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On Super (a,d)-Edge Antimagic Total Labeling of Some Generalized Shackle of Fan Graph

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Abstract: *Generalized shackle of fan graph is the development of shackle operation of fan graph $\boldsymbol{F_n}$ by connecting the some vertices in shackle of fan graph. Graph G with the cardinality of vertex p and the cardinality of edges q is called an super $(\boldsymbol{a}, \boldsymbol{d})$ −edge antimagic total labelling if there exist a bijection $\boldsymbol{f}$: $\boldsymbol{V}(\boldsymbol{G}) \cup \boldsymbol{E}(\boldsymbol{G}) \rightarrow \{\boldsymbol{1}, \boldsymbol{2}, \ldots, \boldsymbol{p}+\boldsymbol{q}\}$ such that the edge-weights, $\boldsymbol{w}(\boldsymbol{xy}) = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{f}(\boldsymbol{y}) + \boldsymbol{f}(\boldsymbol{xy}), \boldsymbol{xy} \in \boldsymbol{E}(\boldsymbol{G})$, form an arithmetic sequence with first term a and common difference d. In this paper, we use pattern recognition method and we analyze the super (a,d)-edge antimagic total labeling of some generalized shackle of fan graph. The results show that Generalized Shackle of Fan Graph, GShackle$(\boldsymbol{F_4}, \boldsymbol{v}, \boldsymbol{n})$ and GShackle$(\boldsymbol{F_3}, \boldsymbol{v}, \boldsymbol{n})$ admit super (a,d)- edge antimagic total labelling for $\boldsymbol{d} \in \{\boldsymbol{0}, \boldsymbol{1}, \boldsymbol{2}\}$.*

Keywords—SEATL, Shackle, Fan Graph

1. Introduction (*Heading 1*)

Mathematical concepts and principles are used in daily life activities such as counting and measuring. Mathematics is very important in solving a problem in daily life activities. In the 4.0 industrial era, a person will be so difficult for facing normal life without using mathematics. There are many interesting branches of mathematics which has various applications in many fields of science. One of the interesting branches of mathematics is discrete mathematics and one of the topic which is discussed is graph theory. Graph theory was introduced by Leonhard Euler (1736) where the first problem raised when he observed the problem related to Konigsberg Bridge (Konigsberg, east of Prussia, Germany). Konigsberg city has a river known as the Pregal River. The Pregal River divides the city of Konigsberg into four main landmasses and seven bridges connecting the four landmasses. Euler presents the problem of how to cross the seven bridges exactly once from a certain land and return to the previous land. In order to solve these problems, Euler use graph theory, where until now many other scientists have contributed to the development of graph theory to solve problems in daily life.

Graph is a branch of discrete mathematics that is widely applied in various fields of life. Graphs can be represented by representing discrete objects as vertices while the relationship between each vertices we called it as edges. Graphs according to [3] provide mathematical models that are useful for every problem, such as monitoring systems for communication networks, data security, and various coding problems. The problem then developed very rapidly because many scientists developed graph theory to solve problems related to real life problem in daily life. One of the topic in graph theory is graph labeling.

The definition of magic labeling on a graph G which is a mapping from the set of edges to the set of real numbers which was firstly introduced by Sedl´acˇek since 1963. Meanwhile, in 1966, Stewart said that if a labeling is mapped to a set of consecutive integers, then the labeling is called as magic labeling. In 1970, Kotzig and Rosa introduced another type of graph labeling which is a vertex labeling. Based on the labeled elements, labeling is divided into three types, namely vertex labeling, edges labeling, and total labeling. Vertex labeling on a graph is a labeling with the domain is a vertex. Edge labeling is labeling with the domain is an edges. Meanwhile, total labeling is a labeling with the domain is both vertex and edges. The labeling concept has also developed very rapidly, from Edge Antimagic Vertex Labeling (EAVL) to Super (a, d)-Edge Antimagic Total Labeling. Dian Anita Hadi et al [4] in their research entitled Super (a,d)- Edge Antimagic Total Labeling of Slikworm Graph explained that Silkworm Graph, Sw_n with $n \geq 2$ admit a Super (a, d) – Edge Antimagic Total Labeling with different value $d \in \{0,1,2\}$. Several other studies related to labeling can be seen in: [1,2,5,6,7]. This study shows the Generalized Shackle of Fan Graph is admit Super (a, d)- Edge Antimagic Total Labeling (SEATL).

