International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 6 Issue 6, June - 2022, Pages: 46-56

Derivation of a Numerical Method for Calculating the Singularity Triple Integrals in the Lower Term of the Integral Numerically

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Abstract: The main objective of this research is to derive a rule for calculating the values of triple integrals numerically with Singularity integrals and partial derivatives in the lower term of the region $\left[x_0, x_n\right]\times\left[y_0, y_1\right]\times\left[z_0, z_n\right]$ of integration using the quasi-burning rule on the three dimensions, z x, y, and how to find the correction limits for them (error formula) and improve these results using the method Romberg's acceleration through these correction limits, when the number of sub-periods into which the integration period is segmented on the inner dimension x is equal to the number of sub-intervals into which the integration period is segmented on the outer dimension y. We will symbolize this method with the symbol Rttt and it is reliable because it gave high accuracy in the results compared to the analytical values of the integrals with a few partial periods.

Keywords: Numerical integration ; triple integrals ; Romberg's acceleration ; continuous integrand:.

1.Introducation

Numerical analysis is characterized by the creation of diverse methods for finding approximate solutions to certain mathematical issues in an effective manner. The efficiency of these methods depends on both the accuracy and the ease with which they can be implemented. The numerical analysis is the numerical interface of the wide field of applied analysis. Tripartite integrations are of great importance in finding sizes and middle positions and the determination of the inertia of the volumes and finding the blocks with variable density, for example the size of the inside and above and below and the calculation of the average position of the size of the impact in and above the level and below the level, Such as a piece of thin wire or a thin sheet of metal. Frank Ayers [8], prompting a number of researchers to work in the field of triple integrals. Which led many researchers to work in the field of tripartite integrations, including Hans Jarr and Jacobsen [1] in 1973, Frank Ayers [2] in 1975, Muhammad [3] in 1984and Hilal [4] in 2081.

In this paper we present a theorem with proof to derive a new base for calculating approximate values of triple integrations with constant inversions and their error formula. This rule is the result of the application of the Rumbark acceleration method to the values resulting from the use of the two point bases on the internal x and outer dimensions and the trapezoid base on the middle dimension (The number of partial periods divided by the internal dimension period, the number of partial periods divided by the middle dimension period and the number of partial periods divided by the external dimension period and we will mark this method with the symbol where the method of accelerating Rumbark and Several derivative we have got good results in terms of accuracy and speed of approaching the number of partial periods of relatively few and very short time

1.1 Triple Integrals For Continuous Integrands With Singularity In Partial Derivatives

Let's say the triple integral is defined by the formula:

$$I=\int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f(x,y,z)dxdydz$$

Which can be written using the trapezoid rule on the three dimensions in the following form:

$$I=\int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f(x,y,z)dxdydz = ttt\left(h\right)+E_{ttt}\left(h\right) \qquad \cdots\left(1-1\right)$$

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Whereas $x_i = a + ih, (i = 1,2,3\cdots,n-1)$ & $h = \dfrac{b-a}{n}$ & $y_j = c + jh, j = 1,2,3,\cdots$ $z_k = e + kh, k = 1,2,3,\cdots$

Note that $ttt(h)$ it represents the approximate value of the integral using the trapezoidal rule, and that $E_{ttt}(h)$

A string of correction terms that can be added to values $ttt(h)$.

Theorem :-Let the function $f(x,y,z)$ be continuous and differentiable at every point in the region $[x_0,x_n]\times[y_0,y_1]\times[z_0,z_n]$. at least one of its derivatives is not differentiable at the point $(x,y,z)=(x_0,y_0,z_0)$ Tthe approximate value of the triple integral

$$I = \int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f(x,y,z)dxdydz$$

$$ttt = \int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f(x,y,z)dxdydz$$

$$= \frac{h^3}{8}[f(x_0,y_0,z_0) + f(x_0,y_0,z_n) + f(x_0,y_n,z_0) +$$
$$f(x_0,y_n,z_n) + f(x_n,y_0,z_0) + f(x_n,y_0,z_n) + f(x_n,y_n,z_0) +$$
$$f(x_n,y_n,z_n) + 2\sum_{i=1}^{n-1}(f(x_0,y_0,z_i) + f(x_0,y_n,z_i) + f(x_n,y_0,z_i) +$$
$$f(x_0,y_i,z_0) + f(x_0,y_i,z_n) + f(x_n,y_i,z_0) + f(x_n,y_i,z_n) + f(x_i,y_0,z_0)$$
$$+ f(x_i,y_0,z_n) + f(x_i,y_n,z_n) + 2\sum_{j=1}^{n-1}(f(x_0,y_i,z_j) + f(x_n,y_i,z_j) +$$
$$f(x_i,y_0,z_j) + f(x_i,y_n,z_j) + f(x_i,y_j,z_0) + f(x_i,y_j,z_n)$$
$$+2\sum_{k=1}^{n-1} f(x_i,y_j,z_k))]$$

