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Develop A New Approach For Solving Np Hard Optimization Problem

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Abstract *— The aim of this paper is to develop a novel approach to solve an optimization problem in a Semidefinite domain Programming. In fact, the K-CLUSTER problem has been identified as an optimization problem, which falls into the category of Np hard problems. Our technical approach is to rely on the bound of the semidefinite programming to find the bound of the optimization problem and then obtain approximate solution. In addition, the penalty method and the Lagrangian method were adopted to find the numerical results. Moreover, a hybrid method was devised that combines the two methods. The basis of its work is to pick up the solution from the faster method. Finally, highly efficient results are obtained, and the hybrid method can be adopted as a basis for solving other NP hard optimization problems.*

Keywords— Optimization technique, K-CLUSTER problem, Lagrangian method and penalty method.

1. INTRODUCTION

Optimization is a fundamental mathematical strategy for determining the values of variables that yield the smallest or greatest value for a mathematical function. The structure of optimization issues is separated into two types: continuous problems with continuous variables and discrete problems with discrete variables, often known as combinatorial optimization. Optimization theory and methods is a relatively new branch of applied mathematics. Though optimization may have originated with extreme-value issues, it did not become a distinct field until the late 1940s, when G.B [1]. Dantzig developed the well-known simplex algorithm for linear programming. Nonlinear programming advanced significantly after the 1950s, when conjugate gradient methods and quasi-Newton ethods were introduced. Modern optimization methods can now address challenging and large-scale optimization issues, making them a vital tool for tackling difficulties in a variety of sectors. Optimization algorithms are a fundamental and effective technique in mathematical programming for arriving at a solution, typically with the assistance of a computer. Optimization algorithms begin with an initial estimate of the value of the variables and generate a series of better estimates, or iterates, until an optimal solution is obtained [2]. A good algorithm should be precise, fast, efficient, and robust. Nonlinear programming advanced significantly after the 1950s, when conjugate gradient methods and quasi-Newton methods were introduced. Modern optimization methods can now address challenging and large-scale optimization problems, making them a vital tool for problem solving in a variety of fields. Optimization algorithms are a fundamental and effective technique in mathematical programming for arriving at a solution, typically with the assistance of a computer. Optimization algorithms begin with an initial estimate of the value of the variables and generate a series of better estimates, or iterates, until an optimal solution is obtained. A good algorithm should be precise, fast, efficient, and robust. Finally, semidefinite programming is a relatively recent topic from which we will begin to locate the starting point for finding an approximate solution to the optimization problem [3,4].

2. NP-HARD IN CLUSTERS

When each connected component of G is a complete graph a graph $G = (V, E)$ is called a cluster graph. For example, a complete sub graph of a graph (V, E) is called a clique. The decision problem is to find out if the graph has a clique of size k and the optimization problem is to find the max clique in the graph. A clique decision problem is NP hard problem for large graphs according to the following proof: If we are given a clique, we can easily determine if it is a clique of size k by counting the number of vertices in the clique; normally this answer cannot be verified in polynomial-time for a large graphs, hence determining the size of a clique is an NP problem [4-6].

3. 1. SEMIDEFINITE PROGRAMMING

The semidefnite programming in combinatorial optimization is subfield of optimization. Actually, in recent years semidefinite programming has become a widely used tool for designing more efficient algorithms for approximating hard combinatorial optimization problems. Also, semidefinite programming, as opposed to linear programming, allows to work with positive semidefinite matrix variables. This field has recently generated a lot of interest, such as applications in optimization theory and control theory as well as the creation of effective internal point methods. However, semidefinite programming is not a novel concept in combinatorial optimization. Since the late 1960s, eigenvalue bounds have been given for combinatorial optimization problems. Semidefinite programs . may typically be recast as eigenvalue bounds. This reformulation is beneficial because it takes use of convex programming traits like duality and polynomial-time solvability while avoiding eigenvalue optimization problems. As shown in the figure below, the semidefinite program (SDP) field and its importance in that it includes several basic fields, the first of which is the linear programming field [5-8].

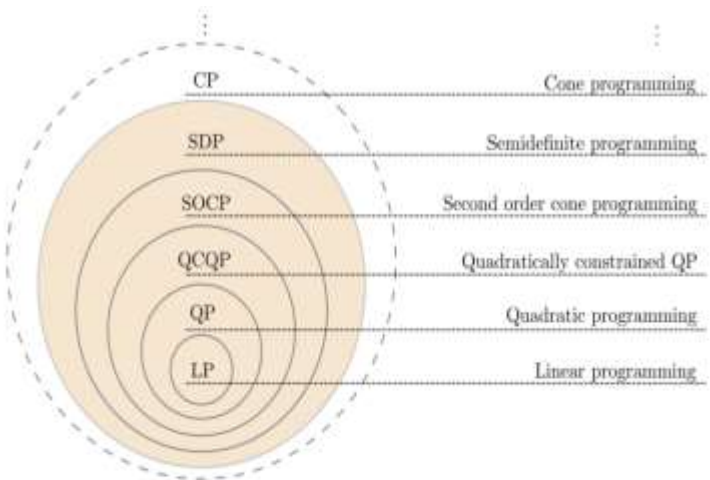

Figure 1: (SDP) and its position at the top of the convex optimization hierarchy

4. K-CLUSTER PROBLEM

The K-CLUSTER problem entails finding a subgraph with the heaviest weight and exactly k nodes$(1 < k < n - 1)$. In fact, this is a classical combinatorial optimization problem and has been shown to be NP-hard and also hard to approximate in (Deogun et al., 1997). Previously, the K-CLUSTER problem was solved to optimality by the semidefinite-based branch-and-bound algorithm, which compared favorably with the convex quadratic relaxation method. Generally, clustering is a common multivariate data analysis procedure [5,8,9]. The general Formula of K-CLUSTER problem is :

