

Numerical Analysis For Stability Control Of Quadcopter Using Extremum Seeking Based Proportional Integral Derivative Controller

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Abstract: Application of autonomous unmanned aerial vehicle is an essential part of modern era control and monitoring devices especially as quadcopter becomes household names in achieving crucial tasks. Therefore, the stability of such device is crucial to prevent or minimize risk of human lives and financial implication that may arise from improper monitoring of the system dynamics. This study acquired raw aerodynamic data of quadcopter in terms of Altitude, Roll, Pitch and Yaw using X-Plane 11 flight simulator after establishing a bidirectional serial communication between X-Plane 11 and Matlab/Simulink 2018a. This study adopted Extremum Seeking based PID controller as designed in Matlab/Simulink environment to tune the system dynamics for stability purposes. The system stability was characterized using rise time, settling time, percentage overshoot, steady state errors performance metrics. The accuracy of the extremum seeking based PID is dependent on objective function and filter of the controller plant, as appropriate selection of objective and filter function reduce percentage overshoot and bring about fast response and settling time. It was found that the ES-PID controller was effective after the system attained stability within shortest period of time compared to the dynamic transient of the raw data..ES-PID controller is recommended for tuning the quadcopter dynamics to achieve the best stability in real time.

Keywords: Aerial, Autonomous, Bidirectional, Extremum, Transient, Unmanned.

INTRODUCTION

Background of the Study

Control and system monitoring has become integral part of society daily tasks and typical application in this modern world is Autonomous Unmanned Aerial Vehicles (UAV) which are becoming more and more popular in a wide field of applications such as intelligence gathering, surveillance, reconnaissance missions, power line inspection, aerial video, search and rescue and several other applications (Ossa-Gomez, Moarref and Rodrigues, 2015). The most useful tool in autonomous unmanned aerial vehicles (UAV) is a drone like flight in the name of quadcopter. Unlike conventional fixed wing aerial vehicles, quadcopter vehicles possess certain essential characteristics, which highlight their potential for use in a broad range of applications. Characteristics that provide a clear advantage over other flying UAVs includes Vertical Take Off and Landing (VTOL) and hovering capability, low-speed and low-altitude cruise as well as performing aggressive maneuvers (Abbasi, Mahjoob and Yazdanpanah, 2008).

A quadcopter is an under actuated, dynamic vehicle with four input forces (basically, the thrust provided by each of the propellers) and six output coordinates (fully spatial movements). Unlike regular helicopters that have variable pitch angle rotors, a quadcopter helicopter has fixed pitch angle rotors (Hua and Rifai, 2010). The flight behavior of a quadcopter is determined by the speeds of each of the four motors, as they vary in concert, or in opposition with each other (Salih and Moghavvemi, 2010). Hence, based on its inputs, a dynamics model equation of the system can be used to predict the position and orientation of the quadcopter which can be used as a guideline for developing the flight control strategy.

Progress in the development of a quadcopter for sophisticated applications under flight condition is mainly hindered by the complexity and highly nonlinearities characteristics of the vehicle model dynamics in the presence of uncertainties and disturbances (Bouadi and Tadjine, 2007). These characteristics are frequently sources of instability and performance deterioration of the quadcopter flight path control system required to manage, command, and regulate the quadcopter dynamic model in a desired way and at a desired performance.

In recent years, conventional proportional-integral-derivative (PID) controllers have been well developed and adopted for industrial automation and process control today. The main reason is due to their simplicity of operation, ease of design, inexpensive maintenance, low cost, and effectiveness for most linear system. However, it has been known that conventional PID controllers generally do not work well for nonlinear systems, higher order and time-delayed linear systems, and particularly complex system that have no precise mathematical models Tang et al (2022).

In this research work, extremum seeking based PID controller is to be employed in the design of the flight controller for quadcopter system to minimize the cost function and find the optimal set of PID parameters (Kotarski, 2016). The work will commence with

the development of mathematical modeling equation of the quadcopter UAV system and simulation study will be carried out on it to illustrate the usefulness of the control technique to be employed.

