $\overrightarrow{I_{e_4}}^{v} = \{v_3\}$, $\overrightarrow{I_{e_5}}^{v} = \{v_4\}$ i.e $\overrightarrow{I_e}^{v} \neq \overrightarrow{I_{é}}^{v}$ $\forall e, é \in E$, hence digtopology τ_{ID} is discrete topology.

Remark 3.18: If $D = (V, E)$ be reflexive digraph then digtopology τ_{ID} is not discrete topology .

The Example illustrate Remark 3.18 .

Example 3.19: Let $D = (V, E)$ be reflexive digraph show in Figure(9).

We note, $\overrightarrow{E_{v_1}} = \{e_1\}$, $\overrightarrow{E_{v_2}} = \{e_2, e_3\}$, $\overrightarrow{E_{v_3}} = \{e_4, e_5\}$. And $S_{ID} = \{\{e_1\}, \{e_2, e_3\}, \{e_4, e_5\}\}$. $T_{ID} = \{E(D), \emptyset, \{e_1\}, \{e_2, e_3\}, \{e_4, e_5\}, \{e_1, e_2, e_3,\}\}$,

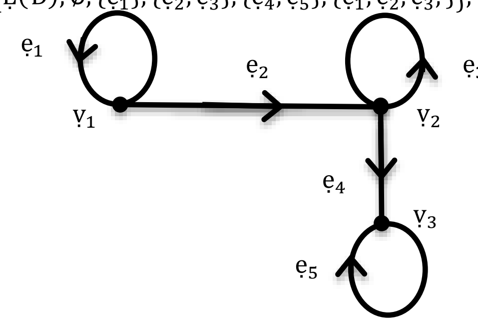

Fig.(9).

$\{e_1, e_4, e_5\}$, $\{e_2, e_3, e_4, e_5\}\}$.We get the digtopology τ_{ID} is not discrete topology.

Proposition 3.20: Let $D = (V, E)$ be reflexive digraph and $d(v) \leq 2$ then the digtopology τ_{ID} is discrete topology.

Prove : Since the $D = (V, E)$ reflexive digraph and $d(v) \leq 2$ for all $v \in V$ there exist only loop on every vertex and hence $\forall\, v \in V$ we get $E_v = \{e\}$, where $e = (v, v)$ and by definition digtopology τ_{ID} implies $\{e\} \in \tau_{ID}$ for all $e \in E$, thus digtopology τ_{ID} is discrete topology. The Example illustrate proposition 3.20.

Example 3.21: Let $D = (V, E)$ be reflexive digraph, show in Figure (10).

We note, $\overrightarrow{E_{v_1}} = \{e_1\}$, $\overrightarrow{E_{v_2}} = \{e_2\}$, $\overrightarrow{E_{v_3}} = \{e_3\}$, $\overrightarrow{E_{v_4}} = \{e_4\}$.

And $S_{ID} = \{\{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}\}$.

$\tau_{ID} = \{E(D), \emptyset, \{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_1, e_2\}, \{e_1, e_3\}, \{e_1, e_4\}, \{e_2, e_3\}, \{e_2, e_4\}, \{e_3, e_4\}, \{e_1, e_2, e_3\}, \{e_1, e_2, e_4\}, \{e_1, e_3, e_4\}, \{e_2, e_3, e_4\}\}$

We get the digtopology τ_{ID} is discrete topology.

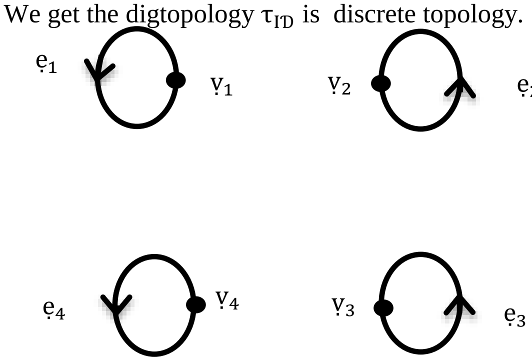

Fig.(10).

Remark 3.22: Let $D = (V, E)$ be a symmetric digraph, then digtopology τ_{ID} not necessary to be discrete topology in general .the following Example shows Remark 3.22.

