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Parameter and Reliability Estimation Left-truncated Normal Distribution using Bayesian Method on The Recovery Time of Covid-19 Patients

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Abstract: *The Bayesian method is a method to produce parameter estimates by combining sample information and other previously available information. In this study, the Bayesian method was used to estimate the parameters and reliability of the left-truncated normal distribution. The results of parameter estimation and reliability are applied to the recovery time of Covid-19 patients sourced from the Jemursari Public Health Center in January 2021. Furthermore, the number of data used was 15 data out of a total of 24 data, due to the left-truncated at point 13, the data used was data which was more than 13 days. The results of this study are the estimated average recovery time for Covid-19 patients is 15.17 days with a standard deviation of 4.19 days and the faster a patient's recovery time indicates that the patient's reliability or survival against Covid-19 disease is higher.*

Keywords—bayes estimation; left truncuted; normal distribution; reliabilitas; recovery time

1. INTRODUCTION

Phenomena which occur in life are always changing in a span of time. Changes which occur tend to be uncertain so that predictions are needed in order to get the best decisions. Changes in phenomena are called as event distributions or commonly known as probability distributions [1]. The thing which cannot be separated from the study of probability distributions is the estimation of parameters. Parameter estimation is a part of statistical inference. Statistical inference is making a decision in a situation [2].

Estimation is used to find the parameters of the distribution related to the data. Furthermore, estimation can be conducted by using two methods, including classical and Bayesian [2]. In the estimation process, both classical and Bayesian methods utilize sample data information. Meanwhile, the difference is that the Bayesian method utilizes sample data information and utilizes previous data information or prior distribution [3]. The prior distribution is divided into two, which are informative prior and non-informative prior with unknown parameter information. One of the methods used in determining non-informative priors is that Jeffrey's method [4].

The live test data analysis aims to estimate the unknown parameters of the data distribution model [5]. One of the distributions used in the analysis of the live test data was normal distribution. Meanwhile, the analysis of live test data has been widely developed, one of which is data truncating, namely data limitation to a certain value. As a result of data truncation, there are three possible forms of distribution, which are left-truncated distribution, right-truncated distribution, and left-right truncated distribution [6].

In addition to parameter estimation, live test data analysis aims to determine the reliability of a system, namely the probability that the system will work for at least one period of time without failure [7]. Meanwhile, in the field of medicine, reliability analysis is used to determine the patient's resistance to a disease.[8] Furthermore, study related to the estimation and reliability of a distribution had been conducted previously by Adel which aims to determine the estimation and reliability of the two-parameter Weibull distribution based on Bayesian with Jeffrey's priors [9].

Therefore, a study was conducted with a different distribution from the previous study that was by estimating the parameters and reliability of the left-truncated normal distribution. In this study, parameters and reliability of the left-truncated normal distribution were estimated based on the Bayesian method with the assumption that the priors used were Jeffrey's priors from the normal distribution. Moreover, the estimation results will be applied to the recovery time of Covid-19 patients sourced from Jemursari Public Health Center in January 2021. Referring to the World Health Organization's appeal, the mandatory isolation time for Covid-19 patients is the first 10 days plus 3 days without symptoms so that the total mandatory isolation time is 13 days, then the data used is left truncated data $(X > a)$ with a =13, which is data that is more than 13 days.

2. LITERATURE REVIEW

2.1 Bayesia Estimation

Parameter estimation can be conducted classical and Bayesian [2]. Classical estimation only uses the results of observations from the sample, and the parameter is considered as a fixed quantity whose value is unknown. Meanwhile, in Bayesian, in order to determine the estimation of the parameters besides using sample information, we also use the prior distribution. Based on the Bayesian method, parameters are random variables which have a distribution which is usually known as the prior distribution [2]. Bayesian estimation for parameter is:

$$\theta^* = E(\theta|x) = \int_{-\infty}^{\infty} \theta\, p(\theta|\underline{x})\, d\theta \tag{1}$$

where $p(\theta|\underline{x})$ is the marginal distribution.

2.2 Jeffrey's Prior

The prior distribution is the initial distribution which should be determined before determining the posterior distribution. Moreover, the use of the Bayesian method in the absence of prior information is called as non-informative prior, that is a prior without description θ, which means the type of distribution of θ is unknown [4]. One way to approach the prior distribution is that by using Jeffrey's prior. It is assumed that the prior used in this study is Jeffrey's prior normal distribution. Jeffrey's prior distribution of the normal distribution $p(\theta, \sigma^2)$ where θ and σ^2 are independent is:

$$p(\theta, \sigma^2) \propto p(\theta)\, p(\sigma^2) \propto \frac{1}{\sigma^2} \tag{2}$$

where $p(\theta)$ is Jeffrey's prior of parameter with $p(\theta) = c(constan)$ and $p(\sigma^2)$ is Jeffrey's prior of the parameter θ with $p(\theta) = c(constan)$ and $p(\sigma^2)$ is Jeffrey's prior of the parameter σ^2 with $p(\sigma^2) = \frac{1}{\sigma^2}$.

