International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 6 Issue 8, August - 2022, Pages: 160-163

Queue For Fuel Filling At Hasanuddin Spbu Using Non-Homogenous Poisson Process

Maktisen Ena[1], Nirwan [2], Nurtiti Sunusi[3], Hedi Kuswanto[4]

[1,2,3,4] Department of Statistics, Faculty of Mathematics and Natural Sciences, Hasanuddin University
Makassar 90245, Indonesia
email: enatisen06@gmail.com

Abstract : *People arriving at the Hasanuddin gas station can be observed at certain time intervals so that the probability of the number of attendance at the Hasanuddin gas station can be observed. The non-homogeneous Poisson process (NHPP) is a process that depends on parameters, time, and exponential distribution which have unequal parameter values and do not depend on each other. The probability that an event does not occur in an early condition is one and the probability of an event in an early condition is zero. The use of non-homogeneous Poisson in this arrangement aims to calculate the number of attendance at the Hasanuddin gas station over a period of 6 months with the number of working hours/day 15 hours equal to 900 minutes, with time intervals that are inaugurated every day. The information used in this research is the number of attendance at the Hasanuddin SBPU for 6 months. The amount of information collected is 141 information, with a total of 15 hours/day. The results of this research obtained the calculation of the probability of the number of attendance at the Hasanuddin gas station.*

Keywords - Hasanuddin gas station, Estimation Parameters, Non-Homogeneous Poisson Process

1. INTRODUCTION

The number of attendance at the Hasanuddin gas station which was inaugurated where the entire process of attendance at the Hasanuddin gas station was on a hierarchical model that was included in the queuing process so that data could be collected to estimate parameters [1]. It is estimated that the presence of people when refueling at the Hasanuddin gas station is more in line for refueling the oil. The non-homogeneous Poisson process can be used as a solution procedure to analyze information within a certain period of time [2]. The non-homogeneous Poisson Process (NHPP) hierarchical model can also be used to determine the number of attendance/day at the Hasanuddin gas station, which is modeled by the non-homogeneous Poisson process, which has a serious meaning. In the Poisson process, it is provided to record each number of events at a certain time interval with the parameter. The Poisson calculation process does not depend on the first interval process or depends on each other or is stationary and is related to the exponential distribution process consisting of homogeneous Poisson and nonhomogeneous Poisson [3].

In practice, the Poisson process is very widely used in statistics, or in calculations for the prediction and implementation of other problems. disease by analysis to increase the non-homogeneous use of the Poisson process in everyday problems [6]. In terms of climatology, rainfall information modeling can also be used to study the influence of time and trends in modeling events on daily rainfall that exceeds the predetermined limit value [7]. In geostatistical modeling, the process of calculating information with a space-time approach uses a non-homogeneous Poisson process, linking 2 components, namely the Gaussian Spatial component and the accounting component of its temporal impact, the goal is the suitability of the information and the identification of the zone with the highest level. Pollution, namely Southwest, Central, and Northwest Mexico City[8]. In the work of the non-homogeneous Poisson process using the traditional Poisson base to maximize the performance of the software in presenting the process in detail to ensure that the resulting model is efficient in improving and maximizing the performance of the traditional non-homogeneous Poisson process model and for distribution [9]. So the author also uses a non-homogeneous Poisson process for the number of attendance or queues at the Hasanuddin gas station. In this research, we try to model the collection of information from the presence of people at the Hasanuddin gas station for 1 semester from January 2020 to June 2020, with as much as 141 information. In terms of the number of attendance/day at the Hasanuddin gas station obtained, the number of people attending the gas station can be modeled at a certain time using a non-homogeneous Poisson process.

2. MATERIALS AND METHODS

2.1. Ingredient

In this paper, we obtained data on the number of people arriving at the Hasanuddin gas station, for 1 semester or 6 months starting from January 2021 to June 2020, the data obtained was 141 data, with 15 working hours or 900 working minutes at the Hasanuddin gas station starting at 07.00 until 22.00.

2.2. Method

2.2.1. Count

In the, culation $\{N(t); t \geq 0\}$ is efined as the counting process if $N(t)$ or N, represents the number of events that occurred during time t [11]. If it fulfills;

a. $N(t) \geq 0$
b. $N(t)$ is an integer
c. If $s < t$, maka $N(s) \leq N(t)$
d. For $s < t, N(t) - N(s)$ represents the number of events that occur in the time interval $(s, t]$.

