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Calculation the Energy Levels, Charge Density and Quadruple Electrical Transmissions B (E2) for $^{65\text{-}66}$Ni Isotope

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Abstracts: *The shell model used to calculate energy levels and reduced electric quadrupole transition probability B(E2) and , Charge Density for* ***$^{65\text{-}66}$Ni*** *isotope using OXBASH code within the f5p shell and using the f5pvh effective interaction.* ***$^{65\text{-}66}$Ni*** *isotope contain 8 proton out side* ***$^{65\text{-}66}$Ni*** *core in shell .The results of the energy levels and values of B(E2) and Charge Density were reasonably consistent with the experimental data available.*

Keywords: Charge Density , OXBASH Code. Isotope **$^{65\text{-}66}$Ni** Energy Levels , B(E2)

1-Introduction:

The basic assumption in the nuclear shell model is that, to first order, each nucleon (proton or neutron) is moving in an independent movement in a mean field. This is not so, a priori, since the nucleus constitutes an A-body problem interacting via the nucleon-nucleon force in the nuclear medium[1]. It is clear from the very beginning that this nucleon-nucleon force will be different from the free nucleon-nucleon interaction (Bohr, Mottelson 1969). The first person to present the idea of closed nuclear shell was the scientist W. W. Classer in 1934, as some studies on the rate of binding energy as well as the properties of nuclei led to the fact that the nuclei inside the nucleus are connected to the orbits of the atomic electrons, and it was called the cortical structure or the structure of the levels that The nucleus is stable[2]. The formulation of the shell model was based on a prototype called the single particle model, in which it is assumed that nucleons move free movement within the nucleus. Whereas, the basic hypothesis of the cortex model is the presence of a nuclear potential in which all the nucleons move, and that the average potential of all the nucleons is what controls the movement of any nucleon separately and that each nucleon represents orbit with a precisely defined angular momentum[3].

2-Theory:

2-1 Single particle shell model

The basic hypothesis of any shell model is the movement of each nucleon that does not depend on the movement of the other nucleon, meaning that each nucleon moves independently, despite the strong overall interaction between the nucleons that generate the binding energy. Neglecting double nucleon, this model is called a single particle shell model. As the Schrödinger equation can be written as follows[4] .

$$-\frac{\hbar^2}{2m_\circ}\nabla^2\Psi + V\Psi = E\Psi, \tag{1}$$

where m_0 is the mass of nucleon, V is the potential and E is the energy of incident particle. As we imagine that every nucleon moves within the same effort, that this potential is spherical in the simplified case, but there is good evidence that nucleons far from the closed shell are under the influence of an oval potential. In the independent particle model, one can interact between one particle and the other particles in the nuclei using a Central Potential potential. The simplest option for a central potential is the harmonic oscillator potential[5]:

$$H^{(0)}\phi(\vec{r}) = E^{(0)}\phi(\vec{r}) \tag{2}$$

$$H^{(0)} = T + V = \frac{P^2}{2M} + \frac{1}{2}M\ \omega^2 r^2 \tag{3}$$

Since M denotes the mass of the nucleon, the angular frequency, r the distance between both the nucleon and the center of the coordinates. The spectrum of the particle energy levels in the harmonic oscillator potential consists of a series of levels that are separated from each other by the amount ,i.e., they are separable in the angular and diagonal coordinates of that self-function can be written as follows[6]:

$$\phi_{nlm}(r,\theta,\phi) = R_{nl}(r)Y_{lm}(\theta,\phi) \tag{4}$$

The diagonal waveform represents the harmonic oscillator. Spherical wave function of harmonic oscillator. l, m is the number of quantum and angular momentum, respectively.

The diagonal waveform functions of the harmonic oscillator (r) are given by,

$$R_{nl}^2(r) = \left[\left(\pi^{1/2}b^3\right).\left\{(2l+1)!!\right\}^2.(n-1)!\right]^{-1} 2^{l-n+3}(2l+2n-1)!! \times\left(\frac{r}{b}\right)^{2l}\sum_{k=0}^{n-1}\left[\ (-1)^k\left\{(n-k-1)!k!(2k+2l+1)!!\right\}^{-1} \times 2^k(n-1)!(2l+1)!!\left(\frac{r}{b}\right)^{2k}\ \right]^2.\exp(-r^2/b^2) \tag{5}$$