2.3 Reliability

An effective method for estimating reliability is the Bayesian method since it can produce more information related to parameter estimation [7]. If t is the failure time, thus, the reliability function is :

$$R(t) = 1 - F(t) = P(T > t) \tag{3}$$

Bayesian estimation for the reliability of a system at time t is also called as probability estimation with the Bayesian approach on a system which works according to its function without failure, at least at time t [10]. In addition, reliability estimation using Bayesian method on a certain distribution with parameters θ and σ^2 is defined as:

$$R(t)^* = \iint R(t)\, g(\theta|\underline{x})\, d\theta\, d\sigma^2 \tag{4}$$

with,

$R(t)$ = Reliability Function

$g(\theta|\underline{x})$ = Posterior Distribution

3. MATERIAL AND MATHOD

3.1 Data Sources

The data used in the application of the paramater and reliability estimation results was data on the recovery time of Covid-19 patients. This data sourced from Jemursari Public Health Center, Surabaya City in January 2021. In addition, the number of patients which were exposed during that time was 24, but only 15 which were used since the left-truncated was conducted on the data, which only used data on the recovery time of patients who were more than 13 days.

3.2 Step of Analysis

The analysis step begins with determining the form of parameter and reliability estimation of the left-truncated normal distribution. Then, the estimation results obtained will be applied to the data data on the recovery time of Covid-19 patients. Completely, the analysis steps are as follows:

1. Determining the form of the likelihood function of the left-truncated normal distribution.
2. Determining the type of prior which was used, that was Jeffrey's prior from the normal distribution.
3. Determining the posterior distribution of the left-truncated normal distribution.
4. Determining the marginal distribution of the left-truncated normal distribution.
5. Determinng the estimation left-truncated normal distribution parameter.
6. Determining the reliability estimate of the left-truncated normal distribution.
7. Creating a program to complete steps (4), (5), and (6) by using Mathematica software.
8. Testing the normality of data recovery Covid-19 patients by using SPSS software.
9. Implementing the program on data recovery Covid-19 patients.

4. RESULT AND DISCUSSION

4.1 Left-truncated Normal Distribution Function

Assumed that $X \sim N(\theta, \sigma^2)$ is a random variable with a normal distribution with an average θ and variance σ^2. The form of the Probability Density Function (PDF) of random variable X is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\theta}{\sigma}\right)^2}$$

if assumed $\phi\left(\frac{x-\theta}{\sigma}\right) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\theta}{\sigma}\right)^2}$, so

$$f(x) = \frac{1}{\sigma}\phi\left(\frac{x-\theta}{\sigma}\right) \tag{5}$$

Cumulative Density Function (CDF) of random variable X is:

$$F(x) = P(X \le x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\theta}{\sigma}\right)^2} dx$$

if assumed $z = \frac{x-\theta}{\sigma}$, so

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$$F(x) = \int_{-\infty}^{\frac{x-\theta}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$$
$$= \int_{-\infty}^{\frac{x-\theta}{\sigma}} \phi(z)\, dz$$
$$= \Phi\left(\frac{x-\theta}{\sigma}\right) \quad (6)$$

Furthermore, determine the PDF of the left-truncated normal distribution by previously finding the probability $X > a$ is:

$$P(X > a) = 1 - P(X \leq a)$$
$$= 1 - \Phi\left(\frac{a-\theta}{\sigma}\right) \quad (7)$$

Thus, the PDF form of the left-truncated normal distribution is the division of the PDF in equation (5) with probability $X > a$ in equation (7):

$$f(x|X > a;\ \theta, \sigma^2) = \frac{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\theta}{\sigma}\right)^2}}{1 - \Phi\left(\frac{a-\theta}{\sigma}\right)} \quad (8)$$

and the form of the likelihood function is:

$$L(\theta, \sigma^2\ |\underline{x}) = \prod_{i=1}^{n} f(x_i|x_i > a\ ;\theta, \sigma^2)$$
$$= \frac{\left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{-\frac{1}{2\sigma^2}\Sigma(X_i-\theta)^2}}{\left(1-\Phi\left(\frac{a-\theta}{\sigma}\right)\right)^n} \quad (9)$$

4.2 Prior Distribution

The prior distribution used in this study was Jeffrey's prior normal distribution with θ and σ^2 are independent. Jeffrey's prior distribution of the normal distribution $p(\theta, \sigma^2)$ where θ and σ^2 are independent according to equation (2).

