The Cyclic Decomposition of The Factor Group $cf(Q_{2m}\times D_4,Z)/\bar{R}(Q_{2m}\times D_4)$ When m is an odd Number

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Abstract : The purpose of this paper is to find The Cyclic Decomposition of The Factor Group $cf(Q_{2m}\times D_4,Z)/\bar{R}(Q_{2m}\times D_4)$ when m is a prime number, where Q_{2m} is denoted to Quaternion group of order 4n, such that for each positive integer n, there are two generators x and y for Q_{2m} satisfies $Q_{2m}=\{x^h y^k, 0\leq h \leq 2m-1, k=0,1\}$which has the following properties$\{x^{2m}=y^4=I, yx^m y^{-1}=x^{-m}\}$ and D_4 is the Dihedral group of order 8 is generate by a rotation r of order 4 and reflection s of order 2 then 8 elements of D_4 can be written as: $\{I^*,r,r^2, r^3,s,sr,sr^2,sr^3\}$.

Keywords: Cyclic Decomposition ,Factor Group, Q_{2m} , D_4 ,odd number.

1.Introduction :

Let F be a field .The general linear group GL (n,F) is a multiplicative group of all non-singular $n\times n$ matrices over F [2].

Let F be a field .A matrix representation of G is homomorphism T: Gr→GL (n , F), n is called ***the degree of representation*** T .T is called a unit representation(principal) if T(g)=1 for all g$\in$Gr[2].

Let T be a matrix representation of Gr over the field F. The ***character*** χ of a matrix representation T is the mapping χ : Gr→F defined by $\chi(g)=tr(T(g))$ for all g $\in$Gr .The degree of T is called the degree of χ .Recall that the trace of an $n\times n$ matrix A is the sum of main diagonal elements : $tr(A)=\sum_{i=1}^{n} a_{ii}$ [1].

Two elements of Gr are said to be ***Γ- conjugate*** if the cyclic subgroups they generate are conjugate in G ; this defines an equivalence relation on Gr .These classes are called ***Γ- classes***[4].

In this work we find The Cyclic Decomposition of The Factor Group $cf(Q_{2m}\times D_4,Z)/\bar{R}(Q_{2m}\times D_4)$ When m is a prime Number .

2. The Group $\bar{R}$(Gr) :

Definition (2.1):[9]

A class function f on the group Gr with values in C is called a **complex – valued class function on Gr**, the set of all complex - valued class functions will be denoted by cf(G,C).

Definition (2.2):[6]

The group generated by all generalized characters on C is called **the group of the generalized characters of G** and it is denoted by R (Gr).

Definition (2.3):[10]

The intersection of cf (Gr , Z) with R (G) forms an abelian group which is called the group of Z–valued generalized characters of G, denoted by $\overline{R}$(G) and the cf (G,Z)/ $\overline{R}$(Gr) is a finite abelian factor group denoted by K(Gr).

Definition (2.4):[5]

Let K a subfield of a field F . The Galios group of F over K ,denoted by Gal(F/K) , is the set of all those automorphisms of F that fix K. If f(x)$\in$K[x] , and if F=K(Z_1,Z_2,…,Z_n) is a splitting field , then the Galois group of f(x) over K is defined to be Gal(F/K) .

3. ***The Rational Valued Characters Table:***

Definition (3.1):[8]

A rational valued character θ of Gr is a character whose values are in Z, which is θ(g) $\in$ Z, for all g $\in$ Gr .

Proposition (3.2):[9]

The rational valued characters $\theta_i = \sum_{\sigma \in Gal(Q(\chi_i)/Q)} \sigma(\chi_i)$ form basis for $\overline{R}$(Gr), where χ_i are the irreducible characters of G and their numbers are equal to the number of all distinct Γ- classes of Gr.

Proposition (3.3):[6]

The number of all rational valued characters of a finite group Gr is equal to the number of all distinct Γ-classes on Gr .

Definition (3.4):[9]

The information about rational valued characters of a finite group Gr is displayed in a table called **the rational valued characters table of Gr**.

We denote it by $\equiv^*$(Gr) which is $l\times l$ matrix whose columns are Γ-classes and rows are the values of all rational valued characters of G, where l is the number of Γ-classes.