It can be calculated from the following rule:-

And that the error formula is

$$\{-h^5[\frac{1}{12}D_x^{(2)} + \frac{1}{12}D_y^{(2)} + \frac{1}{12}D_z^{(2)}] +$$
$$+h^6[(\frac{1}{24}D_x^{(3)} + \frac{1}{24}D_y^{(3)} + \frac{1}{24}D_z^{(3)}) + \frac{1}{24}(D_x^{(2)}D_y + D_y^{(2)}D_x + D_x^{(2)}D_z +$$
$$D_z^{(2)}D_x + D_y^{(2)}D_z + D_z^{(2)}D_y)] - h^7[\cdots]\} f(x_1,y_1,z_1) + A_{TTT}h^2 + B_{TTT}h^4 + \cdots$$

Since $A_{TTT}, B_{TTT},\cdots$ the constants depend on the values of the partial derivatives of the function f

Proof:

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We can write the above integral as follows

$$I=\int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f\left(x,y,z\right)dxdydz=\int_{z_0}^{z_1}\int_{y_0}^{y_1}\int_{x_0}^{x_1} f\left(x,y,z\right)dxdydz$$

$$+\sum_{t=1}^{n-1}\int_{z_t}^{z_{t+1}}\sum_{s=1}^{n-1}\int_{y_s}^{y_{s+1}}\int_{x_0}^{x_1} f\left(x,y,z\right)dxdydz+\sum_{t=1}^{n-1}\int_{z_t}^{z_{t+1}}\int_{y_0}^{y_1}\sum_{r=1}^{n-1}\int_{x_r}^{x_{r+1}} f\left(x,y,z\right)dxdydz+$$

$$\int_{z_0}^{z_1}\sum_{s=1}^{n-1}\int_{y_s}^{y_{s+1}}\sum_{r=1}^{n-1}\int_{x_r}^{x_{r+1}} f\left(x,y,z\right)dxdydz+\sum_{t=1}^{n-1}\int_{z_t}^{z_{t+1}}\int_{y_0}^{y_1}\int_{x_0}^{x_1} f\left(x,y,z\right)dxdydz+\int_{z_0}^{z_1}\sum_{s=1}^{n-1}\int_{y_s}^{y_{s+1}}\int_{x_0}^{x_1} f\left(x,y,z\right)dxdydz$$

$$+\int_{z_0}^{z_1}\int_{y_0}^{y_1}\sum_{r=1}^{n-1}\int_{x_r}^{x_{r+1}} f\left(x,y,z\right)dxdydz+\int_{z_1}^{z_n}\int_{y_1}^{y_n}\int_{x_1}^{x_n} f\left(x,y,z\right)dxdydz \qquad (1-2)$$

Since the function is defined at the point then we can spread it in Taylor series around the point (x_1,y_1,z_1) so we have

$$f(x,y,z)=1+(x-x_1)D_x+(y-y_1)D_y+(z-z_1)D_z+\frac{(x-x_1)^2}{2}D_x^{(2)}$$

$$+\frac{(y-y_1)^2}{2}D_y^{(2)}+\frac{(z-z_1)^2}{2}D_z^{(2)}+(x-x_1)(y-y_1)D_xD_y+(x-x_1)(z-z_1)D_xD_z$$

$$+(y-y_1)(z-z_1)D_yD_z+\frac{(x-x_1)^3}{6}D_x^{(3)}+\frac{(y-y_1)^3}{6}D_y^{(3)}+\frac{(z-z_1)^3}{6}D_Z^{(3)}+$$