International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 6 Issue 8, August - 2022, Pages: 160-163

The enumeration process is called a process with independent increments if the events occurring at separate time intervals are independent of each other [12]. That is, the number of events occurring in the time period t, (i.e. $N(t)$, does not depend on the number of time events between t and $t+s$, (i.e. $N(t+s)-N(t)$).

The number of arrivals is also called a process with stationary increments if the distribution of the number of events occurring at a certain time interval only depends on the length of the interval, not on the location of the interval. That is, the number of events in the time interval $(t_1+s, t_{2+}s]$,, (i.e. $N(t_2+s)-N(t_1+s)$) has the same distribution as the number of events in the time interval (t_1, t_2) (i.e. $N(t_2)-N(t_1)$),, for all $t_1 < t_2, s > 0$, [13].

2.2.2. Poisson Homogeneous

The calculation process with $\{N(t);\ t > 0$ is called a Poisson process with parameter values $\lambda > 0$ and $N(0) = 0$ and the process has a stationary independent increase that satisfies $P(N(h) = 1) = \lambda h + o(h)$ and $P(N(h) \geq 2) = o(h)$, for $t > 0$ as a stationary Poisson process, then: $P(N(s+t)\ N(s) = k) = P(\ N(t) = k|N(0) = 0 = P\_k\ (t)$ for any $s \geq 0, t \geq 0$, represents the probability that k will occur in the interval $(0, t]$.

The Poisson process is a stochastic process that is commonly used to model the time series of the emergence of a system [14]. The calculation process {N(t);t≥0} is called a Poisson process with parameter rate (λ) if:

a. $N(0) = 0$
b. Has a stationary free increment (Independent Increment).
c. The probability that the event will occur in time t

$$P_k(t) = P(N(t+s) - N(s) = k) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}, k = 0,1,2, \dots \quad (1)$$

$$\forall s, t > 0 \Longrightarrow N(s+t) - N(s) \approx POI(\lambda t)$$

Where

$$E[N(t)] = \lambda t, \quad Var[N(t)] = \lambda t \quad (2)$$

$\lambda = \frac{E[N(t)]}{t}$ = speed or average number of events that occur per time t

2.2.3. Non Homogeneous Poisson Process

In the calculation process $\{N(t); t \geq 0\}$ is called a non-homogeneous Poisson process with an intensity function $t \geq 0$ for $t \geq 0$, if;

a. $P(N(0) = 0) = 1$
b. Stochastic processes with independent increments for independent processes $\{N(t); t \geq 0\}$
c. $P(N(t+s) - N(t) = k = \frac{\left(\int_t^{t+s} \lambda x(dx)\right)^k}{k!} e^{-\int_t^{t+s} \lambda x(dx)}$ (3)

Based on equation 3, it can be determined one dimension of the non-homogeneous Poisson process, namely:

$$P(N(t+s) - N(t) = k = \frac{\left(\int_0^t \lambda x(dx)\right)^k}{k!} e^{-\int_0^t \lambda x(dx)}$$

where $k = 0,1,2,3, \dots$ (4)

The probability that an event does not occur at the initial event is one and the number of events that occur in the time interval are independent of each other.

Should $\wedge(t) = \int_0^t \lambda(x)dx$

Where : $P\{N(t+s) - N(t) = k\} = \frac{(\wedge(t+s) - N(t)^k)}{k!} e^{-(\wedge(t+s) - N(t)^k)}$

The non-homogeneous Poisson process distribution has the expectation and variance:

$$E(t) = E[N(t)] = \int_0^t \lambda(x)dx \quad (5)$$

$$V(t) = V[N(t)] = \int_0^t \lambda(x)dx \quad (6)$$

Calculate the standard deviation:

$$D(t) = \sqrt{V[N(t)} = \sqrt{\int_0^t \lambda(x)dx},\ t \geq 0 \quad (7)$$

The expected increase in value for N(s+t)-N(t) is

$$\Delta(t; s) = E(N(s+t) - N(t) = \int_t^{t+s} \lambda(x)dx \quad (8)$$

And that corresponds to the standard deviation is:

$$\sigma(t; s) = \sqrt{\int_t^{t+s} \lambda(x)dx} \quad (9)$$

In a Non-homogeneous Poisson Process if $\lambda(t) = \lambda, t \geq 0$ for each $t > 0$, it is a regular Poisson process. The increments of the Nonhomogeneous Poisson Process are independent, but not necessarily stationary.