That's where n !! = n (n-2) (n-4) ... (2 or 1) and volume information for the harmonic oscillator. And since the momentum operator is given by the following relationship [7].

$$\hat{\mathrm{P}} = -\mathrm{i}\hbar\vec{\nabla} \tag{6}$$

$$H^{(0)}\phi(\vec{r}) = \frac{1}{2}\hbar\omega\left(\ \frac{-\hbar}{M\ \omega}\nabla^2 + \frac{M\ \omega r^2}{\hbar}\ \right)\phi(\vec{r}) = E^{(0)}\phi(\vec{r}) \tag{7}$$

Energy Eigen Values for equation (7) are given as follows:

$$E_N{}^{(0)} = (2n+\ell-\frac{1}{2})\hbar\omega = (N+\frac{3}{2})\hbar\omega, \tag{8}$$

where N = 2 (n-1) + l is the Oscillator Quantum Number.
The subjective values for the effects of the total orbital angular momentum are given by the following equation,

$$\ell^2 Y_{\ell m}(\theta,\phi) = \ell(\ell+1)Y_{\ell m}(\theta,\phi) \tag{9}$$

According to the shell model of a single particle, the nuclear properties are determined by the nucleon outside the core[8]. The structure of the nuclear cortex or the nucleus stabilization relationship to the magic numbers represented by the number of nuclei cannot be easily reached, because we do not fully know the final shape of the nucleus or the nuclear effort. We know that the nuclear force is a strong attraction force and has an effect on small distances in the nucleus, so whatever the shape of this force or effort, the calculations of the single particle model depend on two basic assumptions[9]:

1. Every nucleus moves freely in the field of force expressed in the nuclear effort.
2. Apply Pauli's exclusion principle, meaning that energy levels are filled relative to the exclusion principle.

The nuclear energy levels of single particle states in a central field are arranged in groups, as the converging orbital group is referred to as the Major Shell[10]. Each plane in relation to the cortex model is named by the quantum number n, the value of s (s = ± 1/2) and the value of j and symbolized by it (SPS), and the spectrum resulting from the nucleus in this case is called the spectrum of the levels of one particle[11].

2-3 Nuclear density distribution

The density distribution of the ground plane can be calculated using the wave function of the harmonic oscillator. And the density distribution for a system consisting of A of point nickels can be expressed using the one-particle density factor that is given by the Dirac delta function[12].

$$\hat{\rho}(\vec{r}) = \sum_{i=1}^{A} \delta(\vec{r} - \vec{r}_i) \qquad (10)$$

As for the expected value of the single-particle density effect, it can be written in terms of the multi-particle wave function (ψ) :

$$< \psi(\vec{r}_1, \vec{r}_2 ...., \vec{r}_A) \left| \hat{\rho}(\vec{r}) \right| \psi(\vec{r}_1, \vec{r}_2 ...., \vec{r}_A) > = \sum_{i=1}^{A} \int_0^{\infty} \psi^* (\vec{r}_1, \vec{r}_2 ...., \vec{r}_A) \delta(\vec{r} - \vec{r}_i) \psi (\vec{r}_1, \vec{r}_2 ...., \vec{r}_A) . d\vec{r}_1, d\vec{r}_2, .., d\vec{r}_A$$

(11)

The wave function of a multi-nuclear particle (ψ) can be expressed through a slater determinant, depending on the wave functions of a single particle[13]:

$$\psi(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A) = \frac{1}{\sqrt{A!}} \det \phi_i (\vec{r}_j) \qquad (12)$$

And by taking the integration on equation (10) and using equation (11) we get:

$$\rho_o(r) = \left\langle \psi(\vec{r}_1, \vec{r}_2 ..., \vec{r}_A) \left| \hat{\rho}(\vec{r}) \right| \psi(\vec{r}_1, \vec{r}_2 ..., \vec{r}_A) \right\rangle = \sum_{i=1}^{A} \int_0^{\infty} \phi_i^* . (\vec{r}_l) \delta (\vec{r} - \vec{r}_1) \phi_i (\vec{r}_l) d\vec{r}_1 \qquad (13)$$