$$\frac{(x-x_1)^2(y-y_1)}{2}D_x^{(2)}D_y+\frac{(x-x_1)(y-y_1)^2}{2}D_xD_y^{(2)}+\frac{(x-x_1)^2(z-z_1)}{2}D_x^{(2)}D_z+$$

$$(x-x_1)(y-y_1)(z-z_1)D_xD_yD_z+\frac{(y-y_1)^2(z-z_1)}{2}D_y^{(2)}D_z+\frac{(x-x_1)(z-z_1)^2}{2}D_xD_Z^{(2)}+$$

$$\frac{(y-y_1)(z-z_1)^2}{2}D_yD_Z^{(2)}+\frac{(x-x_1)^4}{24}D_x^{(4)}+\frac{(y-y_1)^4}{24}D_y^{(4)}+$$

$$\frac{(z-z_1)^4}{24}D_z^{(4)}\cdots]f(x_1,y_1,z_1) \qquad (1-3)$$

By integrating equation (1-3) and defining the region $(x_0,x_1)\times(y_0,y_1)\times(z_0,z_1)$, we get

$$\int_{z_0}^{z_1}\int_{y_0}^{y_1}\int_{x_0}^{x_1} f(x,y,z)dxdydz = [(x_1-x_0)(y_1-y_0)(z_1-z_0) - \frac{(x_0-x_1)^2(y_1-y_0)(z_1-z_0)}{2}D_x -$$
$$\frac{(x_1-x_0)(y_0-y_1)^2(z_1-z_0)}{2}D_y - \frac{(x_1-x_0)(y_1-y_0)(z_0-z_1)^2}{2}D_z +$$
$$\frac{(x_0-x_1)^3(y_1-y_0)(z_1-z_0)}{6}D_x^{(2)} - \frac{(x_1-x_0)(y_0-y_1)^3(z_1-z_0)}{6}D_y^{(2)} -$$
$$\frac{(x_n-x_{n-1})(y_n-y_{n-1})(z_n-z_{n-1})^3}{6}D_z^{(2)} + \frac{(x_0-x_1)^2(y_0-y_1)^2(z_0-z_1)^2}{8}D_xD_yD_z +$$
$$\frac{(x_0-x_1)^2(y_0-y_1)^2(z_1-z_0)}{4}D_xD_y + \frac{(x_0-x_1)^2(y_1-y_0)(z_0-z_1)^2}{4}D_xD_z +$$
$$\frac{(z_0-z_1)^2(y_0-y_1)^2(x_1-x_0)}{4}D_zD_y - \frac{(x_0-x_1)^4(y_1-y_0)(z_1-z_0)}{24}D_x^{(3)} -$$
$$\frac{(x_1-x_0)(y_0-y_1)^4(z_1-z_0)}{24}D_y^{(3)} - \frac{(x_1-x_0)(y_1-y_0)(z_0-z_1)^4}{24}D_z^{(3)} +$$
$$\frac{(x_0-x_1)^3(y_0-y_1)^2(z_1-z_0)}{12}D_x^{(2)}D_y + \frac{(x_0-x_1)^2(y_0-y_1)^3(z_1-z_0)}{12}D_xD_y^{(2)} +$$
$$\frac{(x_n-x_{n-1})^3(y_n-y_{n-1})(z_n-z_{n-1})^2}{12}D_x^{\,2}D_Z + \frac{(x_1-x_0)(y_0-y_1)^3(z_0-z_1)^2}{12}D_y^{(2)}D_z +$$
$$\frac{(x_0-x_1)^2(y_1-y_0)(z_0-z_1)^3}{12}D_xD_z^{(2)} + \frac{(x_n-x_{n-1})(y_n-y_{n-1})^2(z_n-z_{n-1})^3}{24}D_z^{(2)}D_y -$$
$$\frac{(x_0-x_1)^5(y_1-y_0)(z_1-z_0)}{120}D_x^{(4)} - \frac{(x_1-x_0)(y_0-y_1)^5(z_1-z_0)}{120}D_y^{\,4} -$$
$$\frac{(x_n-x_{n-1})(y_n-y_{n-1})(z_n-z_{n-1})^5}{120}D_z^{(4)}\cdots\ ]\, f(x_1,y_1,z_1) \qquad (1-4)$$

Substituting for each of $(x_1-x_0)\ (y_1-y_0)$ for h and of $(z_0-z_1)\ (x_0-x_1)\ (y_0-y_1)$