$$\begin{aligned} \text{maximize} \quad & \frac{1}{2} z^T W z \\ \text{subject to} \quad & \mathrm{e}^T z = k, \\ & z \in \{0,1\}^n, k \in z. \end{aligned}$$

5. APPROXIMATE METHOD AUGMENTED LAGRANGIAN AND PENALTY METHODS

Approximation methods are a very active area in optimization. Consider the minimizing convex function $\mathcal{G}: R^n \to R$ on a convex set X. The goal of approximation methods is to replace $\mathcal{G}$ and X with approximation $\mathcal{G}^{\mathrm{k}}$ and X^{k}. The approximation method works only if the approximation is easier than the original problem . For each iteration k we tried to find

$$\mathrm{X}^{\mathrm{k+1}} = \arg min_{x \in X^k} \mathcal{G}^{\mathrm{k}}(x)$$

then at the next iteration, $\mathcal{G}^{\mathrm{k+1}}$ and $\mathrm{X}^{\mathrm{k+1}}$ are generated by the approximation which depends on the new point $\mathrm{x}^{\mathrm{k+1}}$. Many great approximation methods are based on this idea, such as polyhedral approximation, the penalty method, the augmented Lagrangian method and interior point methods. Our work focuses on the penalty and augmented Lagrangian methods 11. In general, the existence of constraints complicates the algorithmic solution and narrows the range of viable methods in optimization problems. As a result, it's only reasonable to try to remove constraints by approximating the relevant indicator function. For example, replace restrictions with penalty functions that impose a significant cost for breaking them. The linear equality constrained problem is given by

$$\begin{aligned} \text{minimize} \quad & \langle c, x\rangle \\ \text{subject to} \quad & \langle a_i, x\rangle = b_i, i = 1, \dots, m, \\ & x \in X. \end{aligned}$$

replaced above problem with a penalized version

$$\begin{aligned} \text{minimize} \quad & \langle c, x\rangle + \alpha^k \sum_{i=1}^{m} P_q(\langle a_i, x\rangle - b_i) \\ \text{subject to} \quad & x \in X. \end{aligned}$$

The scalar α^k is a positive penalty parameter, and decreasing α^k to 0 , the solution x^k of the penalized problem tends to decrease the constraint violation, thereby providing an increasingly accurate approximation to the original problem. An important practical point here is that α^k should be decreased gradually, using the optimal solution of each approximating problem to start iteration of the next approximating problem . One choice for P_q can be the quadratic penalty function, where the penalized problem (1.3)

takes the form 12,98 ,

$$\begin{aligned} \text{minimize} \quad & \langle c, x\rangle + \frac{1}{2\alpha^k} \| Ax - b \|^2 \\ \text{subject to} \quad & x \in X. \end{aligned}$$

where $Ax = b$ represents the system of equation $\langle a_i, x\rangle = b_i, i = 1, \dots, m$. The augmented Lagrangian method is a significant improvement over the penalty function approach, where we add a linear term to $P_q(y)$, involving a multiplier vector $y^k \in \mathbb{R}^n$. Then instead of problem (1.3), we solve the problem

$$\begin{aligned} \text{minimize} \quad & \langle c, x\rangle + \left(y^k\right)^T (Ax - b) + \frac{1}{2\alpha^k} \| Ax - b \|^2 \\ \text{subject to} \quad & x \in X. \end{aligned}$$

After the solution of the above problem x^k is obtained, the multiplier vector y^k is updated by some formula that seeks to approximate an optimal dual solution [6],such that

$$y^{k+1} = y^k + \frac{1}{\alpha^k}\left(Ax^k - b\right).$$

This is called The first order augmented Lagrangian methods is what it's called (the first order method of multipliers). For both inequality and equality requirements, penalty and augmented Lagrangian methods can be employed [5-13].

