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Solving Mathematical Optimization Models by Differential Equations

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Abstract— *A particular emphasis will be placed on optimum control problems that are subject to differential algebraic equations, which will be the subject of this paper. It is our goal to determine the minimum power and launch time required for rockets and satellite-carrying spacecraft while maintaining the highest possible level of performance quality. Finally, the numerical results were obtained through the use of the Python programming language.*

Keywords— Optimization, Differential Equations, Optimal Control, Python language.

1. Introduction

All aspects of human endeavor are now subject to optimization. As early as the prehistoric period, some type of optimization was being used. The history of optimal control theory can be traced back to the calculus of variations, which has been around for more than three centuries, also, the optimum control problems are distinguished by the separation of control from state variables and the acceptance of control constraints. [7,11,17,18] Since 1950, advances in optimal control theory and solution approaches have been made, primarily as a result of military applications. Russian mathematician V. G. Chernyshev and his colleagues V. G. Chernyshev and V. G. Chernyshev made the seminal discovery in the late 1920s and early 1930s. Most subjects, including the natural sciences, engineering sciences, and economics, can benefit from the study of optimal control issues in some way. [8,9] In addition to the traditional optimal control problems involving ordinary differential equations (ODEs), researchers have recently begun to investigate optimal control problems involving partial differential equations (PDEs), differential-algebraic equations (DAEs), and stochastic optimal control problems (SOC). It will be given particular consideration when searching for optimal control problems that are subject to differential algebraic equations, and it will be given particular consideration when searching for optimal control problems that are not subject to differential algebraic equations. Difference equations on manifolds (DEM) are referred to as differential equations on manifolds in the mathematical literature and applications. A differential equation on manifold is a composite system of differential equations and algebraic equations that is a composite system of differential equations and algebraic equations [19-21].

2. New Approach of Optimization Problem

The new optimization problem was set up as shown in Eq. (1)which called the objective function. Then Eq. (2) shows and Eq. (2) and Eq. (3) describe the state EOMs for this problem. Eq. (4)and Eq. (5) specify the boundary conditions for the states. Eq. (6) shows the constraint on the state x.

Minimize : $$g = \frac{1}{2}\int_0^1 (a(t) + (\tfrac{1}{2}))^2 dt \qquad (1)$$

subject to : $$\frac{dx(t)}{dt} = s(t) \qquad (2)$$

$$\frac{ds(t)}{dt} = a(t) \qquad (3)$$

$$X(0) = 0 = x(1) \qquad (4)$$

$$S(0) = 1 = -s(0) \qquad (5)$$

$$X(t) \le k, \quad k = \frac{1}{11} \qquad (6)$$

The parameter a (acceleration) is placed on the time horizon from a starting time of zero to a final time of one, where a represents the acceleration. Then x and s variables denote the position and velocity, respectively.

2.1 Objective Functions with Integral Objectives

We created a program to track our progress and show the results. 2 minutes is the estimated time. The optimal control problem has an integral objective function, which means it has a single objective function. In many systems, integrals are a naturally occurring expression of how to keep the accumulation of a quantity to an absolute minimum.

$$\text{Min :} \quad \frac{1}{2}\int_0^2 x_1^2\, exp(x_1 ln(x_1))(t)dx$$

$$\text{Subject to:} \quad \frac{dx_1}{dt} = u$$

$$X_1(0) = 1$$

$$-1 \leq u(t) \leq 1$$

3. Basic Concepts

3.1 Optimization problem

Finding the most efficient method of completing a task is a common theme in many of the world's most difficult practical problems. The most common application of this is to determine the maximum or minimum value of a function, such as the shortest time required to complete a journey or the lowest cost associated with completing a task, or the maximum amount of power that can be generated by a device, among other things. [1,10,15,16] Identifying the appropriate function and then applying calculus techniques to determine the maximum or minimum value required can solve many of these problems. The problem described as following:

Maximize or minimize : Objective function,

Subject to : Constraints

The general optimization problem form

Minimize F(X) Objective function

Subject to M(X) = 0 Equality Constraints,

N(X) ≥ 0 Inequality Constraints .