Example 3.23: Let $D = (V, E)$ be symmetric digraph in Figure (11) such that $V = \{v_1, v_2, v_3\}$, $E = \{e_1, e_2, e_3, e_4\}$

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We have, $\overrightarrow{E_{v_1}} = \{e_1\}$, $\overrightarrow{E_{v_2}} = \{e_2, e_3\}$, $\overrightarrow{E_{v_3}} = \{e_4\}$. .
$S_{ID} = \{\{e_1\}, \{e_4\}, \{e_2, e_3\}\}$.
$\tau_{ID} = \{ E(D), \emptyset, \{e_1\}, \{e_4\}, \{e_2, e_3\}, \{e_1, e_4\}, \{e_1, e_2, e_3\}, \{e_4, e_2, e_3\}\}$. We note the digtopology τ_{ID} is not discrete topology.

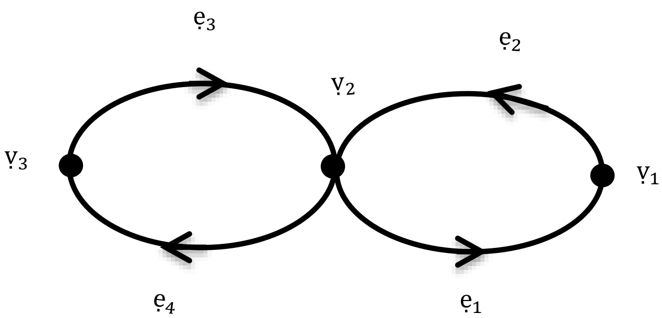

Fig.(11)

Proposition 3.24: If $D = (V, E)$ be a symmetric digraph and $d(v) \le 2$ for all $v \in V$ then the digtopology τ_{ID} is discrete topology.
Prove: Since $D = (V,E)$ be a symmetric digraph and $d(v) \le 2$ implies we get for all $v \in V$ there exist at most one edge is directed to v for all $v \in V$ [because if there exsit two edge are directed to v and $d(v) \le 2$ implies $D = (V,E)$ is not symmetric] and hence, $\overrightarrow{I_e}^{v} \ne \overrightarrow{I_{é}}^{v}$ for all $é \in E$, by proposition 3.14 then the digtopology τ_{ID} is discrete topology .
the following Example shows the Proposition3.24 .
Example 3.25 : Let $D = (V,E)$ be symmetric digraph in Figure (12) such that $V=\{v_1, v_2, v_3, v_4, v_5\}$, $E = \{e_1, e_2, e_3, e_4, e_5\}$.
We note ,$\overrightarrow{E_{v_1}} = \{e_1\}$, $\overrightarrow{E_{v_2}} = \{e_2\}$, $\overrightarrow{E_{v_3}} = \{e_3\}$, $\overrightarrow{E_{v_4}} = \{e_4\}$, $\overrightarrow{E_{v_5}} = \{e_5\}$.And $S_{ID} = \{\{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_5\}\}$.
$\tau_{ID} = \{ E(D), \emptyset, \{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_5\}, \{e_1, e_2\}, \{e_1, e_3\}, \{e_1, e_4\}, \{e_1, e_5\}, \{e_2, e_3\}, \{e_2, e_4\}, \{e_2, e_5\}, \{e_3, e_4\}, \{e_3, e_5\}, \{e_4, e_5\}, \{e_1, e_2, e_3\}, \{e_1, e_2, e_4\}, \{e_1, e_2, e_5\}, \{e_1, e_3, e_4\}, \{e_1, e_3, e_5\}, \{e_1, e_4, e_5\}, \{e_2, e_3, e_4\}, \{e_2, e_3, e_5\}, \{e_2, e_4, e_5\}, \{e_3, e_4, e_5\}, \{e_1, e_2, e_3, e_4\}, \{e_1, e_2, e_3, e_5\}, \{e_1, e_2, e_4, e_5\}, \{e_1, e_3, e_4, e_5\}, \{e_2, e_3, e_4, e_5\}\}$.We get the digtopology τ_{ID} is discrete topology.