Because of the above mentioned attributes regarding quadcopter, it has become very essential to monitor, control and maneuver the dynamics of this system for optimum results. Majority of the tasks embarked upon by quadcopter are very delicate and requires high degree of accuracy, stability and delivery time. Therefore, quadcopter stability would be optimized by utilization of Extremum seeking based proportional integral derivative controller, this would solve the problem of system instability that results in wastage of human and financial resources.

2.0 METHODOLOGY

This study adopted the mathematical frame work of quadcopter model as references and background to the origin and relationship between the numerical aerodynamic data that were acquired from the flight simulator and the governing equations of the system, Benick et al. (2016) gives the full account of the mathematical model of unmanned aerial vehicles with four rotors as explored from the Newton-Euler's equations for any frame of reference.

The bench work was carried out in virtual laboratory for visual simulation of quadcopter dynamics monitoring. System communication and data exchange was done using MATLAB R2018a and X-Plane 11 on HP laptop, AMD Quad-Core E2-7110 APU, 4GB DDR3L System memory, 500GB Hard Drive storage, 1.80 GHz processor, 64-bit operating system. MATLAB R2014a and X-Plane 11 are run concurrently on this computer system for data generation, simulation and stability visualization.

2.1 Mathematical Framework for a Quadcopter

Base on Newton- Euler's equations, the dynamics of quadcopter can be studied in any frame of reference considering the system inputs such as thrusts generated by the Four rotors (Altitude, Roll, Pitch and Yaw), moment of inertia, mass, half-length and scaling factor of the quadcopter. The rotational dynamic model is given in the following form as:

$$\begin{aligned}\ddot{\phi} &= \dot{\theta}\dot{\psi} \left(\frac{I_{yy} - I_{zz}}{I_{xx}} \right) - \frac{\dot{\theta}\omega J_r}{I_{xx}} + \frac{lk(\omega_4^2 - \omega_2^2)}{I_{xx}} \\ &= \dot{\theta}\dot{\psi} \left(\frac{I_{yy} - I_{zz}}{I_{xx}} \right) - \frac{\dot{\theta}\omega J_r}{I_{xx}} + \frac{l}{I_{xx}} u_2\end{aligned}\quad (1)$$

$$\begin{aligned}\ddot{\theta} &= \dot{\phi}\dot{\psi} \left(\frac{I_{zz} - I_{xx}}{I_{yy}} \right) + \frac{\dot{\phi}\omega J_r}{I_{yy}} + \frac{lk(\omega_3^2 - \omega_1^2)}{I_{yy}} \\ &= \dot{\phi}\dot{\psi} \left(\frac{I_{zz} - I_{xx}}{I_{yy}} \right) + \frac{\dot{\phi}\omega J_r}{I_{yy}} + \frac{l}{I_{yy}} u_3\end{aligned}\quad (2)$$

$$\begin{aligned}\ddot{\psi} &= \dot{\phi}\dot{\theta} \left(\frac{I_{xx} - I_{yy}}{I_{zz}} \right) - \frac{\dot{\omega} J_r}{I_{zz}} + \frac{\sum_{i=1}^4 [(-1)^i b \omega_i^2]}{I_{zz}} \\ &= \dot{\phi}\dot{\theta} \left(\frac{I_{xx} - I_{yy}}{I_{zz}} \right) - \frac{\dot{\omega} J_r}{I_{zz}} + \frac{b}{I_{zz}} u_4\end{aligned}\quad (3)$$

where ω = rotor velocity, J_r is the moment of inertia about Z-axis, I_{xx} , I_{yy} and I_{zz} are the moment of inertial, l is the half length of the quadcopter, b is the scaling factor, Euler angles are roll angle ϕ , pitch angle θ and yaw angle ψ , while u_1 will control the altitude (z-axis), u_2 is to control the rotation in the roll angle, u_3 = Pitch angle input control, u_4 = yaw angle input control.