(z_1-z_0) For-h

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$$\int_{z_0}^{z_1}\int_{y_0}^{y_1}\int_{x_0}^{x_1} f(x,y,z)dxdydz = [h^3 - \frac{h^4}{2}D_x - \frac{h^4}{2}D_y - \frac{h^4}{2}D_z + \frac{h^5}{6}D_x^{(2)} + \frac{h^5}{6}D_y^{(2)} + \frac{h^5}{6}D_z^{(2)} + \frac{h^5}{4}D_xD_y +$$
$$\frac{h^5}{4}D_xD_z + \frac{h^5}{4}D_yD_z - \frac{h^6}{24}D_x^{(3)} - \frac{h^6}{24}D_y^{(3)} - \frac{h^6}{24}D_z^{(3)} -$$
$$\frac{h^6}{12}D_x^{(2)}D_y - \frac{h^6}{12}D_xD_y^{(2)} - \frac{h^6}{12}D_x^{(2)}D_z - \frac{h^6}{8}D_xD_yD_z - \frac{h^5}{12}D_y^{(2)}D_z +$$
$$\frac{h^6}{12}D_xD_z^{(2)} - \frac{h^6}{4}D_yD_z^{(2)}$$
$$\frac{h^7}{120}D_x^{(4)} + \frac{h^7}{120}D_y^{(4)} + \frac{h^7}{120}D_z^{(4)} \cdots]f(x_1,y_1,z_1) \qquad (1-5)$$

To find the base of the trapezoid on the three dimensions, we substitute of x for x_0 and y of y_0 and z of z_0 In the equation $(1-3)$ we get

$$f(x_0,y_0,z_0) = [1 + (x_0 - x_1)D_X + (y_0 - y_1)D_y + (z_0 - z_1)D_z$$
$$+ \frac{(x_n - x_{n-1})^2}{2}D_X^{(2)} + \frac{(y_n - y_{n-1})^2}{2}D_y^{(2)} + \frac{(z_n - z_{n-1})^2}{2}D_z^{(2)}$$
$$+(x_0 - x_1)(y_0 - y_1)D_xD_y + (x_0 - x_1)(z_0 - z_1)D_xD_z$$
$$+(y_0 - y_1)(z_0 - z_1)D_yD_z + \frac{(x_0 - x_1)^3}{6}D_x^{(3)} + \frac{(y_0 - y_1)^3}{6}D_y^{(3)}$$
$$+ \frac{(z_0 - z_1)^3}{6}D_Z^{(3)} + \frac{(x_0 - x_1)^2(y_0 - y_1)}{2}D_x^{(2)}D_y$$
$$+ \frac{(x_0 - x_1)(y_0 - y_1)^2}{2}D_xD_y^{(2)} +$$
$$\frac{(x_0 - x_1)^2(z_0 - z_1)}{2}D_x^{(2)}D_z(x_0 - x_1)(y_0 - y_1)(z_0 - z_1)D_xD_yD_z$$
$$+ \frac{(y_0 - y_1)^2(z_0 - z_1)}{2}D_y^{(2)}D_z + \frac{(x_0 - x_1)(z_0 - z_1)^2}{2}D_xD_Z^{(2)} + \frac{(y_0 - y_1)(z_0 - z_1)^2}{2}D_yD_Z^{(2)} +$$
$$\frac{(x_0 - x_1)^4}{24}D_x^{(4)} + \frac{(y_0 - y_1)^4}{24}D_y^{(4)} + \frac{(z_0 - z_1)^4}{24}D_z^{(4)} \cdots]f(x_1,y_1,z_1) \qquad (1-6)$$

In compensation for all $(y_0 - y_1)$, $(z_0 - z_1)$, $(x_0 - x_1)$ of -h in equation (1-6) we get