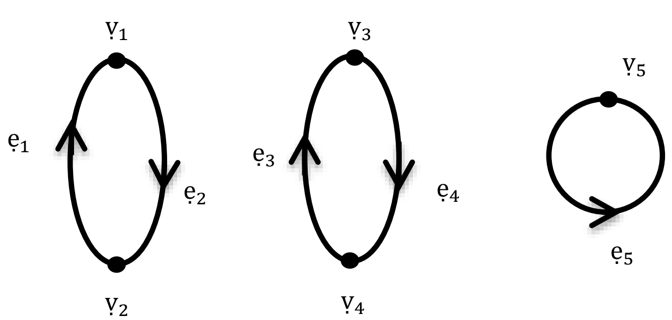

Fig. (12)

Proposition 3.26: The digtopology (E, τ_{ID}) of digraph $D = (V,E)$ is Alexandroff space .
Prove : It is adequate to show that arbitrary intersection of elements of S_{ID} is open , Let $A \subseteq V$: $\bigcap_{v \in A} \overrightarrow{E_v} = \begin{cases} \overrightarrow{E_v} & \textit{if A contin one vertex } v \\ \emptyset & \textit{owther wise} \end{cases}$
And by Definition 3.1 of digtopology τ_{ID} we get $\emptyset$, $\overrightarrow{E_v} \in \tau_{ID}$, then $\bigcap_{v \in A} \overrightarrow{E_v}$ is open .Hence the digtopology τ_{ID} is satisfies property of Alexandroff .
Definition 3.27: In any digraph $D = (V,E)$ since (E, τ_{ID}) is Alexandroff space, for each $e \in E$, the intersection of all open set containing e is the smallest open set containing e and denoted by U_e, Also the family $M_D = \{U_e | e \in E\}$ is the minimal basis for the digtopology τ_{ID} .
Proposition 3.28 : In any digraph $D = (V,E)$, $U_e = \overrightarrow{E_v}$ where $\overrightarrow{I_e}^{v} = \{v\}$ for every $e \in E$.
Prove : $\Rightarrow$ Since every $e \in E$ then $\overrightarrow{I_e}^{v} = \{v\}$ for some $v \in V$ and by Definition3.1.1,of digtopology τ_{ID}, E_v is open contain e and by Definition 3.27,of U_e then we get $U_e \subseteq \overrightarrow{E_v}$.
$\Leftarrow$ since U_e is open set and contain e then by definition digtopology τ_{ID} there exist $\overrightarrow{E_v}$ for some $v \in V$ such that $e \in \overrightarrow{E_v} \subseteq U_e$, and since $é \notin U_e$ implies $é \notin \overrightarrow{E_v}$ when $e \in \overrightarrow{E_v}$, then $é \notin \overrightarrow{E_v}$ when $\overrightarrow{I_e}^{v} = \{v\}$ thus $\overrightarrow{E_v} \subseteq U_e$ when $\overrightarrow{I_e}^{v} = \{v\}$, then $U_e = \overrightarrow{E_v}$ where $\overrightarrow{I_e}^{v} = \{v\}$ for every $e \in E$.
Remark 3.29: Let $D = (V, E)$ be a digraph, if $\overrightarrow{I_e}^{v} \ne \overrightarrow{I_{é}}^{v}$ for all $é \in E$ then $U_e = \{e\}$.
Prove : clear
Theorem 3.30: For any $e, é \in E$ in a digraph $D = (V,E)$ we have $\overrightarrow{I_e}^{v} = \overrightarrow{I_{é}}^{v}$ iff $e \in U_{é}$ i.e $U_e = \{é \in E | \overrightarrow{I_e}^{v} = \overrightarrow{I_{é}}^{v}\}$.
Prove : $\Rightarrow$ Let $\overrightarrow{I_e}^{v} = \overrightarrow{I_{é}}^{v}$ to prove $e \in U_{é}$. Since $\overrightarrow{I_e}^{v} = \overrightarrow{I_{é}}^{v} = \{v\}$, implies $e, é \in \overrightarrow{E_v}$ and by Properties 3.28 $U_e = \overrightarrow{E_v}$ and $U_{é} = \overrightarrow{E_v}$, and hence $U_e = U_{é} = E_v$ then $e \in U_{é}$.
$\Leftarrow$ let $e \in U_{é}$ and by Proposition 3.28, we get $U_{é} = \overrightarrow{E_v}$ where $\overrightarrow{I_{é}}^{v} = \{v\}$,since $e \in U_{é}$ we get $e \in \overrightarrow{E_v}$ where $\overrightarrow{I_e}^{v} = \overrightarrow{I_{é}}^{v} = \{v\}$ and hence $\overrightarrow{I_e}^{v} = \overrightarrow{I_{é}}^{v}$.
Remark 3.31: The digtopology τ_{ID} in any digraph $D = (V, E)$ is not necessary T_0 in general.
Example 3.32: According to Example 3.12 we get the digtopology $\tau_{ID} = \{ \emptyset , E(D)\}$ is not T_0 because $e_2 , e_3 \in E(D)$ But $\nexists$ open set A such that $e_2 \in A$ *and* $e_3 \notin A$ or $e_3 \in A$ *and* $e_2 \notin A$.
Remark 3.33 : Let C_n be a cyclic digraph such that every edges are in the same direction then we get the digtopology τ_{ID} is T_0 and if the edges are not all in the same direction we get the the digtopology τ_{ID} is not T_0.
The next Example are illustrates the remark 3.33 .
Example 3.34 :
(i) By according Example 3.5 we note that the cyclic digraph C_5 all edges in the same direction