2.2 Control of the Dynamic Model of the Quadcopter

The dynamics of quadcopter can be controlled and monitored using the proportional integral derivative (PID) controller to stabilize the flight characteristics stated in Equations (1 – 3) in the quadcopter model earlier. The general form of the PID controller is written as Equation (4), (Jose et al., 2013):

$$e(t) = x_d(t) - x(t)$$

$$u(t) = K_p e(t) + K_I \int_0^t e(\tau) d\tau + K_D \frac{d e(t)}{dt} \quad (4)$$

i. Altitude control

Equation for the thrust force control variable,

$$U_1 = K_{PZ} e_Z + K_{IZ} \int e_Z - K_{DZ} \frac{d}{dt} e_Z \quad (5)$$

Where K_{PZ} , K_{IZ} and K_{DZ} are three altitude PID controller parameters. e_Z is the altitude error, where $e_Z = Z_{des} - Z_{mes}$. Z_{des} is desired altitude and Z_{mes} is the measured altitude.

ii. Roll control Equation for the roll moment control variable U_2 is:

$$U_2 = K_{P\phi} e_\phi + K_{I\phi} \int e_\phi - K_{D\phi} \frac{d}{dt} e_\phi \quad (6)$$

where $K_{P\phi}$, $K_{I\phi}$ and $K_{D\phi}$ are three roll angle PID controller parameters. e_ϕ is the roll angle error, where $e_\phi = \phi_{des} - \phi_{mes}$. ϕ_{des} is the desired roll angle and ϕ_{mes} is the measured roll angle.

iii. Pitch control

Equation for the pitch moment control variable,

$$U_3 = K_{P\theta} e_\theta + K_{I\theta} \int e_\theta - K_{D\theta} \frac{d}{dt} e_\theta \quad (7)$$

Similar to the roll control, $K_{P\theta}$, $K_{I\theta}$ and $K_{D\theta}$ are three pitch angle PID controller parameters. e_θ is the roll angle error, where $e_\theta = \theta_{des} - \theta_{mes}$. θ_{des} is the desired pitch angle and θ_{mes} is the measured pitch angle.

iv. Yaw control

Equation for the yaw moment control variable,

$$U_4 = K_{P\psi} e_\psi + K_{I\psi} \int e_\psi - K_{D\psi} \frac{d}{dt} e_\psi \quad (8)$$

Where $K_{P\psi}$, $K_{I\psi}$ and $K_{D\psi}$ are three yaw angle PID controller parameters. e_ψ is the yaw angle error, where $e_\psi = \psi_{des} - \psi_{mes}$. ψ_{des} is the desired yaw angle and ψ_{mes} is the measured.

2.3 Tuning of the PID controller using Extremum Seeking Tuning Scheme

For adjusting the PID controller parameters the Extremum Seeking (ES) method is adopted to minimize the integrated square error (ISE) cost function:

$$J(\theta) = \frac{\Delta}{T - t_0} \int_{t_0}^T e^2(t, \theta) dt \quad (9)$$

where the error $e(t, \theta) = r(t) - y(t, \theta)$ is the difference between the reference and the output signal of the closed-loop system, and contains the PID parameters.

$$\theta = [K, T_i, T_d]^T \quad (10)$$

The overall ES PID tuning scheme is shown in Figure 2.1. The ES algorithm uses the value $J(\theta(K))$ of the cost function to compute new controller parameters $\theta(K)$. A typical aerodynamic data (altitude) is fed in as input for tuning the quadcopter to achieve stability, by designing appropriate extremum seeking based proportional integral derivative controller in Matlab/Simulink as shown in Figure (2.1).