2.2.4. Parameter Model Estimation

The arrival of people per day for 6 months at the Hasanuddin gas station, the data that the author got was 121 data, which was obtained from 121 working days at the Hasanuddin gas station. Then it will be approached to calculate the depth of $\lambda(t)$by estimating the function of simple linear regression is $y = \alpha + \beta x$ satisfactorily [15]:

$$T(\alpha, \beta) = \sum_{i=1}^n (y_i - (\alpha + \beta x_i)^2 \quad (10)$$

From equation 10 it can be derived with the minimum derivative value of the objective function T and shows and [13].

$$\alpha = \bar{y} - \bar{\beta}\mu\bar{x} \quad (11)$$

$$\beta = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} \quad (12)$$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^n x_i, \quad \bar{y} = \frac{1}{n}\sum_{i=1}^n y_i$$

With
$\bar{x}$ and $\bar{y}$ each as an average of x_i and y_i
λ and $\bar{\beta}$ as an approximate parameter

3. Results Obtained

The data obtained through the sales of Pertalite and Pertamax BBM per day for 121 working days at Hasanuddin gas stations can be seen in table 1.

Table 1. Sales Results of Pertalite and Pertamax at Hasanuddin gas stations

No	Time	Interval	Middle value	Number of Arrivals per day	Number of visitors per minute
1	01 January 2020	[0-900)	450	1089	1
2	02 January 2020	[900-1800)	1350	1120	1
3	03 January 2020	[1800-2700)	2250	1299	1
4	04 January 2020	[2700-3600)	3150	1280	1
⋮	⋮	⋮	⋮	⋮	⋮
141	26 Juny 2020	[126,000-126,900)	126450	1527	2

In table 1 above, it is shown that n = arrivals per day, can be calculated computationally by looking for the values of the following variables:

$$T_x = \bar{x} = 63{,}450 \quad S_y = \bar{y} = 1{,}70$$
$$T_{xx} = S_{xy} - (S_x * S_y) = 3{,}089.36$$
$$T_{yy} = \left(\frac{1}{n}\sum_{i=1}^{n} x_i^2\right) - S_x^2 = 1{,}341{,}900{,}000$$
$$T_{xy} = \frac{1}{n}\sum x_i \; y_i = 110{,}639.36$$

This calculation will be used to calculate the estimated regression coefficient:

$$\beta = \frac{T_{xx}}{T_{yy}} = 0.000002303$$
$$\alpha = T_y - (\beta * T_x) = 1{,}55$$

To get a linear regression from the arrival of people at the Hasanudin gas station, then and are added together

$$\lambda(x) = 1.55 + 0.000002303x, \quad x \geq 0 \qquad (13)$$

Figure 1. Plot of interval with intensity (daily arrivals at Hasanuddin gas station)

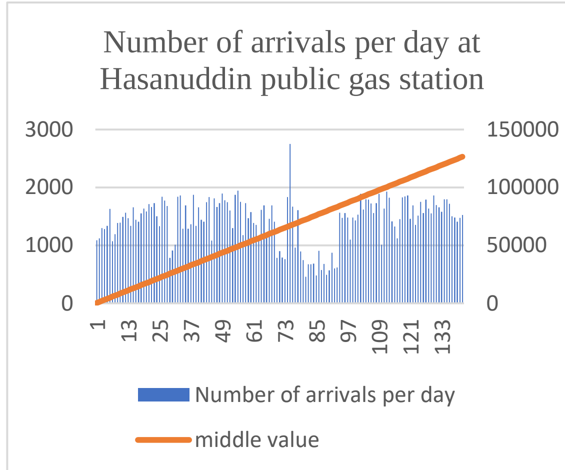

Figure 1 shows that we get the linear regression equation (λ(x)) as in equation (13). The plot is carried out in order to know the number of arrivals per minute at the Hasanuddin gas station.
Using equation (5), then obtained:

$$\wedge(t) = \int_0^t (1.55 + 0.000002303x)\; dx$$
$$= 1{,}55t + 0.0000011515t^2, \; t \geq 0 \qquad (14)$$