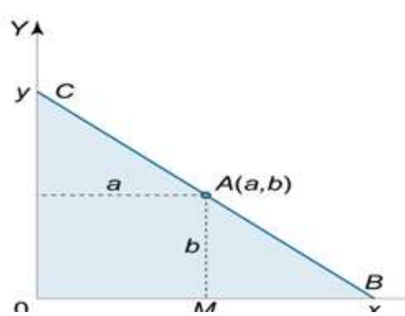


Figure (1): Optimization problem

3.2 Differential equation

It is possible to connect the derivatives of one or more functions in a generalized manner using a differential equation. Differential equations are used to define a relationship between two variables in an application, and functional equations are used to represent physical quantities.[5,6] In a variety of fields, such as engineering and physics, economics, and biology, difference equations play an important role because of the widespread nature of the relationships they represent. Calculus, invented by Newton and Leibniz, is credited with the first appearance of differential equations. Differential equations are classified as follows by Newton in Chapter 2 of his 1671 work:

$$\frac{dy}{dx} = f(x)$$

$$\frac{dy}{dx} = f(x, y)$$

$$x_1 \frac{\partial y}{\partial x_1} + x_2 \frac{\partial y}{\partial x_2} = y$$

If y is an unknown function of x (or x_1 and x_2), then f is the supplied function in all of these circumstances.

3.3 Optimal command and control

Due to its origins in the calculus of variations, optimal control is inextricably linked to that theory. Johann Bernoulli, for example, had a significant impact on the development of optimal control and variational calculus in the early years. [2,3] Richard Bellman (1920-1984) developed dynamic programming in the 1950s, Lev Pontryagin (1908-1988) developed the minimum principle, and Richard Bellman (1920-1984) developed the minimum principle in the 1960s. These were all major milestones in the development of optimal control during the twentieth century. It's a hot topic in the realm of control theory since it has applications in so many different fields, from aerospace to process control to robotics to bioengineering to economics to finance to management science.[4,12] With the advent of the digital computer in the 1950s, only the most straightforward optimum control problems could be solved.

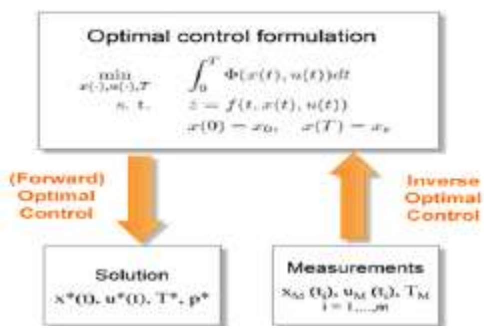


Figure (2): Optimal control formulation

4. Python Language

This programming language is designed to be used for a wide range of applications and is interpreted at a high level of abstraction. Code readability is highly valued in its design philosophy, which is reflected in the extensive use of indentation in the code. Its object-oriented approach and language elements are meant to help programmers write clear and logical code and will be advantageous to both small and large-scale projects. Python is a dynamically typed, garbage-collecting programming language. Structured (particularly procedural), object-oriented, and functional programming paradigms can all be supported by this programming language[13]. The term "batteries included" is frequently used to describe the language because of its extensive standard library.

4.1 GEKKO Optimization

In addition to other mathematical problems, the GEKKO Python package is used for machine learning and optimization of mixed-integer and differential algebraic equations, as well as other mathematical problems. [6,9] Integrated into the system are large-scale linear programming, quadratic programming, nonlinear programming, and mixed integer programming solvers that operate on massively parallel computers (LP, QP, NLP, MILP, MINLP). Some of the modes of operation available to you consist of parameter regression, data reconciliation, real-time optimization, dynamic simulation, and nonlinear predictive control, among other things, to name a few examples. This package, GEKKO, is an object-oriented Python library that makes it easier to run the application AP Monitor on a local computer.

5. Application of New Approach

Optimal control and mathematical programming approaches can be used to solve a wide variety of difficulties that arise during space transportation. It is possible to distinguish three types of problems depending on the point of departure and the point of arrival:

ascent from the Earth's surface to an orbit, reentry from an orbit to the Earth's surface (or to another body in the solar system), and transfer from one spacecraft to another spacecraft. Ascent, transfer, and descent are the traditional divisions of a space mission. It takes a lot of propellant mass to get big payloads like communications satellites into orbit and deliver them, which is why ascent missions are so critical. The thrust required for such a large lift-off mass can only be achieved by chemical rocket engines. These missions, which last about 30 minutes, are focused on reducing the amount of energy they use. Heavy launchers take off vertically from a fixed ground launch pad, and this function is used in the best control of the least amount of energy and time spent on the launch pad during the launch.