and hence we note the digtopology τ_{ID} on C_5 is T_0 .

(ii) By according example 3.6 we note that the cyclic digraph C_6 all edges are not in the same direction and hence we note the digtopology τ_{ID} is not T_0 , because $ẹ_3, ẹ_4 \in Ẹ$ but $\nexists A \in \tau_{ID}$ such that $ẹ_3 \in A$ *and* $ẹ_4 \notin A$ *or* $ẹ_3 \notin A$ *and* $ẹ_4 \in A$.

Remark 3.35 : Let P_n be a path digraph such that every edges are in the same direction then we get the digtopology τ_{ID} is T_0, and if the edges are not all in the same direction we get the digtopology τ_{ID} is not necessary T_0.

The next two Example are illustrates the Remark 3.35 .

Example 3.36:

(i) By according Example 3.8 we note that the path digraph P_5 all edges in the same direction and hence we note the digtopology τ_{ID} on P_5 is T_0.

(ii) By according Example 3.9 we note that the path digraph P_5 all edges are not in the same direction and hence we note the digtopology τ_{ID} is T_0.

(iii) By according Example 3.10 we note that the path digraph P_5 all edges are not in the same direction and hence we note the digtopology τ_{ID} is not T_0, because $ẹ_2, ẹ_3 \in Ẹ$ but $\nexists A \in \tau_{ID}$ such that $ẹ_2 \in A$ *and* $ẹ_3 \notin A$ *or* $ẹ_3 \notin A$ *and* $ẹ_2 \in A$.

Remark 3.37: Let S_n be a star digraph such that every edges are indgree to the center vertex then we get the digtopology τ_{ID} is not T_0, and if the every edges are not indgree to the center vertex we get the digtopology τ_{ID} is T_0. Also, if only one edge from set edges is directed in to center vertex then we have the digtopology τ_{ID} is T_0 .

The next Example are illustrates the Remark 3.37.

Example 3.38 :

(i) By according Example 3.12 in Figure (9)(A) we note that the star digraph S_5 every edges are indgree to the center vertex and hence we note the digtopology τ_{ID} is not T_0 ,because $ẹ_2, ẹ_4 \in Ẹ$ but $\nexists A \in \tau_{ID}$ such that $ẹ_2 \in A$ *and* $ẹ_4 \notin A$ *or* $ẹ_2 \notin A$ *and* $ẹ_4 \in A$.

(ii) By according Example 3.12 in figure (7)(B) we note that the star digraph S_5 every edges are not indgree to the center vertex we get the digtopology τ_{ID} is T_0.

Proposition 3.39: The digtopology τ_{ID} in any digraph $Ɗ = (Ṿ, Ẹ)$ is T_0 if and only if $\overrightarrow{I_{ẹ}} \neq \overrightarrow{I_{é}}$ for every distinct pair of edges $ẹ, é \in Ẹ$.

Prove: $\Longrightarrow$ Suppos the digtopology τ_{ID} is T_0 to prove $\overrightarrow{I_{ẹ}} \neq \overrightarrow{I_{é}}$ for every distinct pair of edges $ẹ, é \in Ẹ$.

If $\overrightarrow{I_{ẹ}} = \overrightarrow{I_{é}}$ then by Theorem 3.30 $ẹ \in U_{é}$ and we get there exist u is open set such that $é \in u$ and $ẹ \in u$ implies the digtopology τ_{ID} is not T_0 this contradiction .then $\overrightarrow{I_{ẹ}} \neq \overrightarrow{I_{é}}$ for every distinct pair of edges $ẹ, é \in Ẹ$.