2. METHODOLOGY

In this research, we used deductive axiomatic method. The method is used by deriving the existing lemma related to the determination of the upper bound of the difference d, then applied in the process of determining the upper bound of the difference d in the Generalized Shackle of Fan Graph. Then give a label to the vertex and edge by looking for patterns that can form an arithmetic sequences. The indicators for super edge antimagic total labeling in the Generalized Shackle of Fan Graph are:

a. All the vertices labels are different. The super edge antimagic total labelling on the generalized shackle of fan graph is a bijective function from the set of vertices to integers from one to a number of vertices.

b. All the edges labels are different. The super edge antimagic total labeling on the generalized shackle of fan graph is a bijective function from the set of edges to the integers from the number of vertices plus one to the number of vertices and edges.

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c. The total weight is the sum of the vertex labels and the edge labels on each edges. In this research, the total weight of the edges must form an arithmetic sequence with a difference of d and the initial term a. We can see the research methodology ini the Figure 1.

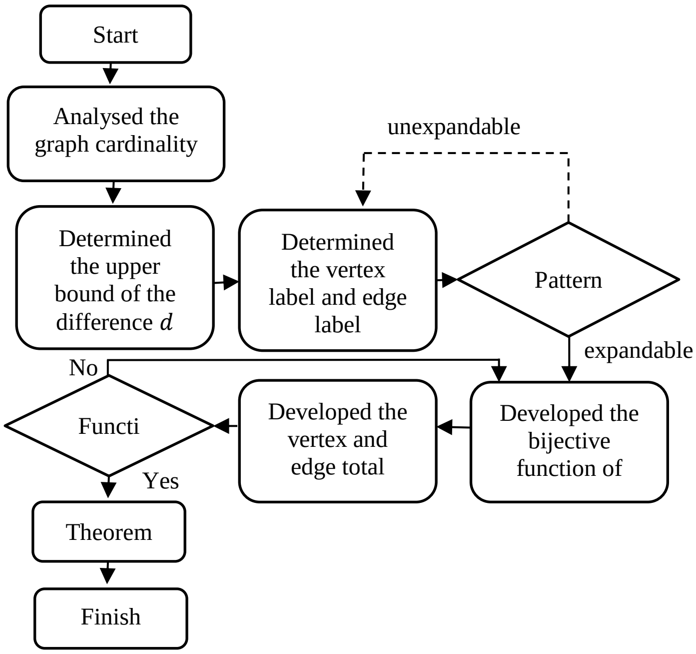

Fig 1. Research Methodology

3. RESULTS AND DISCUSSION

In this section, it will be explained about the results and discussion of super edge antimagic total labeling research on the generalized shackle of fan graph. The first step in this research is determining the cardinality of vertices and edges in each graphs, the upper bound value of d, determining the labels of vertices, edges, and edge weights, and determining the super edge antimagic total labeling on the generalized shackle of fan graph. In this research results, we will describe the super edge antimagic total labeling of generalized shackle of fan graph in some lemma and theorem together with the proofs.

Lemma 1. If a Generalized Shackle of Fan Graph, $GShackle(F_4,v,n)$, is super (a,d)-edge-antimagic total labeling then $d \le 2$.

Proof. Assume that a $GShackle(F_4,v,n)$ is super (a,d)-edge-antimagic total labeling with a bijective function: $f: V(G) \cup E(G) \to \{1,2, \dots, p+q\}$ and the set of edge weights:$\{a, a+d, a+2d, \dots a+(q-1)d\}$. $GShackle(F_4,v,n)$ is the development of shackle operation of fan graph f_n with four vertices and we connect the vertex x_{i+1} and x_{i+2}. $GShackle(F_4,v,n)$ has vertex set and edge set as follows: $V=\{x_i;1\le i\le 2n\}\cup\{y_i;1\le i\le 2n+1\}$ and $E=\{x_ix_{i+1};1\le i\le 2n-1\}\cup\{y_iy_{i+1};1\le i\le 2n\}\cup\{x_iy_i;1\le i\le 2n\}\cup\{x_iy_{i+1};1\le i\le 2n\}$. Then, the cardinality of the vertices and the edges are $|V|=p=4n+1$ and $|E|=q=8n-1$. Thus, we can analyse the lower bound of d in the following formula:

$$d\le\frac{2p+q-5}{q-1}\Leftrightarrow d\le\frac{2(4n+1)+(8n-1)-5}{(8n-1)-1}\Leftrightarrow d\le 2\ldots\square$$

Lemma 2. If a Generalized Shackle of Fan Graph, $GShackle(F_3,v,n)$, h is super (a,d)-edge-antimagic total labeling then $d\le 2$.