6. THE MAIN IDEA

The semidefinite programming field is a very important field because it contains many fields, which can be seen clearly as in the figure. Therefore, the idea came to obtain a solution to the NP hard optimization problem, where the semidefinite (SDP) bound will be adopted as a bound for the K-CLUSTER (KC) problem, based on the mathematical relationship that states

$$KC \ bond \leq SDP \ bond$$

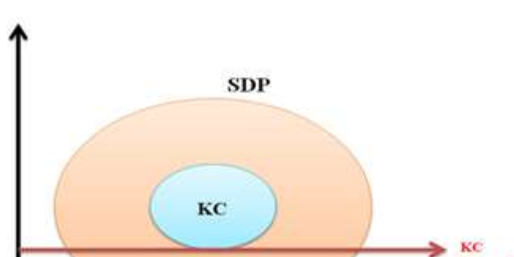

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Figure 2: The idea of the solution according to the simple scheme

7. Numerical Resultes

As shown in these results, the Figure (3) of graphs types (be120.3 and be120.8) depicts the convergence of a solution . Actually, the sample (be120.8.2) was the focus of the test, the hybrid method clearly has better convergence characteristics than the other two methods (penalty and augmented Lagrangian). The exact solution of KCLUSTER in this figure appears to be equal to 1882, while the semidefinite bound is equal to 1902. The hybrid method algorithm attained this bound faster at 444 function calls while the Augmented Lagrangian method reach that by 449 function calls, but the penalty method reach that by 1802 function calls. In this graph, the penalty method gave a slower convergence time to attain the same bound. Getting to the bound with the fewest number of function calls is saving time, so the method is judged to be the fastest. Finally, in this test the hybrid approach is deemed to be the quickest since it gets to the bound with the fewest amount of function calls. These graphs types (be120.3 and be120.8) selected from the Biq Mac library [14,15].

Our results					
Graphs	**Optimal value**	**Penalty**	**Augmented**	**Hybrid**	**The goal bound**
be120.3.1	13067	2200	2191	**2049**	13100.8
be120.3.2	13046	1200	1100	**1054**	13046.2
be120.3.3	12418	1173	1062	**1000**	12418.3
be120.3.4	13867	2922	2200	**1512**	13873
be120.3.5	11403	1589	**1009**	1321	11403
be120.3.6	12915	987	**900**	1804	12915
be120.3.7	14068	758	770	**749**	14068
be120.3.8	14701	**689**	748	825	14701
be120.3.9	10458	2002	1660	**1559**	10486
be120.3.10	12201	1520	1195	**1192**	12203.8
Total winner fcalls		1	2	7	

Table (1): The test types of graphs (be120.3 which has n=120 and d=0.3) which was selected from the Biq Mac library by the function calls of (augmented Lagrangian , penalty, hybrid) methods.

Our results					
Graphs	**Optimal value**	**Penalty**	**Augmented**	**Hybrid**	**The goal bound**
be120.8.1	18691	2321	2550	**2000**	18989.2
be120.8.2	18827	1802	449	**444**	19022.7
be120.8.3	19302	1846	**1408**	1533	19458.4
be120.8.4	20765	**1760**	1955	2473	20791
be120.8.5	20417	2090	2058	**2004**	20440.9
be120.8.6	18482	1789	**1270**	1510	18602.5
be120.8.7	22194	1900	2056	**1899**	22432.2
be120.8.8	19534	2077	2300	**2022**	19907.1
be120.8.9	18195	1983	**1095**	1890	18357.9
be120.8.10	19049	2230	**2080**	2381	19128
Total winner fcalls		1	4	5	

Table (2): The test types of graphs (be120.8 which has n=120 and d=0.3) which was selected from the Biq Mac library by the function calls of (augmented Lagrangian , penalty, hybrid) methods.

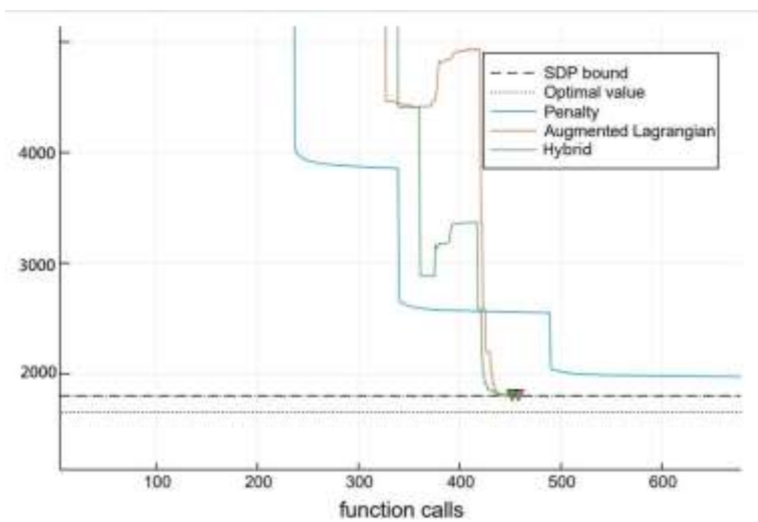

Figure 3: On graph (be120.8.2), bounds versus function calls for Augmented Lagrangian, penalty methods, and hybrid technique where y-axis represented the bound.

8. CONCLUSION

The new approach was evaluated using a variety of graphs from the Biq Mac library. These graphs included a variety of characteristics, including a huge number of nodes and edges.The results indicated that the Augmented lagrangian method was preferable than the Penalty method in terms of achieving the goal bound. We also put the Hybrid technique to the test, which alternates between the penalty and inequality Augmented lagrangian method. Finally, the Hybrid technique performed better than the two strategies independently, according to the findings.

9. ACKNOWLEDGMENT

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