4.3 Distribusi Posterior

The posterior distribution of the left-truncated normal distribution is:

$$g(\theta, \sigma^2|\underline{x}) = \frac{L(\theta, \sigma^2|\underline{x})\, p(\theta, \sigma^2)}{\int L(\theta, \sigma^2|\underline{x})\, p(\theta, \sigma^2)}$$

$$= \frac{\left(\frac{(\frac{1}{\sigma\sqrt{2\pi}})^n e^{-\frac{1}{2\sigma^2}\Sigma(X_i-\theta)^2}}{(1-\Phi\left(\frac{a-\theta}{\sigma}\right))^n}\right)(\frac{1}{\sigma^2})}{\int_{\sigma^2=0}^{\infty}\int_{\theta=-\infty}^{\infty}\left(\frac{(\frac{1}{\sigma\sqrt{2\pi}})^n e^{-\frac{1}{2\sigma^2}\Sigma(X_i-\theta)^2}}{(1-\Phi\left(\frac{a-\theta}{\sigma}\right))^n}\right)(\frac{1}{\sigma^2})d\theta\, d\sigma^2}$$

if assumed,

$$u = \int_{\sigma^2=0}^{\infty}\int_{\theta=-\infty}^{\infty}\left(\frac{(\frac{1}{\sigma^{n+2}})\, e^{-\frac{1}{2\sigma^2}\Sigma(X_i-\theta)^2}}{(1-\int_{-\infty}^{\frac{a-\theta}{\sigma}}\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{z^2}{2}}\, dz)^n}\right)d\theta\, d\sigma^2 \text{ , so}$$

$$g(\ \theta, \sigma^2|\underline{x}) = \frac{1}{u}\left(\frac{(\frac{1}{\sigma^{n+2}})\ e^{-\frac{1}{2\sigma^2}\Sigma(X_i-\theta)^2}}{(1-\Phi\left(\frac{a-\theta}{\sigma}\right))^n}\right) \quad (10)$$

4.4 Parameter Estimation

1. Parameter Estimation of θ

Firstly, determine the marginal distribution of θ as follows:

$$p(\theta|\underline{x}) = \int_0^{\infty} g(\ \theta, \sigma^2|\underline{x})\ d\sigma^2$$
$$= \frac{1}{u}\int_0^{\infty}\left(\frac{(\frac{1}{\sigma^{n+2}})\ e^{-\frac{1}{2\sigma^2}\Sigma(X_i-\theta)^2}}{(1-\Phi\left(\frac{a-\theta}{\sigma}\right))^n}\right) d\sigma^2 \quad (11)$$

Thus the parameter estimation of θ is:

$$\theta^* = \int_{-\infty}^{\infty}\theta\, p(\theta|\underline{x})\, d\theta$$
$$= \int_{-\infty}^{\infty}\theta\left[\frac{1}{u}\int_0^{\infty}\left(\frac{(\frac{1}{\sigma^{n+2}})\ e^{-\frac{1}{2\sigma^2}\Sigma(X_i-\theta)^2}}{(1-\Phi\left(\frac{a-\theta}{\sigma}\right))^n}\right) d\sigma^2\right] d\theta$$

(12)

2. Parameter Estimation of σ^2

Firstly, determine the marginal distribution of σ^2 is:

$$p(\sigma^2|\underline{x}) = \int_{-\infty}^{\infty} g(\ \theta, \sigma^2|\underline{x})\ \ d\theta$$
$$= \frac{1}{u}\int_{-\infty}^{\infty}\left(\frac{(\frac{1}{\sigma^{n+2}})\ e^{-\frac{1}{2\sigma^2}\Sigma(X_i-\theta)^2}}{(1-\Phi\left(\frac{a-\theta}{\sigma}\right))^n}\right) d\theta$$

(13)

Thus the parameter estimation of σ^2 is:

$$\sigma^{2*} = \int_0^{\infty}\sigma^2\, p(\sigma^2|\underline{x})\, d\sigma^2$$
$$= \int_0^{\infty}\sigma^2\left[\frac{1}{u}\int_{-\infty}^{\infty}\left(\frac{(\frac{1}{\sigma^{n+2}})\ e^{-\frac{1}{2\sigma^2}\Sigma(X_i-\theta)^2}}{(1-\Phi\left(\frac{a-\theta}{\sigma}\right))^n}\right) d\theta\right] d\sigma^2$$

(14)

Bayesian estimation for parameters and θ dan σ^2 has an implicit form so that the estimation form can be solved by using a numerical integral approach with Mathematica software.