$\Leftarrow$ since $\overrightarrow{I_{ẹ}} \neq \overrightarrow{I_{é}}$ for every distinct pair of edges then by corollary 3.16 digtopology τ_{ID} is discrete implies the digtopology τ_{ID} is T_0 .

The next Example is illusteted this Proposition 3.39.

Example 3.40: Acorroding Example 3.5, we note that $\overrightarrow{I_{ẹ}} \neq \overrightarrow{I_{é}}$ for every distinct pair of edges [since $\overrightarrow{I_{ẹ_1}}^{v} = \{ṿ_5\}, \overrightarrow{I_{ẹ_2}}^{v} = \{ṿ_1\}, \overrightarrow{I_{ẹ_3}}^{v} = \{ṿ_2\}, \overrightarrow{I_{ẹ_4}}^{v} = \{ṿ_3\}, \overrightarrow{I_{ẹ_5}}^{v} = \{ṿ_4\}$]then we get the digtopology τ_{ID} is T_0 .

And we note, that in the Example 3.6 [$\overrightarrow{I_{ẹ_1}}^{v} = \{ṿ_1\}, \overrightarrow{I_{ẹ_2}}^{v} = \{ṿ_3\}, \overrightarrow{I_{ẹ_3}}^{v} = \{ṿ_4\}, \overrightarrow{I_{ẹ_4}}^{v} = \{ṿ_4\}$, $\overrightarrow{I_{ẹ_5}}^{v} = \{ṿ_5\}, \overrightarrow{I_{ẹ_6}}^{v} = \{ṿ_1\}$]there exists $\overrightarrow{I_{ẹ_1}}^{v} = \overrightarrow{I_{ẹ_6}}^{v}$ and hence, the digtopology τ_{ID} is not T_0 .

Corollary 3.41: The digtopology τ_{ID} in any digraph $Ɗ = (Ṿ,Ẹ)$ is T_0 if and only if it is discrete .

Prove : The proof is easy by Properties 3.39 and Corollary 3.16.

Remark 3.42: The digtopology τ_{ID} in any digraph $Ɗ = (Ṿ,Ẹ)$ is not necessary T_1 in general .

Example 3.43: According to Example 3.3 we get the digtopology τ_{ID} is not T_1 because $ẹ_4 , ẹ_6 \in Ẹ(Ɗ)$ But $\nexists$ open set A such tha $ẹ_4 \in A$ *and* $ẹ_6 \notin A$ and $ẹ_6 \in A$ *and* $ẹ_4 \notin A$.

Remark 3.44: Let C_n be a cyclic digraph such that every edges are in the same direction then we get the digtopology τ_{ID} is T_1, and if the edges are not all in the same direction we get the digtopology τ_{ID} is not T_1.

The next Example are illustrates the Remark 3.44.

Example 3.45 :

(i) By according Example 3.5 we note that the cyclic digraph C_5 all edges in the same direction and hence we note the digtopology τ_{ID} on C_5 is T_1.

(ii) By according Example 3.6 we note that the cyclic digraph C_6 all edges are not in the same direction and hence we note the digtopology τ_{ID} is not T_1 ,because $ẹ_1, ẹ_6 \in Ẹ$ but $\nexists A \in \tau_{ID}$ such that $ẹ_1 \in A$ *and* $ẹ_6 \notin A$ and$ẹ_6 \in A$ *and* $ẹ_1 \notin A$.

Remark 3.46 : Let P_n be a path digraph such that every edges are in the same direction then we get the digtopology τ_{ID} is T_1, and if the edges are not all in the same direction we get the digtopology τ_{ID} is not necessary T_0.

This Remark illustrates in the next Example.

Example 3.47:

(i) By according Example 3.8 we note that the path digraph P_5 all edges in the same direction and hence we note the digtopology τ_{ID} on P_5 is T_1.

(ii) By according Example 3.9 we note that the path digraph P_5 all edges are not in the same direction and hence we note the digtopology τ_{ID} is T_1.