$$f(x_0,y_0,z_0)=1-hD_x-hD_y-hD_z+\frac{h^2}{2}D_x^{(2)}+\frac{h^2}{2}D_y^{(2)}+\frac{h^2}{2}D_z^{(2)}+h^2D_xD_y+$$
$$h^2D_xD_z+h^2D_yD_z-\frac{h^3}{6}D_x^{(3)}-\frac{h^3}{6}D_y^{(3)}-\frac{h^3}{6}D_z^{(3)}-\frac{h^3}{2}D_x^{(2)}D_y-\frac{h^3}{2}D_y^{(2)}D_x-\frac{h^3}{2}D_x^{(2)}D_z$$
$$-h^3D_xD_yD_z-\frac{h^3}{2}D_y^{(2)}D_z-\frac{h^3}{2}D_xD_z^{(2)}-\frac{h^3}{2}D_yD_z^{(2)}$$
$$-\frac{h^3}{2}D_xD_z^{(2)}-\frac{h^3}{2}D_yD_z^{(2)}+\frac{h^4}{24}D_x^{(4)}$$
$$+\frac{h^4}{24}D_y^{(4)}+\frac{h^4}{24}D_z^{(4)}+\cdots]f(x_1,y_1,z_1) \qquad (1-7)$$

Substituting the rest of the points in the above way, we get the following results

$$f(x_0,y_1,z_0)=[1-hD_x-hD_z+\frac{h^2}{2}D_x^{(2)}+\frac{h^2}{2}D_z^{(2)}+h^2D_xD_z+$$
$$-\frac{h^3}{6}D_x^{(3)}-\frac{h^3}{6}D_z^{(3)}-\frac{h^3}{2}D_x^{(2)}D_z-\frac{h^3}{2}D_xD_z^{(2)}+\frac{h^4}{24}D_x^{(4)}+\frac{h^4}{24}D_z^{(4)}+\cdots]f(x_1,y_1,z_1) \qquad (1-8)$$

And also

$$f(x_1,y_0,z_0)=[1-hD_y+hD_z+\frac{h^2}{2}D_y^{(2)}+\frac{h^2}{2}D_z^{(2)}+h^2D_yD_z$$
$$-\frac{h^3}{6}D_y^{(3)}-\frac{h^3}{6}D_z^{(3)}-\frac{h^2}{2}D_y^{(2)}{}_z-\frac{h^2}{2}D_yD_z^{(2)}+$$
$$\frac{h^4}{24}D_y^{(4)}+\frac{h^4}{24}D_z^{(4)}+\cdots]f(x_1,y_1,z_1) \qquad (1-9)$$

And also

$$f(x_1,y,z_0)=[1-hD_z+\frac{h^2}{2}D_z^{(2)}-\frac{h^3}{6}D_z^{(3)}+\frac{h^4}{24}D_z^{(4)}\cdots]f(x_1,y_1,z_1) \qquad (1-11)$$

$$f(x_1,y_0,z_1)=[1-hD_{1,y}+\frac{h^2}{2}D_y^{(2)}-\frac{h^3}{6}D_y^{(3)}+\frac{h^4}{24}D_y^{(4)}+\cdots]f(x_1,y_1,z_1) \qquad (1-12)$$

$$f(x_1,y_1,z_1)=1 \qquad (1-13)$$

$$f(x_0,y_0,z_1)=[1-hD_x-hD_y+\frac{h^2}{2}D_x^{(2)}+\frac{h^2}{2}D_y^{(2)}+h^2D_xD_y+$$
$$\frac{h^3}{6}D_x^{(3)}-\frac{h^3}{6}D_y^{(3)}-\frac{h^3}{2}D_x^{(2)}D_y-\frac{h^3}{2}D_xD_y^{(2)}+\frac{h^4}{24}D_x^{(4)}+\frac{h^4}{2}D_y^{(4)}+\cdots]f(x_1,y_1,z_1) \qquad (1-14)$$

And multiplying the equations $(1-12),(1-13),(1-14),(1-15)$ of $\frac{-h^3}{8}$ combine it with the equation $(1-4)$ We get

$$\int_{z_0}^{z_1}\int_{y_0}^{y_1}\int_{x_0}^{x_1} f(x,y,z)=\frac{h^3}{8}[f(x_1,y_1,z_1)+f(x_1,y_0,z_1)+f(x_1,y_0,z_0)+$$
$$f(x_0,y_1,z_1)+f(x_0,y_1,z_0)+f(x_0,y_0,z_0)+f(x_0,y_0,z_1)+f(x_1,y_1,z_0)]$$
$$-[h^5(\frac{1}{12}D_x^{(2)}+\frac{1}{12}D_y^{(2)}+\frac{1}{12}D_z^{(2)})+$$
$$\frac{h^6}{24}((D_x^{(3)}+D_y^{(3)}+D_z^{(3)})+(D_x^{(2)}D_y+D_xD_y^{(2)}+$$
$$D_zh_y^{(2)}+D_z^{(2)}D_y+D_xD_z^{(2)}+D_x^{(2)}D_z))]-h^7[\cdots]f(x_1,y_1,z_1) \qquad (1-15)$$