Because of the orthogonal functions and using the properties of the Dirac delta function, the density distribution for a system containing A of nucleons, we get :

$$\rho_o(r) = \sum_{i=1}^{A} \left| \phi_i(\vec{r}) \right|^2 \qquad (14)$$

In order to obtain a clear expression for the elements of the one-particle density matrix, the harmonic oscillator basis will be used to define the states of the single particle[14] :

$$\phi_i(\vec{r}) = \psi_{nlm}(\vec{r}) \chi_{s_i} \chi_{t_i} \qquad (15)$$

Where $i = n, l, m, s_i, t_i$ while , $\psi_{nlm}(\vec{r})$, χ_{S_i} , χ_{t_i} , space function, spin, isospin respectively Each of n , l , m represents the main quantum count and the orbital angular momentum and its projections. As for the spin, it represents the single-particle spin and isotopic spin.

The space vector function is as a function of the harmonic oscillation and is given by the following relationship[15] :

$$\psi_{nlm}(\vec{r}) = R_{nl}(r)\, Y_{lm}(\theta_i, \varphi_i) \qquad (16)$$

Represents the

$R_{nl}(r)$: diagonal wave function of the harmonic oscillator.

$Y_{lm}(\theta_i, \phi_i)$: The spherical wave function of the harmonic oscillator

Substituting equations (15) and (16) into equation (14), [16] we get

$$\rho_o(r) = \sum_{nlm} \left| R_{nl}(r)\, Y_{lm}(\theta_i, \varphi_i) \right|^2 \sum_{s_i t_i} \left| \chi_{s_i} \chi_{t_i} \right|^2 \qquad (17)$$

$$\rho_o(r) = 4\sum_{nlm} \left| R_{nl}(r)\, Y_{lm}(\theta_i, \varphi_i) \right|^2 = 4\sum_{nl} \left| R_{nl}(r) \right|^2 \sum_{m} \left| Y_{lm}(\theta_i, \phi_i) \right|^2 \qquad (18)$$

The factor 4 in equation (18) is taken from the isotopic twist calculation, and the second term in equation (18) can be written as follows

$$\sum_m |\, Y_{lm}(\theta_i, \varphi_i)\,|^2 = \frac{2l+1}{4\pi} \qquad (19)$$

From the substitution of equation (19) in equation (18), the formula for the density of a system of point nucleons, A, can be obtained:

$$\rho_o(r) = 4\sum_{nl} \left| R_{nl}(r) \right|^2 \frac{2l+1}{4\pi} \qquad (20)$$

And by replacing the term $2(2l+1)$ with $(2j+1)$ in equation (20), we get the equation for mass density when Z=N

$$\rho_o(r) = \frac{2}{4\pi} \sum_{nlj} (2j+1) \left| R_{nl}(r) \right|^2 \qquad (21)$$

Or in the case of Z≠N we get:

$$\rho_o(r) = \frac{2}{4\pi} \sum_{nlj \in I} (2j+1) \left| R_{nl}(r) \right|^2 + \frac{1}{4\pi} \sum_{nlj \notin I} N \left| R_{nl}(r) \right|^2 \qquad (22)$$

where I represents the closed orbit

N represents the number of nucleons in the unfilled orbital $.nlj$

$$\int \rho_\circ(r)\ d\tau = 4\pi \int \rho_\circ(r)\ r^2 dr = A \qquad (23)$$

where A is the mass number of the nucleus[17].

As for the density of the charge distribution, it is given by the following equation $\rho_{ch}(r) = \frac{1}{4\pi}\sum_{nlj\in I}(2j+1)\left|R_{nl}(r)\right|^2$

$$+\frac{1}{4\pi}\sum_{nlj\notin I}N_P\left|R_{nl}(r)\right|^2 \qquad (24)$$

$$\int \rho_{ch}(r)\ d\tau = 4\pi\int \rho_{ch}(r)\ r^2 dr = z \qquad (25)$$

Where Z represents the number of protons in the nucleus [18].

3-Results and Discussion:

3.1.1 Energy levels of ^{65}Ni nucleus:

By applying the shell model and using the OXBASH program, the ground state of the ^{65}Ni nucleus is a closed nucleus ^{56}Ni with 9 neutrons distributed in three orbits (1f5/2, 2p3/2 and 2p1/2) representing the model space. The expected states are formed through the presence of these nucleons in a model space, and to calculate their energy levels, the interaction (f5pvh) was used. Table (1) shows the comparison between the theoretical and experimental values available for the nucleus of ^{65}Ni [8,19].