Lemma (3.5):[8]

Let A, L and W be matrices with entries in the principal ideal domain R .Let L and W are invertible matrices, then :

D_k(L A W)= D_k (A).

Theorem (3.6):[3]

Let M $\in M_{n\times m}$(A) be a matrix with entries in a principle ideal domain. Then there exist two invertible matrices L$\in GL_n$(A) , W$\in GL_m$(A) and a quasi-diagonal matrix D$\in M_{n\times m}$(A) (that is , d_{ij} =0 for $i \neq j$)such that

1- M=LDW .
2- $d_1 \mid d_2, \ldots, d_i \mid d_{i+1}, \ldots$, where the d_j are the diagonal entries of D .

And then , D_k(LDW)= D_k(M) modulo the group of unites of A .

Theorem (3.7):[9]

K(G) = $\oplus \sum C_{d_i}$

Such that $d_i = \pm D_i(\equiv^*(Gr)) / \pm D_{i-1}(\equiv^*(Gr))$.

Theorem (3.8):[4]

$|K(G)| = \det(\equiv^*(Gr))$.

Proposition(3.9):[9]

If A and B two matrices of the degree n and m respectively, then

$\det(A \otimes B) = (\det(A))^m . (\det(B))^n$

Proposition(3.10):[7]

Let A and B be two non-singular matrices of the rank n and m respectively ,over a principal domain R .

And let $L_1AW_1 = D(A) = \mathrm{diag}\{d_1(A), d_2(A),\ldots\ldots, d_n(A)\}$

$L_2BW_2 = D(B) = \mathrm{diag}\{d_1(B), d_2(B),\ldots\ldots, d_m(B)\}$

be the invariant factor matrices of A and B , then

$$(L_1 \otimes L_2).(A \otimes B).(W_1 \otimes W_2) = D(A) \otimes D(B)$$

and from this we can write down the invariant factor matrix of $A \otimes B$.

Let H_1 and H_2 be P_1-group and P_2-group respectively ,where P_1 and P_2 are distinct primes .We know that

$\equiv(H_1 \times H_2) = \equiv(H_1) \otimes \equiv(H_2)$

$(P_1, P_2) = 1$,so we have

$\equiv^*(H_1 \times H_2) = \equiv^*(H_1) \otimes \equiv^*(H_2)$.

Theorem(3.11):[6]

Let H_1 and H_2 be p_1-group and p_2-group respectively ,where

$(p_1, p_2) = 1$, let $\equiv^*(H_1)$ and $\equiv^*(H_2)$ be of the ranks n , m respectively.

$$K(H_1 \times H_2) = \underbrace{K(H_1) \oplus \cdots \cdots \oplus K(H_1)}_{m\text{-times}} \oplus \underbrace{K(H_2) \oplus \cdots \cdots \oplus K(H_2)}_{n\text{-times}}$$

Proposition(3.11):[9]

Let $n = \prod_{i=1}^{k} P_i^{\alpha_i}$,where P_i are distinct primes ,then :

$$K(C_n) = \oplus \sum_{i=1}^{k} \left(\oplus \sum K\left(C_{P_i^{\alpha_i}}\right)\right) \left[\prod_{\substack{j \neq i \\ j=1}}^{k} (\alpha_j + 1) \right] time.$$

Example(3.12) :

$$K(C_{165}) = K(C_{5.3.11}) = \bigoplus_{i=1}^{2} C_5 \bigoplus_{i=1}^{2} C_3 \bigoplus_{i=1}^{2} C_{11}$$

Theorem(3.13) : [11]

If m is an odd number , then

$K(Q_{2m}) = K(C_{2m}) \oplus C_4$.

4 *The Main Results*

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To calculate two matrices N and R.

First we will define two matrices n_1 and r_1 are the same degree of $\equiv^*(Q_{2m})$.

$$n_1 = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad r_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & 1 \end{bmatrix}$$

Second we will define two matrices n_2 and r_2 such that

$$n_2 = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad r_2 = \begin{bmatrix} (p-1) & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & (p-1) & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Third we will define two matrices D_1 and D_2 where

$$D_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 1 & -1 & -1 & 1 & 0 \end{bmatrix} \qquad D_2 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & -1 & 1 & 1 & 1 \\ 0 & 1 & -1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

From first,second and third we have

$N = (n_2, n_1)\, D_1$

$R = (r_1, r_2)\, D_2$.