Proof. Assume that a $GShackle(F_3,v,n)$ is super (a,d)-edge-antimagic total labeling with a bijective function: $f: V(G)\cup E(G)\to\{1,2, \dots, p+q\}$ an the set of edge weights: $\{a, a+d, a+2d, \dots a+(q-1)d\}$. $GShackle(F_3,v,n)$ is the development of shackle operation of fan graph f_n with three vertices and we connect the vertexz_i dan x_{i+2}. $GShackle(F_3,v,n)$ has vertex set and edge set as follows:

$V=\{x_i;1\le i\le n\}\cup\{y_i;1\le i\le n+1\}\cup\{z_i;1\le i\le n\}$

and $E=\{x_iy_i;1\le i\le n\}\cup\{y_iy_{i+1};1\le i\le n\}\cup\{x_iy_{i+1};1\le i\le n\}\cup\{z_ix_{i+1};1\le i\le n-1\}\cup\{y_iz_i;1\le i\le n\}\cup\{z_iy_{i+1};1\le i\le n\}$ Then, the cardinality of the vertices and the edges are $|V|=p=3n+1$ and $|E|=q=6n-1$. Thus, we can analyse the lower bound of d as follow:

$$d\le\frac{2p+q-5}{q-1}\Leftrightarrow d\le\frac{2(3n+1)+(6n-1)-5}{(6n-1)-1}\Leftrightarrow d\le 2\ldots\square$$

Theorem 1. If $n\ge 2$ then $GShackle(F_4,v,n)$ is super $(4n+5,2)$-edge-antimagic total labeling.

Proof. For $d=2$, the first step to prove this theorem is determining the vertex label and edges label of $GShackle(F_4,v,n)$ graph. The vertex and edge label is a bijective function which is mapped the vertex and edge set of $GShackle(F_4,v,n)$ to the integer. From $GShackle(F_4,v,n)$ graph, we got the vertex label and edges label as follow:

$$f_1(x_i)=2i \qquad f_1(x_iy_i)=4(n+i)-2$$
$$f_1(y_i)=2i-1 \qquad f_1(y_iy_{i+1})=4(n+i)-1$$
$$f_1(x_iy_{i+1})=4(n+i)$$
$$f_1(x_ix_{i+1})=4(n+i)+1$$

Furthermore, we are able to determine the edge weight label f_1 which can be obtained from the sum of two adjacent vertex label with the appropriate edge label. The bijective function wf_1 can be obtained from the pattern recognition observation by using the arithmetic sequence concept. Here are the edge weight function:

$$wf_1(x_i y_i) = 4(n+2i) - 3$$
$$wf_1(y_i y_{i+1}) = 4(n+2i) - 1$$
$$wf_1(x_i y_{i+1}) = 4(n+2i) + 1$$
$$wf_1(x_i x_{i+1}) = 4(n+2i) + 3$$

Based on the edge weight function of $GShackle(F_4, v, n)$, it can be seen that the smallest edge weight lies on $wf_1(x_1 y_1)$, which is $4n + 5$. And the biggest edge weight lies on $wf_1(x_4 y_5)$, which is $4n + 16$. Therefore, it can be shown that the edge weight function of $GShackle(F_4, v, n)$ form the arithmetic sequence with the ordered as follow: $wf_1 = \{4n+5, 4n+7, 4n+9, \dots, 4n+16\}$ and $= GShackle(F_4, v, n)$ had the value of $d = 2$. Thus, it can be concluded that $GShackle(F_4, v, n)$ admit super $(4n+5, 2)$-edge-antimagic total labelling □. The labelling illustration of $GShackle(F_4, v, n)$ can be seen in figure 2.