Table (1) is a comparison between the theoretical values of the energy levels relative to the ground state of the nickel ^{65}Ni nucleus with the experimental results using the f5pvh interaction.

Theoretical values of f5pvh interaction		**Experimental values**	
J^-	E (MeV)	E (MeV)	J^π
$5/2_1$	0.0000	0.0000	5/2-
$1/2_1$	0.0240	0.0634	1/2-
$3/2_1$	0.2780	0.3101	3/2-
$3/2_2$	0.5030	0.6932	3/2-
$1/2_2$	0.9230	1.4176	1/2-
$7/2_1$	0.9690	1.1417	(7/2-,9/2-)
$5/2_2$	1.1500	1.2741	(5/2-)
$9/2_1$	1.4600	----	----
$3/2_3$	1.6710	1.7720	(3/2)-
$5/2_3$	1.6990	----	----
$7/2_2$	1.9360	1.5940	7/2-
$3/2_4$	2.2420	2.1468	3/2-
$11/2_1$	2.2600	----	----

$9/2_2$	2.3220	----	----
$5/2_4$	2.4790	----	----
$7/2_3$	2.7210	----	----
$5/2_5$	2.7420	----	----
$3/2_5$	2.7950	3.1080	(1/2-,3/2-)
$9/2_3$	2.9240	----	----
$5/2_6$	2.9380	----	----
$9/2_4$	2.9760	----	----
$7/2_4$	3.2400	----	----
$1/2_3$	3.2630	----	----
$5/2_7$	3.2990	----	----
$7/2_5$	3.5310	----	----
$3/2_6$	3.5520	----	----
$9/2_5$	3.5910	----	----
$5/2_8$	3.7020	----	----
$3/2_7$	3.7170	----	----
$7/2_6$	3.7390	----	----
$1/2_4$	3.7610	----	----
$3/2_8$	3.8980	----	----
$5/2_9$	4.0360	----	----
$1/2_5$	4.2370	----	----
$5/2_{10}$	4.3140	----	----
$3/2_9$	4.5150	----	----
$3/2_{10}$	5.0680	----	----

Through Table (1) and comparing the results of this analogue, it is clear that:

1- An agreement was obtained for the ground condition of the level $5/2^-_1$ when compared with the confirmed experimental value.
2- Good agreement was obtained in levels ($1/2^-_1$, $3/2^-_1$, and $3/2^-_2$) with energy values (0.0240, 0.2780, and 0.5030) respectively in MeV, with corresponding experimental values.

3- The total angular momentum and parity of the level (7/2-,9/2-), (5/2-), and (1/2-,3/2-) with an uncertain value of (1.1417, 1.2741, and 3.1080) MeV have been confirmed in practice by close matching of theoretical values.
4- The total angular momentum of the level (3/2)- with an uncertain value of 1.7720 MeV have been confirmed in practice by close matching of theoretical values.
5- We did not notice from table (1) that we obtained 27 new levels, which are experimentally undefined for the purpose of discussing this study.

3.1.2 Energy levels of ^{66}Ni nucleus:

By applying the shell model and using the OXBASH program, the ground state of the ^{66}Ni nucleus is a closed nucleus ^{56}Ni with 10 neutrons distributed in three orbits (1f5/2, 2p3/2 and 2p1/2) representing the model space. The expected states are formed through the presence of these nucleons in a model space, and to calculate their energy levels, the interaction (f5pvh) was used. Table (2) shows the comparison between the theoretical and experimental values available for the nucleus of ^{66}Ni [9].

Table (2) is a comparison between the theoretical values of the energy levels relative to the ground state of the nickel ^{66}Ni nucleus with the experimental results using the f5pvh interaction.

Theoretical values of f5pvh interaction		**Experimental values**	
J^+	E (MeV)	E (MeV)	J^π
0_1	0.0000	0.0000	0+
2_1	1.4340	1.4248	2+
4_1	2.6220	----	----
2_2	2.7820	2.9160	2+
3_1	2.9910	2.6708	(3+)
4_2	3.0300	3.1854	(4+)
0_2	3.1520	2.4450	0+
1_1	3.2910	----	----
2_3	3.3880	3.2306	2+
2_4	3.5870	3.7460	2+
3_2	3.6090	----	----
0_3	3.8670	2.9650	0+
1_2	4.0300	----	----

Through Table (2) and comparing the results of this analogue, it is clear that:

1- A match was obtained for the ground state of the level 0^+_1 when compared with the confirmed experimental value.
2- Good agreement was obtained at the level 2^+_1 with energy value 1.4340 MeV, with the corresponding experimental value.
3- The total angular momentum and parity of the level (3+), and (4+) with an uncertain value of (2.6708, and 3.1854) MeV were confirmed in practice by a close match of the theoretical values.