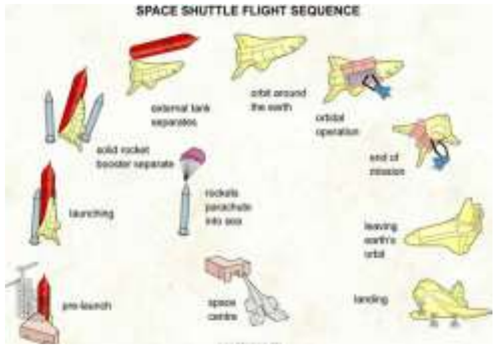


Figure (3): Bryson-Denham optimal control problem

6. Numerical Results in New Approach

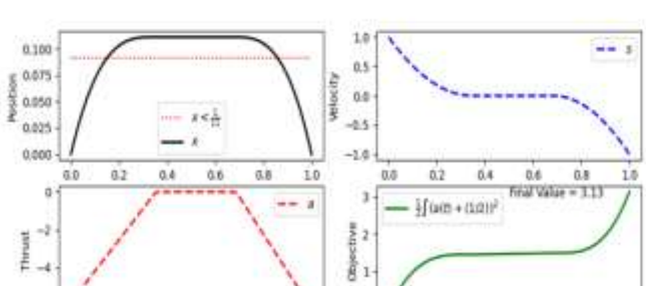
When applying the new approach from the two previous objective functions, we obtained the optimum time and energy to solve the problem, as shown in the figure(4,5) and table(1,2):

iter	objective	convergence	iter	objective	convergence
0	4.03129E+05	3.72500E+03	11	-5.70170E+01	4.62700E-01
1	1.75732E+05	9.60840E+02	12	-5.70607E+01	3.10217E-03
2	6.90421E+04	6.78857E+02	13	3.12315E+00	6.82940E-03
3	-5.88901E+02	2.91141E+01	14	3.12260E+00	1.32320E-03
4	-5.91530E+02	1.89425E+00	15	3.12263E+00	1.58776E-03
5	-5.91704E+02	7.17573E-02	16	3.12251E+00	1.34267E-01
6	-5.91711E+02	1.19843E-04	17	3.71023E-01	3.79003E-02
7	-3.96052E+01	7.15202E-03	18	3.12248E+00	1.48173E-05
8	-5.62293E+01	1.36105E-02	19	3.12248E+00	1.90261E-06
9	-5.63841E+01	1.95298E-02	20	3.12253E+00	5.77179E-09
10	2.52136E+01	6.16765E+00			

Table (1) New Approach

Solution time : 0.2943 sec

Objective : 3.132975568965791



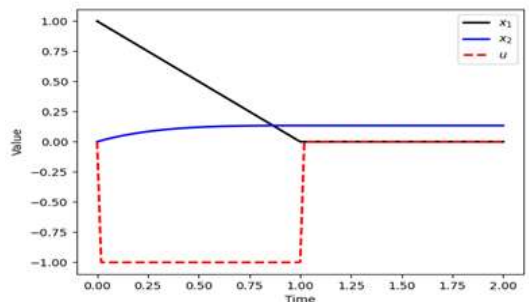
Figure(4) New Approach

	scaled	unscaled
Objective	1.3410879988895941e-01	1.3410879988895941e-01
Dual infeasibility	5.5511151231257827e-17	5.5511151231257827e-17
Constraint violation	2.2204460492503131e-16	2.2204460492503131e-16
Complementarity	1.0000000525685841e-11	1.0000000525685841e-11
Overall NLP error	1.0000000525685841e-11	1.0000000525685841e-11

Table (2): Integral Objective Function

Solution time : 0.280599999998230 sec

Objective : 0.134108799888959



Figure(5) Integral Objective Function

6.1 The Description

In the first table, we notice that the numerical value of the objective function in the repetition number (13, 14, 15, 16, 17, 18, 19. 20) depends on the same value and thus we choose it as the smallest value of the objective function at the time (0.2943 sec). As for the fourth figure, The problem can be visualized as a frictionless box moving sideways with the initial velocity of 1 m/s. The box is only allowed to move a maximum of 1/11 m before it has to turn around. The box has to get back to the starting point after 1 second and it has to have the same velocity when it started but in the other direction. In the second table, we got the lowest error rate for the indicated objective function at the time(0.280599999998230sec). And in the fifth figure, The initial condition of the integral is zero, and it becomes the integral between 0 and 2. At the end of the optimum control problem's time horizon, the value that is minimized is the starting value.

7. Conclusion

Two new functions have been developed to control the optimum minimum power and time used to launch rockets for transporting spacecraft. The goal was to reduce the energy used and save time to reach the optimal solution to the problem. The numerical results were obtained using the Python program.