As for the other seven integrals, their integrals are continuous functions in their regions of integration and their partial derivatives are present. So we use theorem (3-2) for continuous integrals by TTT method, so we get

$$\sum_{t=1}^{n-1}\int_{z_t}^{z_{t+1}}\sum_{s=1}^{n-1}\int_{y_s}^{y_{s+1}}\int_{x_0}^{x_1} f(x,,y,z)dxdydz=\frac{h^3}{8}\sum_{t=1}^{n-1}\sum_{s=1}^{n-1}[f(x_0,y_s,z_t)+f(x_0,y_s,z_{t+1})+$$
$$f(x_0,y_{s+1},z_t)+f(x_0,y_{s+1},z_{t+1})+f(x_1,y_s,z_t)+$$
$$f(x_1,y_s,z_{t+1})+f(x_1,y_{s+1},z_t)+f(x_1,y_{s+1},z_{t+1})]$$
$$+A_1h^2+B_1h^4+\cdots \qquad (1-16)$$

also

$$\sum_{t=1}^{n-1}\int_{z_t}^{z_{t+1}}\int_{y_0}^{y_1}\sum_{r=1}^{n-1}\int_{x_r}^{x_{r+1}} f(x,y,z)dxdydz=\frac{h^3}{8}\sum_{t=1}^{n-1}\sum_{r=1}^{n-1}[f(x_r,y_0,z_t)+f(x_r,y_0,z_{t+1})+$$
$$f(x_r,y_1,z_t)+f(x_r,y_1,z_{t+1})+f(x_{r+1},y_0,z_t)+$$
$$f(x_{r+1},y_0,z_{t+1})+f(x_{r+1},y_1,z_t)+f(x_{r+1},y_1,z_{t+1})]$$
$$+A_2h^2+B_2h^4+\cdots \qquad (1-17)$$

Likewise

aalso

$$\int_{z_0}^{z_1}\sum_{s=1}^{n-1}\int_{y_s}^{y_{s+1}}\sum_{r=1}^{n-1}\int_{x_r}^{x_{r+1}} f(x,y,z)dxdydz=\frac{h^3}{8}\sum_{s=1}^{n-1}\sum_{r=1}^{n-1}[f(x_r,y_s,z_0)+f(x_r,y_s,z_1)+$$
$$f(x_r,y_{s+1},z_0)+f(x_r,y_{s+1},z_1)+f(x_{r+1},y_s,z_0)+$$
$$f(x_{r+1},y_s,z_1)+f(x_{r+1},y_{s+1},z_0)+f(x_{r+1},y_{s+1},z_1)]+$$
$$+A_4h^2+B_4h^4+\cdots \qquad (1-19)$$

$$\int_{z_0}^{z_1}\sum_{s=1}^{n-1}\int_{y_s}^{y_{s+1}}\int_{x_0}^{x_1} f(x,y,z)dxdydz=\frac{h^3}{8}\sum_{s=1}^{n-1}[f(x_0,y_s,z_0)+f(x_0,y_s,z_1)+$$
$$f(x_0,y_{s+1},z_0)+f(x_0,y_{s+1},z_1)+f(x_1,y_s,z_0)+$$
$$f(x_1,y_s,z_1)+f(x_1,y_{s+1},z_0)+f(x_1,y_{s+1},z_1)]$$
$$+A_5h^2+B_5h^4+\cdots \qquad (1-20)$$