4.5 Reliability

Assume that X is the lifetime of a left-truncated normal distribution system. In order to determine the reliability of a system, first determine the CDF of the left-truncated normal distribution as follows:

$$F(t|T>a)=\int_a^t \frac{\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\theta}{\sigma}\right)^2}}{1-\Phi\left(\frac{a-\theta}{\sigma}\right)}dx=\int_a^t \frac{\frac{1}{\sigma}\phi\left(\frac{x-\theta}{\sigma}\right)}{1-\Phi\left(\frac{a-\theta}{\sigma}\right)}dx$$

if assumed that $y=\frac{x-\theta}{\sigma}$, thus it can be written:

$$F(t|T>a)=\frac{1}{\sigma\left(1-\Phi\left(\frac{a-\theta}{\sigma}\right)\right)}\int_{\left(\frac{a-\theta}{\sigma}\right)}^{\left(\frac{x-\theta}{\sigma}\right)}\phi(y)dy$$

$$=\frac{\left(\Phi\left(\frac{t-\theta}{\sigma}\right)-\Phi\left(\frac{a-\theta}{\sigma}\right)\right)}{\left(1-\Phi\left(\frac{a-\theta}{\sigma}\right)\right)} \tag{15}$$

Therefore, the form of the left-truncated normal distribution reliability based on equation (3) is:

$$R(t|T>a)=1-\frac{\left(\Phi\left(\frac{t-\theta}{\sigma}\right)-\Phi\left(\frac{a-\theta}{\sigma}\right)\right)}{\left(1-\Phi\left(\frac{a-\theta}{\sigma}\right)\right)} \tag{16}$$

Furthermore, based on equation (4), if X is known to have a posterior distribution of $g\left(\theta,\sigma^2\middle|\underline{x}\right)$ then the Bayesian estimation for reliability in a left-truncated normal distribution is berdasarkan persamaan (4):

$$R(t|T>a)^*=\int_{\sigma^2=0}^{\infty}\int_{\theta=-\infty}^{\infty}\left(1-\frac{\left(\Phi\left(\frac{t-\theta}{\sigma}\right)-\Phi\left(\frac{a-\theta}{\sigma}\right)\right)}{\left(1-\Phi\left(\frac{a-\theta}{\sigma}\right)\right)}\right)\left(\frac{1}{u}\left(\frac{\left(\frac{1}{\sigma^{n+2}}\right)e^{-\frac{1}{2\sigma^2}\Sigma(x_i-\theta)^2}}{\left(1-\Phi\left(\frac{a-\theta}{\sigma}\right)\right)^n}\right)\right)d\theta\, d\sigma^2 \tag{17}$$

4.6 The Application of The Parameter Estimation Results to The Recovery Time of Covid-19 Patients

The data which was used in the application of the parameter estimation results was the recovery time of Covid-19 patients sourced from Jemursari Public Health Center Surabaya in January 2021. The available data was 24, then after the left-truncated was conducted on the data, the number of data became 15. The truncating was conducted at point a = 13.

The data on the recovery time of Covid-19 patients was tested for distribution by using the Shapiro Wilk test on SPPS software with a significant level of 5% as follows:

Table 1: The Shapiro Wilk Test Output

	df	**p-value**
Data lengkap	24	0.146
Data terpotong kiri	15	0.175

Based on the results of the Shapiro Wilk test in Table 1, it shows that the p-value of the recovery time data for Covid-19 patients, both complete and left-truncated are more than a significant level of 5%. Thus, it can be concluded that the recovery time of Covid-19 patients, both complete and left-truncated, are normally distributed.

Furthermore, by using the Mathematica software, Bayesian estimation is obtained from the recovery time of Covid-19 patients, which is normally truncated to the left. The estimated average recovery time of Covid-19 patients is 15.17 days while the estimated standard deviation for the recovery time for Covid-19 patients is 4.19 days.