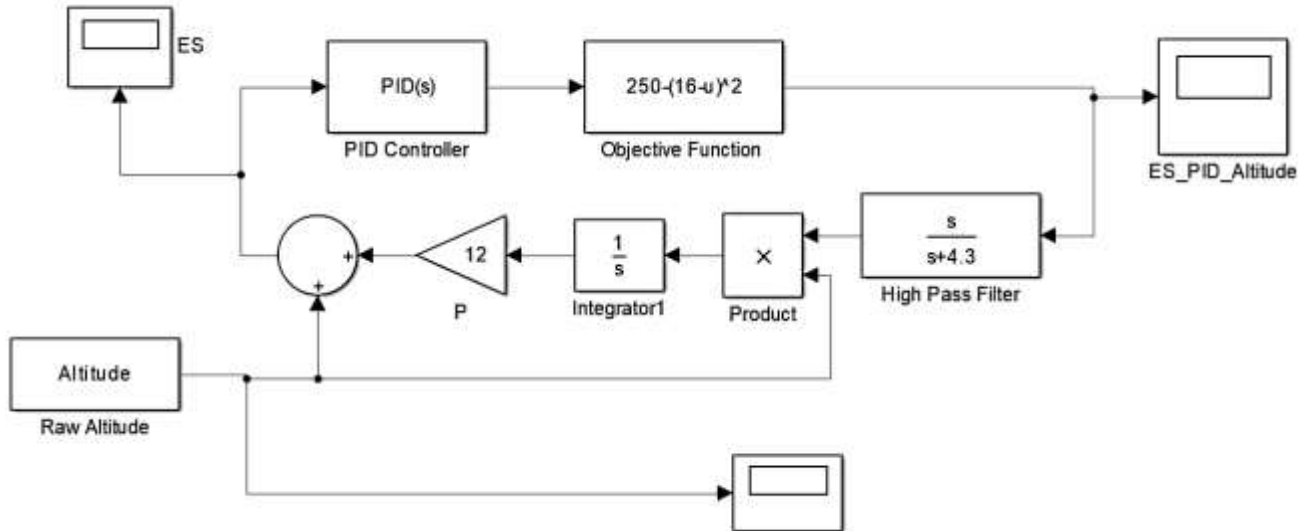


Figure 2.1: Extremum Seeking Based PID Controller Design

The simulink design in Figure (2.1) was used to stabilize the quadcopter by feeding in the aerodynamic data (Altitude, Roll, Pitch and Yaw).

3 Results and Discussion

This section highlights the results of aerodynamic data (Altitude, Yaw, Roll, and Pitch) necessary for quadcopter stability analysis using Extremum seeking based PID controller. After proper configuration of the flight simulator software (X-Plane 11) in order to communicate the flight data with Matlab/Simulink, the flight parameters are generated and analysed in this section.

3.1 Altitudinal Behaviour of the Quadcopter

Setting up the flight parameters after launching of the flight, the altitudinal dynamics of the quadcopter data are generated by X-Plane for a period of the first 120 seconds. The altitude and time relation of the quadcopter are graphically shown in Figure (3.1). The behaviour of the flight with respect to altitude is an analogy of a trainee, therefore these data/results describe the raw values of quadcopter in transient state. The raw results were sent to Matlab/Simulink for stability tuning using Extremum seeking PID controller

designed in Figure (2.1), where Figure (3.1) shows the comparison of raw (transient) altitudinal and ES-PID controller tuned profile.