International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 6 Issue 6, June - 2022, Pages: 46-56

$$\int_{z_0}^{z_1}\int_{y_0}^{y_1}\sum_{r=1}^{n-1}\int_{x_r}^{x_{r+1}} f(x,y,z)dxdydz=\frac{h^3}{8}\sum_{r=1}^{n-1}[f(x_r,y_0,z_0)+f(x_r,y_0,z_1)+$$
$$f(x_r,y_1,z_0)+f(x_r,y_1,z_1)+f(x_{r+1},y_0,z_0)+$$
$$f(x_{r+1},y_0,z_1)+f(x_{r+1},y_1,z_0)+f(x_{r+1},y_1,z_1)]$$
$$+A_6h^2+B_6h^4+\cdots \qquad (1-21)$$

$$\int_{z_1}^{z_n}\int_{y_1}^{y_n}\int_{x_1}^{x_n} f(x,y,z)dxdydz=\frac{h^3}{8}[f(x_1,y_1,z_1)+f(x_1,y_1,z_n)+$$
$$f(x_1,y_n,z_1)+f(x_1,y_n,z_n)+f(x_n,y_1,z_1)+$$
$$f(x_n,y_1,z_n)+f(x_n,y_n,z_1)+f(x_n,y_n,z_n)+$$
$$2\sum_{i=2}^{n-1}(f(x_1,y_1,z_i)+f(x_1,y_n,z_i)+f(x_n,y_1,z_i)+f(x_1,y_i,z_1)+$$
$$f(x_1,y_i,z_n)+f(x_n,y_i,z_1)+f(x_n,y_i,z_n)+f(x_i,y_1,z_1)+f(x_i,y_1,z_n)+$$
$$f(x_i,y_n,z_n)+2\sum_{j=2}^{n-1}(f(x_1,y_i,z_j)+f(x_n,y_i,z_j)+$$
$$f(x_i,y_1,z_j)+f(x_i,y_n,z_j)+f(x_i,y_j,z_1)+f(x_i,y_j,z_n)+$$
$$2\sum_{k=2}^{n-1}f(x_i,y_j,z_k))]+A_7h^2+B_7h^4+\cdots \qquad (1-22)$$

also

whereas $(i=1,2,3,4,5,6,7)$ $A_i, B_i, \cdots$ Constants that depend on the values of the partial derivatives of a function f For the variables z,y,x Adding the equations $(4-21)\cdots(4-14)$, we get

$$\int_{z_0}^{z_n}\int_{y_0}^{y_n}\int_{x_0}^{x_n} f(x,y,z)dxdydz$$
$$=\frac{h^3}{8}[f(x_0,y_0,z_0)+f(x_0,y_0,z_n)+f(x_0,y_n,z_0)+f(x_0,y_n,z_n)+f(x_n,y_0,z_0)+$$
$$f(x_n,y_0,z_n)+f(x_n,y_n,z_0)+f(x_n,y_n,z_n)+$$
$$2\sum_{i=1}^{n-1}(f(x_0,y_0,z_i)+f(x_0,y_n,z_i)+f(x_n,y_0,z_i)+$$
$$f(x_0,y_i,z_0)+f(x_0,y_i,z_n)+f(x_n,y_i,z_n)+f(x_n,y_i,z_n)+f(x_i,y_0,z_0)+f(x_i,y_0,z_n)+f(x_i,y_n,z_n)$$
$$+2\sum_{j=1}^{n-1}(f(x_0,y_i,z_j)+f(x_n,y_i,z_j)+f(x_i,y_0,z_j)+f(x_i,y_n,z_j)+f(x_i,y_j,z_0)+$$
$$f(x_i,y_j,z_n)+2\sum_{k=1}^{n-1}f(x_i,y_j,z_k))]-h^5[\frac{1}{12}D_x^{(2)}+\frac{1}{12}D_y^{(2)}+\frac{1}{12}D_z^{(2)}]+$$
$$+h^6[(\frac{1}{24}D_x^{(3)}+\frac{1}{24}D_y^{(3)}+\frac{1}{24}D_z^{(3)})-\frac{1}{24}(D_x^{(2)}D_y+D_y^{(2)}D_x+D_x^{(2)}D_z+$$
$$D_z^{(2)}D_x+D_y^{(2)}D_z+D_z^{(2)}D_y)]-h^7[\cdots]f(x_1,y_1,z_1)+$$
$$A_{TTT}h^2+B_{TTT}h^4+\cdots \qquad (1-23)$$

whereas $A_{TTT}, B_{TTT}, \cdots$ Constants that depend on the values of the partial derivatives of the function f

Thus, the proof was made.