Based on equations (4), (5) and (14), we can obtain a one-dimensional distribution of the non-homogeneous Poisson process for the number of arrivals at the Hasanuddin gas station above as follows:

$$P(N(t) = k) = \frac{(\wedge(t))^k}{k!} e^{-(\wedge(t))^k}, \quad k = 0,1,2,\ldots \qquad (15)$$
$$= \frac{(1{,}55t+0.0000011515t^2)^k}{k!} e^{-(1{,}55t+0.0000011515t^2)} \qquad (16)$$

By using (t)=(1.55t+0.0000011515t^2) where t≥0, we can get the number of arrivals at Hasanuddin gas station at a certain interval with increasing time interval N(t+s)-N(t) . Equations 8 and 9 are treated in order to determine the expected increase in value and according to the standard deviation, a non-homogenous Poisson process can be calculated with an increase in time intervals, as in this equation:

$$P(N(t+s) - N(t) = k) = \frac{(\wedge(t+s)-\wedge(t))^k}{k!} e^{-(\wedge(t+s)-\wedge(t))} \qquad (17)$$

Example

For example, if we predict the number of people arriving at the Hasanuddin gas station on July 6, 2020, we get an interval of $[131{,}400, 132{,}300]$ the number of arrivals at the Hasanuddin gas station in a time interval of not greater than $g = 2000$ and not less than $h = 1100$ people.

Answer

$$\Delta(131{,}400{,}1) = E(N(132{,}300) - N(131{,}400)$$

$$\int_{131{,}400}^{132{,}300} (1.55 + 0.000002303x)\ dx = 1{,}668.2855$$

and

$$\sigma(131{,}400) = \sigma((N(132{,}300) - N(131{,}400) = \sqrt{1{,}668.2855} = 40.8446$$

The prediction result of the calculation above shows the 147th interval on that date. The customer arrival rate at Hasanuddin public gas station is 1,668 people and the standard deviation is 41.

4. Conclusion

By using the non-homogeneous Poisson process procedure, it is very helpful in predicting the number of attendance at Hasanuddin gas stations over a period of 6 months. The Poisson process with independent improvement is the model that makes sense to calculate the number of attendance at the Hasanuddin gas station. The use of linear regression is one option to ensure the parameters are estimated in the approach to the number of arrivals at the Hasanuddin gas station. With the number of attendance at the gas station that is obtained is different for each set time duration, which is every day working hours from 07. 00 in the morning to 22. 00 at night, with the number of working hours 15 hours being equal to 900 minutes and successively affecting, so the number of attendance at the gas station Hasanuddin can be signaled as a non-homogeneous Poisson process. The implementation of the model obtained can be used to calculate the number of attendance at the Hasanuddin gas station at certain intervals.

REFERENCES

1. LEE, Clement; WILKINSON, Darren J. A Hierarchical model of nonhomogeneous Poisson processes for twitter retweets. *Journal of the American Statistical Association*, 2020, 115.529: 1-15.
2. Munandar, D., Supian, S., & Subiyanto, S. (2020). Probability distributions of COVID-19 tweet posted trends uses a nonhomogeneous Poisson process. *International Journal of Quantitative Research and Modeling*, *1*(4), 229-238.
3. Ross, S. M. (2000). The exponential distribution and the poisson process. *Introduction to Probability Models. San Diego, CA: Academic*, 239-243.
4. Grabski, F. (2018). Nonhomogeneous compound Poisson process application to modeling of random processes related to accidents in the Baltic Sea waters and ports. *Journal of Polish Safety and Reliability Association*, *9*.
5. Grabski, F. (2018). Nonhomogeneous stochastic processes connected to Poisson process. *Zeszyty Naukowe Akademii Marynarki Wojennej*, *59*.
6. Cifuentes-Amado, M. V., & Cepeda-Cuervo, E. (2015). Non-homogeneous Poisson process to model seasonal events: Application to the health diseases. *International Journal of Statistics in Medical Research*, *4*(4), 347-359.
7. Ngailo, T., Shaban, N., Reuder, J., Rutalebwa, E., & Mugume, I. (2016). Non homogeneous poisson process modelling of seasonal extreme rainfall events in Tanzania. *International journal of science and research (IJSR)*, *5*(10), 1858-1868.
8. Morales, F. E. C., Vicini, L., Hotta, L. K., & Achcar, J. A. (2017). A nonhomogeneous Poisson process geostatistical model. *Stochastic Environmental Research and Risk Assessment*, *31*(2), 493-507.