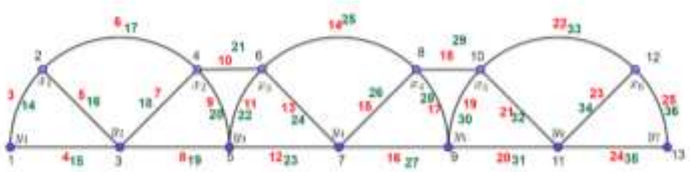

Fig 2. The Super $(4n + 5,2)$-edge-antimagic total labelling of $GShackle(F_4, v, 3)$

Theorem 2. If $n \geq 2$ then $GShackle(F_4, v, n)$ is super $(8n + 4, 1)$-edge-antimagic total labelling.

Proof. For $d = 1$, The first step to prove this theorem is determining the vertex label and edges label of $GShackle(F_4, v, n)$ graph. The vertex and edge label is a bijective function which is mapped the vertex and edge set of $GShackle(F_4, v, n)$ to the integer. From $GShackle(F_4, v, n)$ graph, we got the vertex label and edges label as follow:

$$f_2(x_i) = 2i \qquad f_2(x_i y_i) = 2(4n - i) + 3$$
$$f_2(y_i) = 2i - 1 \qquad f_2(y_i y_{i+1}) = 2(6n - i + 1)$$
$$f_2(x_i y_{i+1}) = 2(4n - i + 1)$$
$$f_2(x_i x_{i+1}) = 2(6n - i) + 1$$

Furthermore, it will be determined the edge weight label f_2 which can be obtained from the sum of two adjacent vertex label with the appropriate edge label. The bijective function wf_2 can be obtained from the pattern recognition observation by using the arithmetic sequence concept. Here are the edge weight function:

$$wf_2(x_i y_i) = 2(4n + i + 1)$$
$$wf_2(y_i y_{i+1}) = 2(6n + i + 1)$$
$$wf_2(x_i y_{i+1}) = 2(4n + i) + 3$$
$$wf_2(x_i x_{i+1}) = 2(6n + i) + 3$$

Based on the edge weight function of $GShackle(F_4, v, n)$, it can be seen that the smallest edge weight lies on $wf_2(x_1 y_1)$, which is $8n + 4$. And the biggest edge weight lies on $wf_2(x_{2n} y_{2n+1})$, which is $16n + 2$. Therefore, it can be shown that the edge weight function of $GShackle(F_4, v, n)$ form the arithmetic sequence with the ordered as follow: $wf_2 = \{8n+4, 8n+5, 8n+6, \dots, 16n+2\}$ with the value of $GShackle(F_4, v, n)$ d is 1. Thus, it can be concluded that $GShackle(F_4, v, n)$ is super $(8n + 4, 1)$-edge-antimagic total labelling □.

Theorem 3. If $n \geq 2$ then $GShackle(F_4, v, n)$ is super $(12n + 3, 0)$-edge-antimagic total labeling.

Proof. For $d = 0$, the first step to prove this theorem is determining the vertex label and edges label of $GShackle(F_4, v, n)$ graph. The vertex and edge label is a bijective function which is mapped the vertex and edge set of $GShackle(F_4, v, n)$ to the integer. From $GShackle(F_4, v, n)$ graph, we got the vertex label and edges label as follow:

$$f_3(x_i) = 2i \qquad f_3(x_i y_i) = 4(3n - i) + 4$$
$$f_3(y_i) = 2i - 1 \qquad f_3(y_i y_{i+1}) = 4(3n - i) + 3$$
$$f_3(x_i y_{i+1}) = 4(3n - i) + 2$$
$$f_3(x_i x_{i+1}) = 4(3n - i) + 1$$

Then, it will be investigated the edge weight label f_3 which can be obtained from the sum of two adjacent vertex label with the appropriate edge label. The bijective function wf_3 can be obtained from the pattern recognition observation by using the arithmetic sequence concept. Here are the edge weight function:

$$wf_3(x_i y_i) = 12n + 3$$
$$wf_3(y_i y_{i+1}) = 12n + 3$$
$$wf_3(x_i y_{i+1}) = 12n + 3$$
$$wf_3(x_i x_{i+1}) = 12n + 3$$

Based on the edge weight function of $GShackle(F_4, v, n)$, it can be seen that all the edge weight are the same, which is $12n + 3$. Therefore, it can be shown that the edge weight function of $GShackle(F_4, v, n)$ form the arithmetic sequence with the ordered as follow: $wf_3 = \{12n+3, 12n+3, 12n+3, \dots, 12n+3\}$ with the value of $d = 1$. Thus, it can be concluded that $GShackle(F_4, v, n)$ is super $(12n + 3, 0)$-edge-antimagic total labelling □.