The following is the application of reliability estimation to the recovery time of Covid-19 patients. By using Mathematica software, the reliability estimation with a recovery time of 17 days or t = 17 based on equation (17) is:

$$R(17|T>a)^*=\int_{\sigma^2=0}^{\infty}\int_{\theta=-\infty}^{\infty}\left(1-\frac{\left(\Phi\left(\frac{17-\theta}{\sigma}\right)-\Phi\left(\frac{13-\theta}{\sigma}\right)\right)}{\left(1-\Phi\left(\frac{a-\theta}{\sigma}\right)\right)}\right)\left(\frac{1}{u}\left(\frac{\left(\frac{1}{\sigma^{n+2}}\right)e^{-\frac{1}{2\sigma^2}\Sigma(x_i-\theta)^2}}{\left(1-\Phi\left(\frac{13-\theta}{\sigma}\right)\right)^{15}}\right)\right)d\theta\, d\sigma^2=0.9067$$

The reliability estimation with a recovery time of 17 days or t = 16 is:

$$R(16|T>a)^*=\int_{\sigma^2=0}^{\infty}\int_{\theta=-\infty}^{\infty}\left(1-\frac{\left(\Phi\left(\frac{16-\theta}{\sigma}\right)-\Phi\left(\frac{13-\theta}{\sigma}\right)\right)}{\left(1-\Phi\left(\frac{a-\theta}{\sigma}\right)\right)}\right)\left(\frac{1}{u}\left(\frac{\left(\frac{1}{\sigma^{n+2}}\right)e^{-\frac{1}{2\sigma^2}\Sigma(x_i-\theta)^2}}{\left(1-\Phi\left(\frac{13-\theta}{\sigma}\right)\right)^{15}}\right)\right)d\theta\, d\sigma^2=0.9297$$

The reliability estimation with a recovery time of 17 days or t = 15 is:

$$R(15|T>a)^*=\int_{\sigma^2=0}^{\infty}\int_{\theta=-\infty}^{\infty}\left(1-\frac{\left(\Phi\left(\frac{15-\theta}{\sigma}\right)-\Phi\left(\frac{13-\theta}{\sigma}\right)\right)}{\left(1-\Phi\left(\frac{a-\theta}{\sigma}\right)\right)}\right)$$

$$\left(\frac{1}{u}\left(\frac{(\frac{1}{\sigma^{n+2}})\ e^{-\frac{1}{2\sigma^2}\Sigma(X_i-\theta)^2}}{(1-\Phi(\frac{13-\theta}{\sigma}))^{15}}\right)\right)d\theta\ d\sigma^2 = 0.9537$$

Based on the reliability estimation, it can be concluded that the faster the recovery time of a patient, the higher the reliability or survival of the patient against Covid-19 disease. In addition, high survivability means having high immunity to be able to recover faster from exposure to Covid-19.

5. CONCLUSION

Based on the results of the study, it shows Bayesian estimation for the parameters θ and σ^2 of the left-truncated normal distribution is as follows:

$$\theta^* = \int_{-\infty}^{\infty}\theta\left[\frac{1}{u}\int_{0}^{\infty}\left(\frac{(\frac{1}{\sigma^{n+2}})\ e^{-\frac{1}{2\sigma^2}\Sigma(X_i-\theta)^2}}{(1-\Phi\left(\frac{a-\theta}{\sigma}\right))^n}\right)d\sigma^2\right]d\theta$$

$$\sigma^{2*} = \int_{0}^{\infty}\sigma^2\left[\frac{1}{u}\int_{-\infty}^{\infty}\left(\frac{(\frac{1}{\sigma^{n+2}})\ e^{-\frac{1}{2\sigma^2}\Sigma(X_i-\theta)^2}}{(1-\Phi\left(\frac{a-\theta}{\sigma}\right))^n}\right)d\theta\right]d\sigma^2$$

The form of the reliability estimation on the left-truncated normal distribution based on the Bayesian method is as follows:

$$R(t|T>a)^* = \int_{\sigma^2=0}^{\infty}\int_{\theta=-\infty}^{\infty}\left(1-\frac{\left(\Phi\left(\frac{t-\theta}{\sigma}\right)-\Phi\left(\frac{a-\theta}{\sigma}\right)\right)}{\left(1-\Phi\left(\frac{a-\theta}{\sigma}\right)\right)}\right)$$
$$\left(\frac{1}{u}\left(\frac{(\frac{1}{\sigma^{n+2}})\ e^{-\frac{1}{2\sigma^2}\Sigma(X_i-\theta)^2}}{(1-\Phi\left(\frac{a-\theta}{\sigma}\right))^n}\right)\right)d\theta\ d\sigma^2$$

The parameter estimation results and the reliability of the left-truncated normal distribution are applied to the recovery time of Covid-19 patients with the help of Mathematica software. Based on the parameter estimation results, the average recovery time for Covid-19 patients is 15.17 days while the standard deviation of recovery time for Covid-19 patients is 4.19 days. In addition, based on the results of the reliability estimation, it can be concluded that the faster the recovery time of a Covid-19 patient, the higher the reliability or survival of the patient against Covid-19 disease.

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