2.EXAMPLS

	Integrals	Values of Approximate Integrals
1	$\int_0^1\int_0^1\int_0^1 \sqrt{x+y+z}\,dxdydz$	1.2056568615166 rounded to thirteen decimal places
2	$\int_0^1\int_0^1\int_0^1 \log(x+y+z)dxdydz$	0.3378332434334 rounded to thirteen decimal places

1-Integration function $I = \int_0^1\int_0^1\int_0^1 \sqrt{x+y+z}\,dxdydz$ Continuous on region $[0,1]\times[0,1]\times[0,1]$ and derivable in the region $(0,1]\times(0,1]\times(0,1]$

But the integrator is defective at the lower end and the type of disorder is radical, so the error formulas (correction limits) for the integral So the error formulas for the integration are mentioned when applying the ttt:-

$$E_{ttt}(h) = A_{ttt}h^2 + a_1h^{3.5} + B_{ttt}h^4 + C_{ttt}h^6 + \cdots$$

We conclude from Table (1) when=64 the value of the integral using the rule is true for sixe decimal places, and when using the Rumberck acceleration with the rule, we get an integer value for thirteen decimal places

2-- Integration function $\int_0\int_0\int^1\int^1\int^1 \log(x+y+z)dxdydz$ Continuous on region $[0,1]\times[0,1]\times[0,1]$ and derivable in the region $(0,1]\times(0,1]\times(0,1]$ But the integral is defective at the lower end and the type is logarithmic, so the error formulas (correction limits) for the integral So the error formulas for the integration are mentioned when applying the ttt

$$E_{ttt}(h) = A_{ttt}h^2 + a_1h^3 + B_{ttt}h^4 + C_{ttt}h^6 + \cdots$$

We conclude from Table (2) when=64 the value of the integral using the rule is true for five decimal places, and when using the Rumberck acceleration with the rule, we get an integer value for thirteen decimal places

International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 6 Issue 6, June - 2022, Pages: 46-56

n	ttt	K=2	K=3.5	K=4	K=6	K=8	
1	1.1218364368360						
2	1.1901503959160	1.2129217156093					
4	1.2023197348487	1.2063761811597	1.2057415370401				
8	1.2048739362637	1.2057253367353	1.2056622319438	1.2056569449374			
16	1.2054658994889	1.2056632205640	1.2056571978835	1.2056568622795	1.2056568609674		
32	1.2056095569371	1.2056574427531	1.2056568825462	1.2056568615237	1.2056568615117	1.2056568615138	
64	1.2056450748019	1.2056569140902	1.2056568628319	1.2056568615176	1.2056568615175	1.2056568615175	1.2056568615175
table(1) $\int_0^1\int_0^1\int_0^1 \sqrt{x+y+z}\,dxdydz$						The real value	1.2056568615166

n	ttt	K=2	K=3	K=4	K=6	K=8	K=10	K=12	K=14
1	0.3972567287935								
2	0.3272497416737	0.3039140793004							
4	0.3310025621034	0.3322535022466	0.3363019912389						
8	0.3354555400613	0.3369398660473	0.3376093465902	0.3376965036136					
16	0.3371353081997	0.3376952309125	0.3378031401790	0.3378160597515	0.3378179574680				
32	0.3376432642149	0.3378125828867	0.3378293474544	0.3378310946061	0.3378313332546	0.3378313857086			
64	0.3377834866478	0.3378302274587	0.3378327481119	0.3378329748224	0.3378330046671	0.3378330112216	0.3378330128106		
128	0.3378204805160	0.3378328118054	0.3378331809978	0.3378332098569	0.3378332135876	0.3378332144069	0.3378332146055	0.3378332146548	
256	0.3378300070954	0.3378331826219	0.3378332355957	0.3378332392355	0.3378332397019	0.3378332398043	0.3378332398291	0.3378332398352	0.3378332434334
table (2) $\int_0^1\int_0^1\int_0^1 \log(x+y+z)\,dxdydz$								The real value	0.3378332434334

3.Discussion

We conclude through the results and tables of this research that this method gives good results and with high accuracy, as this rule gave correct results for four decimal places compared to the analytical value and after using the Rumberck acceleration, I obtained a value close to thirteen decimal places as in the first example at n = 64

And when=256 in the second integration we get an approximation of thirteen decimal places after using the Rumberk acceleration and in a very short time

From observing the results of the above examples, we conclude that the Rttt method has high accuracy, excellent speed, short time, and is reliable in calculating triple integrals

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