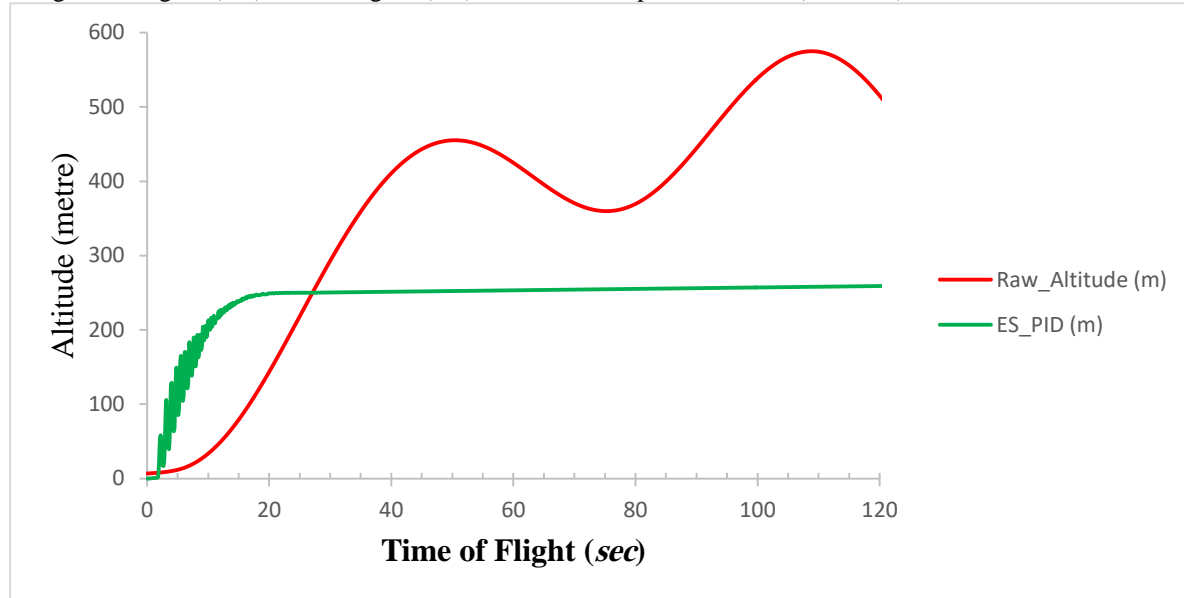


Figure 3.1: Altitudinal (Raw data) and ES-PID controller Dynamics of the Quadcopter

Observing the altitudinal characteristics of the raw data of the quadcopter, the transient behaviour is fed into the ES-PID controller using the parameters in Table (3.1) the system is driven towards stability and the corresponding curve is also presented in Figure (3.1) for comparison.

Table 3.1: Extremum seeking based PID controller Tuning Parameters

ES_PID Parameters	Values
Proportional gain	5
Integrator gain	1
Derivative gain	0.00001
High Pass Filter	$\frac{s}{s + 4.3}$
Objective function	$250 - (16 - u)^2$

By the application of the tuning parameters in Table (3.1) while designing the extremum seeking based PID controller to the raw data depicted in Figure (3.1), the system achieve stability using the metrics (peak response, rise time, settling time and the percentage overshoot). The stability metrics comparisons were drawn in Table 3.2 between the extremum seeking based PID controller tuned quadcopter data and the raw data (transient).

Table 3.2: Summary of Altitudinal Stability Metrics of Raw and ES-PID Controller

Stability Metrics	Raw Quadcopter Data	ES-PID Controller
Rise time (s)	35.02	9.75
Settling time (s)	undefined	20.70
Percentage overshoot (%)	37.40	0.001
Steady state (m)	Unstable	250

Rise time of the flight is the time at the lowest possible altitude when trying to achieve stability.

It can be observed from the profile at lowest altitude around 360.42m that the corresponding time is around 35 seconds.

It can be observed that the system does not achieve stability as there are ripples of altitude variation over time even after altitudinal height of 452 meter above sea level. This system is characterized by poor percentage overshoot of approximately 37.40%. The maximum peak response value being 575m and lowest altitude as approaching steady state is 360m, therefore the overshoot is the difference between highest crest and lowest trough values of the signal, i.e. (575m – 360m). The percentage overshoot is calculated from the profile using the relation in equation (4.1)

$$\% \text{ overshoot} = \frac{H_C - H_T}{H_C} \times 100 \Rightarrow \frac{575 - 360}{575} \times 100 = 37.4\% \quad (3.1)$$

where H_C is the signal value at the highest peak (crest), H_T is the lowest signal value at lowest trough. The 37.4% overshoot is not a good measure considering the stability of a system.

The settling time is undefined base on the altitudinal behaviour of the quadcopter as the system does not attain steady state, therefore does not have steady state response. All these instability measures were corrected with the help extremum seeking based PID controller and were minimized to the barest minimum for system stability.

The objective function and the High Pass Filter (integrator block) in ES-PID controller becomes a paramount tuning parameters having known the altitude of interest during operation of the system, they are responsible for the stability optimization of the quadcopter. For a typical example like in real life application, the objective function was chosen to converge at an altitude of interest around 250m. Around this height above the sea level, the quadcopter would only be hovering without making any significant altitudinal changes but could perform any other dynamical changes such as roll, yaw, and pitch.