3 -2 Reduced Electric Quadrupole Transition Probability B(E2)

We have studied the reduction potential of the quadrupole electrode B(E2) for the isotopes of nickel $^{65\text{-}66}$Ni, and these calculations were carried out using the (OXBASH) program. As transitions are one of the important quantities in nuclear physics to determine the group in nuclei, the transition potential is one of the most sensitive criteria in determining effective interactions. In order to illustrate this sensitivity, the reduced electrode quadrupole transmission potential B(E2) is calculated. These calculations may be performed and compared with the available experimental values. The accuracy of the calculated transitions in this work is done using harmonic oscillating potential (HO, b), the calculated results are in good agreement for all cores in this work with the available experimental values. The effect of closed heart depolarization

A ^{65}Ni nucleus was selected for the F5P model space, and table (3) shows the B(E2) values of ^{65}Ni isotope obtained for the f5pvh interaction with effective charges (ep = 1.65 e and en = 0.65 e) in comparison with the available experimental values.

Table 3: Comparison of B (E2) values for ^{65}Ni nucleus with experimental values are available in e^2fm^4 units.

Ji	→	**Jf**	**Exp. B (E2) (e^2fm^4)**	**f5pvh B (E2) (e^2fm^4)**
$1/2^-_1$	→	$5/2^-_1$	1.847	1.909
$3/2^-_1$	→	$5/2^-_1$	---	1.933
$3/2^-_1$	→	$1/2-_1$	---	14.72
$3/2^-_2$	→	$5/2^-_1$	---	44.69
$3/2^-_2$	→	$1/2^-_1$	---	0.4001
$3/2^-_2$	→	$3/2^-_1$	---	6.28
$1/2^-_2$	→	$5/2^-_1$	---	14.4
$1/2^-_2$	→	$3/2^-_1$	---	36.08
$7/2^-_1$	→	$5/2^-_1$	---	28.43
$7/2^-_1$	→	$3/2^-_1$	---	14.74

A ^{66}Ni nucleus was selected for the F5P model space, and table (4) shows the B(E2) values of ^{66}Ni isotope obtained for the f5pvh interaction with effective charges (ep = 1.5 e and en = 0.5 e) in comparison with the available experimental values.

.**Table 4**: Comparison of B (E2) values for ^{66}Ni nucleus with experimental values are available in e^2fm^4 units

Ji	→	**Jf**	**Exp. B (E2) (e^2fm^4)**	**f5pvh B (E2) (e^2fm^4)**
2^+_1	→	0^+_1	---	14.5
4^+_1	→	2^+_1	---	13.94
2^+_2	→	0^+_1	---	0.1254
2^+_2	→	2^+_1	---	1.746
2^+_2	→	4^+_1	---	0.04739
3^+_1	→	2^+_1	---	0.0008534
3^+_1	→	4^+_1	---	0.7782
3^+_1	→	2^+_2	---	1.201

$4^{+}2 \rightarrow 2^{+}_1$	---	1.019
$4^{+}_2 \rightarrow 4^{+}_1$	---	1.361

3-3 Charge density distribution ρ_{ch}

The nuclear charge density distributions for nickel isotopes $^{65\text{-}66}$Ni were calculated and they are shown in Figures (1) and (2). It is noticed from Figure (1) and (2) that the value of the charge density of the isotopes is in the center of the nucleus ρ_0 (e^2fm^4), then it begins to increase to a limit R_0(fm) which lasted in table (5).

Table 5: Value of ρ_0 (e^2fm^4) and R_0(fm) for nickel isotopes.