(iii) By according Example 3.10 we note that the path digraph P_5 all edges are not in the same direction and hence we digtopology τ_{ID} is not T_1, because $ẹ_2, ẹ_3 \in Ẹ$ but $\nexists A \in \tau_{ID}$ such that $ẹ_2 \in A$ and $ẹ_3 \notin A$ and $ẹ_3 \notin A$ and $ẹ_2 \in A$.

Proposition 3.48: The digtopology τ_{ID} in any digraph Ɗ = (Ṿ,Ẹ) is T_1 if and only if $\overrightarrow{I_ẹ} \neq \overrightarrow{I_é}$ for every distinct pairs of edges $ẹ, é \in Ẹ$ **.**

Prove: $\Longrightarrow$ Suppos the digtopology τ_{ID} is T_1 to prove $\overrightarrow{I_ẹ} \neq \overrightarrow{I_é}$ for every distinct pair of edges $ẹ, é \in Ẹ$.

If $\overrightarrow{I_ẹ} = \overrightarrow{I_é}$ then by Theorem 3.30 $ẹ \in U_é$ and we get u is open set such that $é \in u$ and $ẹ \in u$ implies the digtopology τ_{ID} is not T_1 this contradiction, thus $\overrightarrow{I_ẹ} \neq \overrightarrow{I_é}$ for every distinct pair of edges $ẹ, é \in Ẹ$.

$\Leftarrow$ Let $\overrightarrow{I_ẹ} \neq \overrightarrow{I_é}$ for every distinct pair of edge by Corollary 3.16 implies the digtopology τ_{ID} is discrete and hence τ_{ID} is T_1 .

Corollary 3.49: The digtopology τ_{ID} in any digraph Ɗ = (Ṿ,Ẹ) is T_1 if and only if it is discrete .

Prove : The proof is easy by Properties 3.48 and Corollary 3.16 .

Proposition 3.50: The digtopology τ_{ID} in any digraph Ɗ = (Ṿ,Ẹ) is T_0 if and only if T_1 .

Prove: By Proposition 3.46 and Proposition 3.41.

Corollary 3.51: Let Ɗ = (Ṿ, Ẹ) be a digraph, For every $ẹ \in Ẹ$ we have $\overline{U_ẹ} = \overline{\overrightarrow{Ẹ_ṿ}}$ where $\overrightarrow{I_ẹ}^{v} = \{ṿ\}$.

Prove: Let $ẹ \in Ẹ$ by Proposition 3.28 we get $U_ẹ = \overrightarrow{Ẹ_ṿ}$ where $\overrightarrow{I_ẹ}^{v} = \{ṿ\}$ Therefor $\overline{U_ẹ} = \overline{\overrightarrow{Ẹ_ṿ}}$ where $\overrightarrow{I_ẹ}^{v} = \{ṿ\}$.

Corollary 3.52: Given a digraph Ɗ = (Ṿ,Ẹ). for every $ẹ \in Ẹ$ $\overline{\{ẹ\}} \subseteq \overline{U_ẹ} = \overline{\overrightarrow{Ẹ_ṿ}}$ where $\overrightarrow{I_ẹ}^{v} = \{ṿ\}$.

Prove: Let $u \in \overline{\{ẹ\}}$ this implies $U \cap \{ẹ\} \neq \emptyset$ for all open set U containing ẹ. since $\{ẹ\} \subseteq U_ẹ$ this implies $U \cap U_ẹ \neq \emptyset$ for all open set U containing ẹ. hence $ẹ \in \overline{U_ẹ}$ and so, $\overline{\{ẹ\}} \subseteq \overline{U_ẹ}$. then by Corollary 3.42, $\overline{\{ẹ\}} \subseteq \overline{U_ẹ} = \overline{\overrightarrow{Ẹ_ṿ}}$ where $\overrightarrow{I_ẹ}^{v} = \{ṿ\}$.

Corollary 3.53: For any $ẹ, é \in Ẹ$ in a digraph Ɗ = (Ṿ,Ẹ) we have, $é \in \overline{\{ẹ\}}$ if and only if $\overrightarrow{I_ẹ} = \overrightarrow{I_é}$.

Prove: $é \in \overline{\{ẹ\}} \Leftrightarrow U \cap \{ẹ\} \neq \emptyset$ for all open set U containing $é \Leftrightarrow ẹ \in U_é \Leftrightarrow \overrightarrow{I_ẹ} = \overrightarrow{I_é}$, by corollary 3.28 .

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