Theorem 4. If $n \geq 2$ then $GShackle(F_3, v, n)$ is super $(3n + 5, 2)$-edge-antimagic total labeling.

Proof. The first step to prove this theorem is determining the vertex label and edges label of $GShackle(F_3, v, n)$ graph. The vertex and edge label is a bijective function which is mapped the vertex and edge set of $GShackle(F_3, v, n)$ to the integer. From $GShackle(F_3, v, n)$ graph, we got the vertex label and edges label as follow:

$$f_4(x_i) = 3i - 1 \qquad f_4(z_i y_{i+1}) = 3(n + 2i)$$
$$f_4(y_i) = 3i - 2 \qquad f_4(y_i y_{i+1}) = 3(n + 2i) - 2$$
$$f_4(z_i) = 3i \qquad f_4(x_i y_i) = 3(n + 2i) - 4$$
$$f_4(x_i y_{i+1}) = 3(n + 2i) - 1$$
$$f_4(y_i z_i) = 3(n + 2i) - 3$$
$$f_4(z_i x_{i+1}) = 3(n + 2i) + 1$$

Furthermore, it will be investigated the edge weight function label f_4 which can be obtained from the sum of two adjacent vertex label with the appropriate edge label. The bijective function wf_4 can be obtained from the pattern recognition observation by using the arithmetic sequence concept. Here are the edge weight function:

$$wf_4(z_i y_{i+1}) = 3(n + 4i) + 1$$
$$wf_4(y_i y_{i+1}) = 3(n + 4i) - 3$$
$$wf_4(x_i y_i) = 3(n + 4i) - 7$$
$$wf_4(x_i y_{i+1}) = 3(n + 4i) - 1$$
$$wf_4(y_i z_i) = 3(n + 4i) - 5$$
$$wf_4(z_i x_{i+1}) = 3(n + 4i) + 4$$

Based on the edge weight function of $GShackle(F_3, v, n)$, it can be seen that the smallest edge weight lies on $f_4(y_1 z_1)$, which is $3n + 5$. And the biggest edge weight lies on $wf_4(x_n y_{n+1})$, which is $15n + 1$. Therefore, it can be shown that the edge weight function of $GShackle(F_3, v, n)$ form the arithmetic sequence with the ordered as follow: $wf_4 = \{3n + 5, 3n + 7, 3n + 9, \ldots, 15n + 1\}$ with the value of d in $GShackle(F_4, v, n)$ is 2. Thus, it can be concluded that $GShackle(F_3, v, n)$ is super $(3n + 5, 2)$-edge-antimagic total labelling □. The labelling illustration of $GShackle(F_3, v, n)$ can be seen in figure 3.

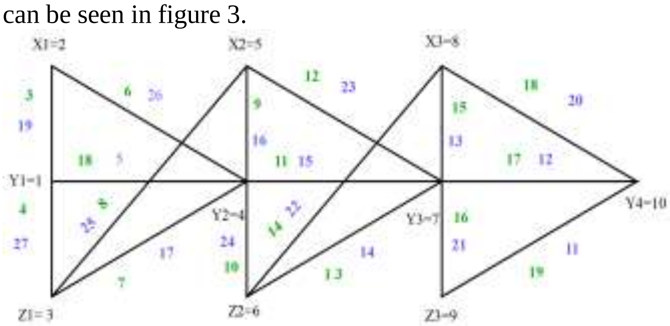

Fig 3. The Super $(3n + 5,2)$-edge-antimagic total labeling of $GShackle(F_3, v, 3)$

Theorem 5. If $n \geq 2$ then $GShackle(F_3, v, n)$ is super $(6n + 4, 1)$-edge-antimagic total labeling.