Nucleus	$\rho_0(e^2fm^4)$	$R_0(fm)$
^{65}Ni	0.100	7.5
^{66}Ni	0.100	7.5

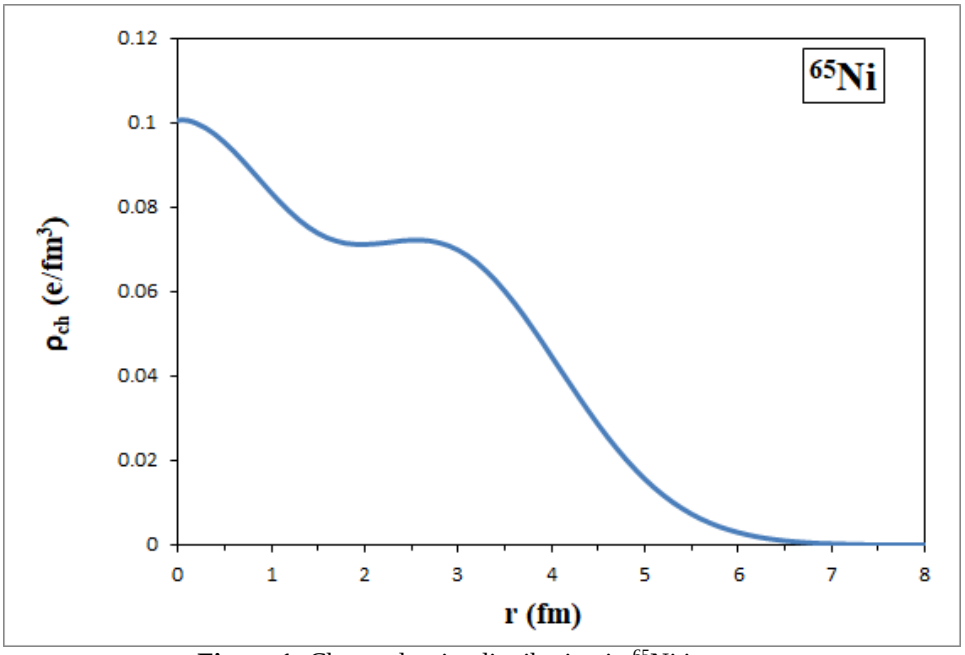

Figure 1: Charge density distribution in ^{65}Ni isotope.

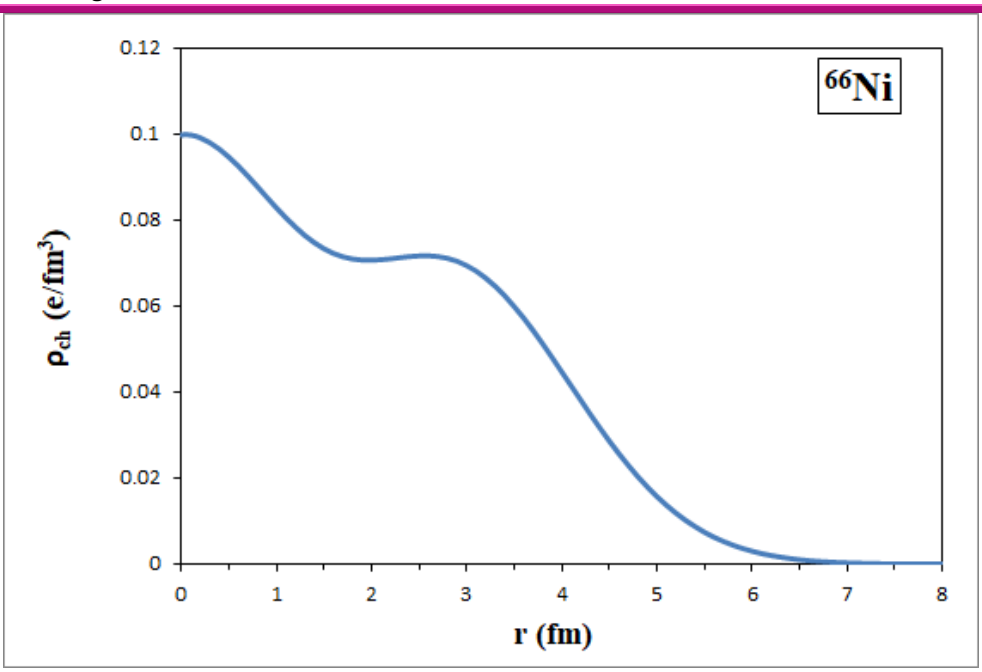

Figure 2: Charge density distribution in ^{66}Ni isotope.

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