Theorem(4.1):

If $m = p_1.p_2 \ldots p_n$, such that p_i's are distints primes, all i=1,2,…,n then the cyclic decomposition of $k(Q_{2m} \times D_4)$ is :

$$K(Q_{2m} \times D_4) = \bigoplus_{i=1}^{3} K(C_{2m}) \oplus K(C_2^{\,4}) \bigoplus_{i=1}^{3} C_4 \bigoplus_{i=1}^{3} C_2 \oplus C_3$$

Proof :

We use theorem(3.6) and theorem(3.7) to prove theorem .

If m is a prime number, we will define two matrices N and R such that

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$$N(Q_{2m}\times D_4) = \begin{pmatrix}
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&1&1&0&-1&0&0&0&0&0&0&1&1&0&-1&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&-1&1&0&0&0&0&0&0&0&0&-1&1&0&0&0&0&0&0\\
0&0&0&0&0&1&-1&-1&1&0&0&0&0&0&0&1&-1&-1&1&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&0\\
0&1&1&0&-1&0&1&1&0&-1&0&1&1&0&-1&0&1&1&0&-1&0&0&0&0&0\\
0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&-1&1&0&0&0&-1&1&0&0&0&-1&1&0&0&0&-1&1&0&0&0&0&0&0\\
1&-1&-1&1&0&1&-1&-1&1&0&1&-1&-1&1&0&1&-1&-1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&1&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&-1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&-1&-1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&1&1&0&-1&0&1&1&0&-1&0&1&1&0&-1&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&-1&1&0&0&0&-1&1&0&0&0&-1&1&0&0&0&0&0&0&0&0&0&0&0\\
1&-1&-1&1&0&1&-1&-1&1&0&1&-1&-1&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&1&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&-1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&-1&-1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{pmatrix}$$

$$\equiv^{*}(Q_{2m}\times D_4) = \begin{pmatrix}
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&1&-1&1&-1&1&1&-1&1&-1&1&1&-1&1&-1&1&1&-1&1&-1&1&1&-1&1&-1\\
1&1&1&-1&-1&1&1&1&-1&-1&1&1&1&-1&-1&1&1&1&-1&-1&1&1&1&-1&-1\\
1&1&-1&-1&1&1&1&-1&-1&1&1&1&-1&-1&1&1&1&-1&-1&1&1&1&-1&-1&1\\
2&-2&0&0&0&2&-2&0&0&0&2&-2&0&0&0&2&-2&0&0&0&2&-2&0&0&0\\
m-1&m-1&m-1&m-1&m-1&-1&-1&-1&-1&-1&m-1&m-1&m-1&m-1&m-1&-1&-1&-1&-1&-1&0&0&0&0&0\\
m-1&m-1&-(m-1)&m-1&-(m-1)&-1&-1&1&-1&1&m-1&m-1&-(m-1)&m-1&-(m-1)&m-1&m-1&-(m-1)&m-1&-(m-1)&0&0&0&0&0\\
m-1&m-1&m-1&-(m-1)&-(m-1)&-1&-1&-1&1&1&m-1&m-1&m-1&-(m-1)&-(m-1)&m-1&m-1&m-1&-(m-1)&-(m-1)&0&0&0\\
m-1&m-1&-(m-1)&-(m-1)&m-1&-1&-1&1&1&-1&m-1&m-1&-(m-1)&-(m-1)&m-1&m-1&m-1&-(m-1)&-(m-1)&m-1&0&0&0&0&0\\
2(m-1)&-2(m-1)&0&0&0&-2&2&0&0&0&2(m-1)&-2(m-1)&0&0&0&2(m-1)&-2(m-1)&0&0&0&0&0&0&0&0\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&-1&-1&-1&-1&-1\\
1&1&-1&1&-1&1&1&-1&1&-1&1&1&-1&1&-1&1&1&-1&1&-1&-1&-1&1&-1&1\\
1&1&1&-1&-1&1&1&1&-1&-1&1&1&1&-1&-1&1&1&1&-1&-1&-1&-1&-1&1&1\\
1&1&-1&-1&1&1&1&-1&-1&1&1&1&-1&-1&1&1&1&-1&-1&1&-1&-1&1&1&-1\\
1&1&-1&-1&1&1&1&-1&-1&1&1&1&-1&-1&1&1&1&-1&-1&1&-2&2&0&0&0\\
m-1&m-1&m-1&m-1&m-1&-1&-1&-1&-1&-1&-(m-1)&-(m-1)&-(m-1)&-(m-1)&-(m-1)&1&1&1&1&1&0&0&0&0&0\\
m-1&m-1&-(m-1)&m-1&-(m-1)&-1&-1&1&-1&1&-(m-1)&-(m-1)&m-1&-(m-1)&m-1&1&1&-1&1&-1&0&0&0&0&0\\
m-1&m-1&m-1&-(m-1)&-(m-1)&-1&-1&-1&1&1&-(m-1)&-(m-1)&-(m-1)&m-1&m-1&1&1&1&-1&-1&0&0&0&0&0\\
m-1&m-1&-(m-1)&-(m-1)&m-1&-1&-1&1&1&-1&-(m-1)&-(m-1)&m-1&m-1&-(m-1)&1&1&-1&-1&1&0&0&0&0&0\\
2(m-1)&-2(m-1)&0&0&0&-2&2&0&0&0&-2(m-1)&2(m-1)&0&0&0&1&1&-1&-1&1&0&0&0&0&0\\
2&2&2&2&2&2&2&2&2&2&-2&-2&-2&-2&-2&-2&-2&-2&-2&-2&0&0&0&0&0\\
2&2&-2&2&-2&2&2&-2&2&-2&-2&-2&2&-2&2&-2&-2&2&-2&2&0&0&0&0&0\\
2&2&2&-2&-2&2&2&2&-2&-2&-2&-2&-2&2&2&-2&-2&-2&2&2&0&0&0&0&0\\
2&2&-2&-2&2&2&2&-2&-2&2&-2&-2&2&2&-2&-2&-2&2&2&-2&0&0&0&0&0\\
4&-4&0&0&0&4&-4&0&0&0&-4&4&0&0&0&-4&4&0&0&0&0&0&0&0&0
\end{pmatrix}$$