Pitch, Roll and Yaw Characteristics of the Quadcopter

Other dimensions that describe a quadcopter dynamics include; Pitch, Roll and Yaw. The four corners of the quadcopter comprises of four rotors that were placed in a counter –clockwise direction to one another. A downwards tilt will move the aircraft (drone in this case) in a forward motion, while an upwards tilt will move it backwards.

The roll is the measure of how the drone navigates sideways causing it to “roll.” However, it does not cause the drone to change its altitude position but function of angles. These “rolls” cause the quadcopter to move to the right and the left on its horizontal axis and the corresponding data generated from X-Plane are sent to Simulink for ES-PID controller tuning. The Yaw is the rotation of the quadcopter along the z-axis on its vertical axis.

The above mention quadcopter dynamic orientations are all functions of angular displacement and time. These parameters are based on the choice of the controller and function towards selected target. In this study, these data were acquired from the flight simulator without any specific coordinate in space, so it an analogy to a trainee trying fly a quadcopter above the sea level with the aim of achieving stability at any altitude in any direction. Figure (3.2), shows the profile for roll, pitch and yaw characteristics of raw data.

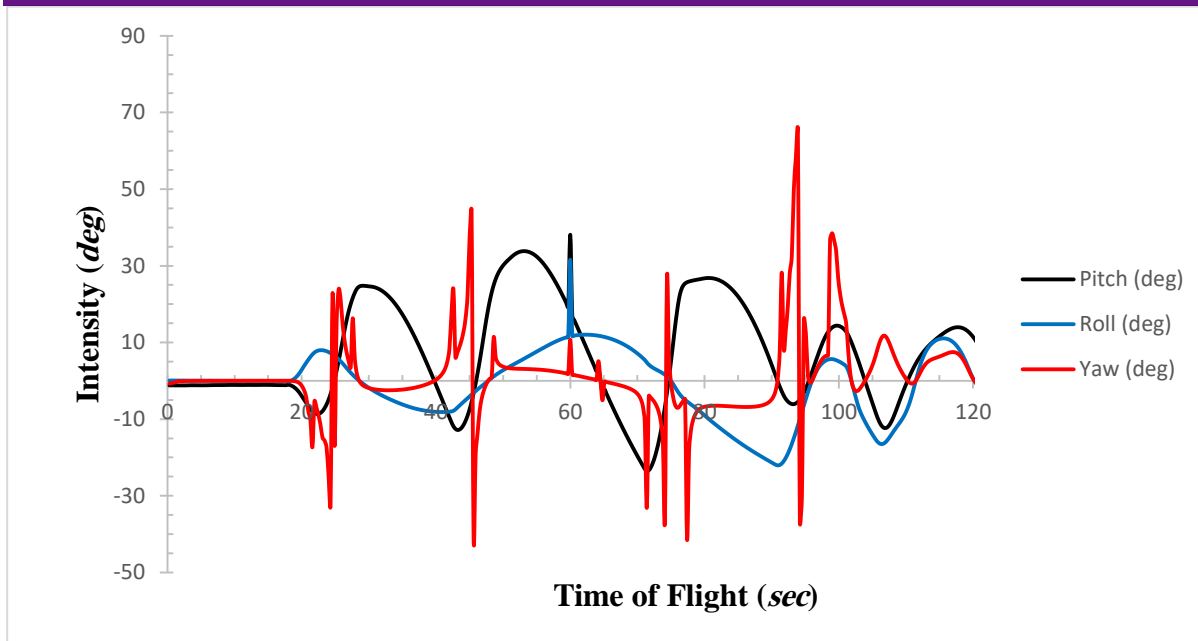


Figure 3.2: RawRoll, Yaw and Pitch of Quadcopter

Figure (3.2) is like a quadcopter taken off the sea level as reference point of pitch angle of approximately zero degree and a normal gradual increase with altitude, it was tilted towards the vertical direction with a decrease in pitch angle as observed in the pitch dynamics profile. The system is seen as not stable as the rise and fall in yaw are seen to be very sharp (Peak response approximately 22.48°) over time with the pitch angle also extending its readings to the negative region, thereby resulting into a relatively high value of percentage overshoot.