Proof. For $d = 1$, the first step to prove this theorem is determining the vertex label and edges label of $GShackle(F_3, v, n)$ graph. The vertex and edge label is a bijective function which is mapped the vertex and edge set of $GShackle(F_3, v, n)$ to the integer. From $GShackle(F_3, v, n)$ graph, we got the vertex label and edges label as follow:

$$f_5(x_i) = 3i - 1 \qquad f_5(z_i y_{i+1}) = 3(2n - i) + 2$$
$$f_5(y_i) = 3i - 2 \qquad f_5(y_i y_{i+1}) = 3(2n - i) + 3$$
$$f_5(z_i) = 3i \qquad f_5(x_i y_i) = 3(2n - i) + 4$$
$$f_5(x_i y_{i+1}) = 3(3n - i) + 2$$
$$f_5(y_i z_i) = 3(3n - i) + 3$$
$$f_5(z_i x_{i+1}) = 3(3n - i) + 1$$

Furthermore, it will be investigated the edge weight function label f_5 which can be obtained from the sum of two adjacent vertex label with the appropriate edge label. The bijective function wf_5 can be obtained from the pattern recognition observation by using the arithmetic sequence concept. Here are the edge weight function:

$$wf_5(z_i y_{i+1}) = 3(2n + i) + 3$$
$$wf_5(y_i y_{i+1}) = 3(2n + i) + 2$$
$$wf_5(x_i y_i) = 3(2n + i) + 1$$
$$wf_5(x_i y_{i+1}) = 3(3n + i) + 2$$
$$wf_5(y_i z_i) = 3(3n + i) + 1$$
$$wf_5(z_i x_{i+1}) = 3(3n + i) + 3$$

Based on the edge weight function of $GShackle(F_3, v, n)$, it can be seen that the smallest edge weight lies on $f_5(y_1 z_1)$, which is $6n + 4$. And the biggest edge weight lies on $wf_5(x_n y_{n+1})$, which is $12n + 2$. Therefore, it can be shown that the edge weight function of $GShackle(F_3, v, n)$ form the arithmetic sequence with the ordered as follow: $wf_5 = \{6n + 4, 6n + 5, 6n + 6, \ldots, 12n + 2\}$ with the value of $GShackle(F_4, v, n)$ d is 1. Thus, it can be concluded that $GShackle(F_3, v, n)$ is super $(6n + 4, 1)$-edge-antimagic total labelling □.

Theorem 6. If $n \geq 2$ then $GShackle(F_3, v, n)$ is super $(9n + 3,0)$-edge-antimagic total labeling.

Proof. For $d = 0$, the first step to prove this theorem is determining the vertex label and edges label of $GShackle(F_3, v, n)$ graph. The vertex and edge label is a bijective function which is mapped the vertex and edge set of $GShackle(F_3, v, n)$ to the integer. From $GShackle(F_3, v, n)$ graph, we got the vertex label and edges label as follow:

$$f_6(x_i) = 3i - 1 \qquad f_6(z_i y_{i+1}) = 3(3n - 2i) + 2$$
$$f_6(y_i) = 3i - 2 \qquad f_6(y_i y_{i+1}) = 3(3n - 2i) + 4$$
$$f_6(z_i) = 3i \qquad f_6(x_i y_i) = 3(3n - 2i) + 6$$
$$f_6(x_i y_{i+1}) = 3(3n - 2i) + 3$$
$$f_6(y_i z_i) = 3(3n - 2i) + 5$$
$$f_6(z_i x_{i+1}) = 3(3n - 2i) + 1$$

Furthermore, it will be investigated the edge weight function label f_6 which can be obtained from the sum of two adjacent vertex label with the appropriate edge label. The bijective function wf_6 can be obtained from the pattern recognition observation by using the arithmetic sequence concept. Here are the edge weight function:

$$wf_5(z_i y_{i+1}) = 9n+3$$
$$wf_5(y_i y_{i+1}) = 9n+3$$
$$wf_5(x_i y_i) = 9n+3$$
$$wf_5(x_i y_{i+1}) = 9n+3$$
$$wf_5(y_i z_i) = 9n+3$$
$$wf_5(z_i x_{i+1}) = 9n+3$$

Based on the edge weight function of $GShackle(F_3, v, n)$, it can be seen that all the edge weight are the same, which is $9n + 3$. Therefore, it can be shown that the edge weight function of $GShackle(F_3, v, n)$ form the arithmetic sequence with the ordered as follow: $wf_6 = \{9n + 3, 9n + 3, 9n + 3, \dots, 9n + 3\}$ with the value of $d = 0$. Thus, it can be concluded that $GShackle(F_3, v, n)$ is super $(9n + 3, 0)$-edge-antimagic total labelling □.

4. CONCLUSION

Generalized Shackle of Fan Graph, $GShackle(F_4, v, n)$ and $GShackle(F_3, v, n)$ admit super (a,d)- edge antimagic total labelling for $d \in \{0,1,2\}$.

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