$$R(Q_{2m}\times D_4)=\begin{pmatrix}
(m-1) & (m-1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & (m-1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-(m-1) & -(m-1) & (m-1) & (m-1) & (m-1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & (m-1) & -(m-1) & (m-1) & 2(m-1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & (m-1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & -1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-(m-1) & -(m-1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (m-1) & (m-1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -(m-1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (m-1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
(m-1) & (m-1) & -(m-1) & -(m-1) & -(m-1) & 0 & 0 & 0 & 0 & 0 & -(m-1) & -(m-1) & (m-1) & (m-1) & (m-1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -(m-1) & (m-1) & -(m-1) & -2(m-1) & 0 & 0 & 0 & 0 & 0 & 0 & (m-1) & -(m-1) & (m-1) & 2(m-1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -(m-1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (m-1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & -1 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -m & -m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m & m & -m & -m & -m & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -m & m & -m & -2(m-1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

Then

$N.\equiv^{*}(Q_{2m}\times D_4).\ R=\text{diag}\{84m,44m,-24m,-20m,26m,4m,2,-4m,8m,-2,42m,-6,2m,1,-1,-11,12,20,-20,-12,6,-13,11,1,-1\}$

Example(4.2):

Find the cyclic decomposition to the group($Q_{22}\times D_4$) .

$$N.\equiv^{*}(Q_{22}\times D_4).\ R = \begin{pmatrix}
924 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 484 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -264 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -220 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 286 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 44 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -44 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 88 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 462 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -66 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -20 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -12 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -13 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 11 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1
\end{pmatrix}$$

= diag{924,484,-264,-220,286,44,2,-44,88,-2,462,-66,22,1,-1,-11,12,20,-12,6, -13,11,1,-1}.

$$K(Q_{22}\times D_4) = \bigoplus_{i=1}^{3} K(C_{22}) \oplus K(C_2^{4}) \bigoplus_{i=1}^{3} C_4 \bigoplus_{i=1}^{3} C_2 \oplus C_3 .$$

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