This instability could be minimized by treating the signal with Extremum seeking base PID controller with target coordinates. Without any specific coordinates towards target, the acquired raw data were subjected to ES-PID for stability purpose and the corresponding profile for the tuned system is shown in Figure (3.3).

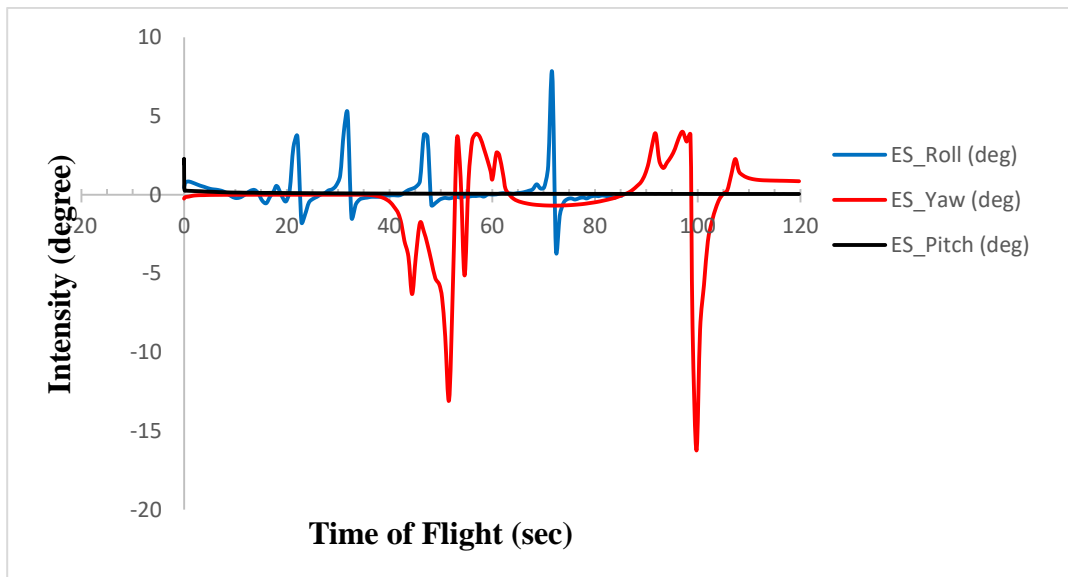


Figure 3.3: Tuned Roll, Yaw and Pitch of Quadcopter

Comparatively, it can be observed from the profile in Figure (3.3) that the instabilities along all possible orientations have been minimized drastically, as the raw counterpart profile in Figure (3.2) indicate overlapping and many consecutive crest/trough orientations over time. The system does not attain stability during the course of its flight because the system rolls away from

equilibrium position (reference angle, 0°) and never negotiate towards it. By the application of ES-PID controller scheme the roll, pitch and yaw dynamics orient within acceptable weight of stability.

Conclusion

This study was able to acquire aerodynamic data of quadcopter dynamics in terms of Altitude, Roll, Pitch and Yaw through flight simulator (X-Plane11), the raw data were communicated in real time with Matlab/Simulink. Extremum seeking base PID controller was designed on this platform to monitor and tune the stability metrics of the unstable/raw model of the quadcopter. With appropriate objective function selection through trial by error technique, the system performance was stabilized compared to the transient nature of the acquired raw model of the quadcopter. By characterizing the performance metrics in terms of rising time, percentage overshoot, rise time and steady state, extremum seeking base PID controller was found to be effective for stability and monitoring of quadcopter system. The system stability was optimized using appropriate metrics in real time. It can be concluded that knowing the coordinates of operation within which the quadcopter must operate, extremum seeking base PID is a suitable algorithm to achieve stability.

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