8. REFERENCES

[1] Albert, A., Imsland, L., & Haugen, J. (2016). Numerical optimal control mixing collocation with single shooting: a case study. IFAC-PapersOnLine, 49(7), 290-295.
[2] Alridha, A., Wahbi, F. A., & Kadhim, M. K. (2021). Training analysis of optimization models in machine learning. International Journal of Nonlinear Analysis and Applications, 12(2), 1453-1461.
[3] Andersson, L. E., Scibilia, F., & Imsland, L. (2016). An estimation-forecast set-up for iceberg drift prediction. Cold Regions Science and Technology, 131, 88-107.
[4] Bartels, S. (2015). Error control and adaptivity for a variational model problem defined on functions of bounded variation. Mathematics of Computation, 84(293), 1217-1240.
[5] Beal, L. D., Hill, D. C., Martin, R. A., & Hedengren, J. D. (2018). Gekko optimization suite. Processes, 6(8), 106.
[6] Bremer, J., Rätze, K. H., & Sundmacher, K. (2017). CO2 methanation: Optimal start-up control of a fixed-bed reactor for power-to-gas applications. AIChE Journal, 63(1), 23-31.
[7] Gesenhues, J., Hein, M., Habigt, M., Mechelinck, M., Albin, T., & Abel, D. (2016, June). Nonlinear object-oriented modeling based optimal control of the heart: Performing precise preload manipulation maneuvers using a ventricular assist device. In 2016 European Control Conference (ECC) (pp. 2108-2114). IEEE.
[8] Gillis, J. (2015). Practical Methods for Approximate Robust Periodic Optimal Control ofNonlinear Mechanical Systems.
[9] Hedengren, J. D., Shishavan, R. A., Powell, K. M., & Edgar, T. F. (2014). Nonlinear modeling, estimation and predictive control in APMonitor. Computers & Chemical Engineering, 70, 133-148.
[10] Longuski, J. M., Guzmán, J. J., & Prussing, J. E. (2014). Optimal control with aerospace applications (pp. 193-212). New York: Springer.
[11] Kadhim, M. K., Wahbi, F. A., & Hasan Alridha, A. (2022). Mathematical optimization modeling for estimating the incidence of clinical diseases. International Journal of Nonlinear Analysis and Applications, 13(1), 185-195.
[12] Grant, M. J., & Antony, T. (2016). Rapid indirect trajectory optimization of a hypothetical long range weapon system. In AIAA Atmospheric Flight Mechanics Conference (p. 0276).
[13] Patterson, M. A., & Rao, A. V. (2014). GPOPS-II: A MATLAB software for solving multiple-phase optimal control problems using hp-adaptive Gaussian quadrature collocation methods and sparse nonlinear programming. ACM Transactions on Mathematical Software (TOMS), 41(1), 1-37.
[14] Singh, S., & Grant, M. J. (2018). The use of homotopy analysis method for indirect trajectory optimization. In 2018 AIAA Guidance, Navigation, and Control Conference (p. 0864).
[15] Antony, T., & Grant, M. J. (2018). Path constraint regularization in optimal control problems using saturation functions. In 2018 AIAA Atmospheric Flight Mechanics Conference (p. 0018).
[16] Antony, T., & Grant, M. J. (2017). Rapid indirect trajectory optimization on highly parallel computing architectures. Journal of Spacecraft and Rockets, 54(5), 1081-1091.
[17] Al-Jilawi, A. S., & Abd Alsharify, F. H. (2022). Review of Mathematical Modelling Techniques with Applications in Biosciences. Iraqi Journal For Computer Science and Mathematics, 3(1), 135-144.
[18] Alridha, A., & Al-Jilawi, A. S. (2022, October). K-cluster combinatorial optimization problems is NP_Hardness problem in graph clustering. In AIP Conference Proceedings (Vol. 2398, No. 1, p. 060034). AIP Publishing LLC.
[19] ALRIDHA, A. H., Mousa, E. A., & Al-Jilawi, A. S. (2022). Review of Recent Uncertainty Strategies within Optimization Techniques. Central Asian Journal of Theoretical and Applied Science, 3(6), 160-170.
[20] ALRIDHA, A. H., Salman, A. M., & Al-Jilawi, A. S. (2022). Numerical Optimization Approach for Solving Production Planning Problem Using Python language. CENTRAL ASIAN JOURNAL OF MATHEMATICAL THEORY AND COMPUTER SCIENCES, 3(6), 6-15.
[21] Alridha, A. H., & Al-Jilawi, A. S. (2022). Solving NP-hard problem using a new relaxation of approximate methods. International Journal of Health Sciences, 6, 523-536.