International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 6 Issue 8, August - 2022, Pages: 195-200

Artin's cokernel of the group ($Q_{2m} \times C_4$) When m= $2^h . p_1^{r_1} . p_2^{r_2} \ldots . p_n^{r_n}$,h,r ∈ Z^+ and p is prime Number

Rajaa Hasan Abbas

Department of Mathematics, Faculty of Education for Girls, University of Kufa, Iraq.
Email :rajaah.alabidy@uokufa.edu.iq

Abstract— *The main purpose of this paper is to find The Cyclic decomposition of the group ($Q_{2m} \times C_4$) when m=* $2^h . p_1^{r_1} . p_2^{r_2} \ldots . p_n^{r_n}$ *,h,r ∈ Z^+ and p is prime number, which is denoted by AC ($Q_{2m} \times C_4$) where Q_{2m} is the Quaternion group and C_4 is the cyclic group of order 4.*

Keywords— Quaternion group; the cyclic group; Artin's characters table ;the cyclic decomposition**.**

1. INTRODUCTION

This matrix is very important to find the cyclic decomposition of the factor group AC(G) and Artin's exponent A(G). In 1981 C.Curits and I. Reiner[3] studied Methods of Representation Theory with Application to Finite Groups. In 2009 S.J. Mahmood [6] studied the general from of Artin's characters table Ar(Q_{2m}) when m is an even number. The aim of this paper is to find the general from of The Cyclic decomposition and the Artin's characters table of the group ($Q_{2m} \times C_4$) when m= $2^h . p_1^{r_1} . p_2^{r_2} \ldots . p_n^{r_n}$,h,r ∈ Z^+ and p is prime number.

2. PRELIMINARIES

This section introduce some important definitions and basic concepts the factor group AC(G) of a group G and the matrix M(G), M(Q_{2m}),P(Q_{2m}) and W(Q_{2m}).

2.1 Proposition :[5] The rational valued characters table of the group ($Q_{2m} \times C_4$) when m is an even number is equal to the tensor product of the rational valued characters table of Q_{2m} when m is an even number and the rational valued characters table of C_4 that is: $\overset{*}{\equiv}(Q_{2m} \times C_4) = \overset{*}{\equiv}(Q_{2m}) \otimes \overset{*}{\equiv}(C_4)$.

2.2 Definition :[4] Let T(G) be the subgroup of $\overline{R}$(G) generated by Artin's characters .T (G) is normal subgroup of $\overline{R}$(G) and denotes the factor abelian group $\overline{R}$(G)/T(G) by AC(G) which is called **Artin cokernel of** G.

2.3 Definition :[3] Let M be a matrix with entries in a principal domain R. **A k-minor of M** is the determinant of k×k sub matrix preserving row and column order.

2.4 Definition :[3] A k-th determinant divisor of M is the greatest common divisor (g.c.d) of all the k-minors of M.This is denoted by D_k (M)

2.5 Lemma :[3] Let M, P and W be matrices with entries in a principal ideal domain R, let P and W be invertible matrices ,Then D_k (P M W)= D_k (M) module the group of unites of R.

2.6 Theorem :[3] Let M be an n × n matrix with entries in principal ideal domain R, then there exist two matrices P and W such that:

1. P and W are invertible.
2. P M W = D.
3. D is diagonal matrix.
4. if we denote D_{ii} by d_i then there exists a natural number m ; 0 ≤ m ≤ n such that j > m

implies d^j = 0 and j ≤ m implies $d^j \neq 0$ and 1 ≤ j ≤m implies $d^j | d^{j+1}$.

2.7 Definition :[3] Let M be matrix with entries in a principal domain R, be equivalent to a matrix D = diag {d_1, d_2 , … ,d_m , 0, 0, … , 0} such that $d_j | d_{j+1}$ for 1 ≤ j < m**.** We call D **the invariant factor matrix of** M and d_1, d_2 , … ,d_m the invariant factors of M.

2.8 Theorem :[3] Let K be a finitely generated module over a principal domain R, then K is the direct sum of cyclic sub module with an annihilating ideal $<d_1>, <d_2>, \ldots, <d_m>$, $d_j \mid d_{j+1}$ for $j = 1, 2, \ldots, K-1$.

2.9 Proposition:[4] AC(G) is a finitely generated Z- module .Let m be the number of all distinct Γ-classes then Ar(G) and ≡*(G) are of the rank l. There exists an invertible matrix M(G) with entries in rational number such That: $\equiv^*(G)=M^{-1}(G).Ar(G)$ and this implies $M(G)=Ar(G).(\equiv^*(G))^{-1}$

2.10 Theorem:[2] $AC(G) = \bigoplus_{i=1}^{l} C_{d_i}$ where $d_i = \pm D_i(G) / D_{i-1}(G)$ where l is the number of all distinct Γ-classes.

2.11 Corollary :[4] $|AC(G)| = |\det(M(G))|$

2.12 Lemma :[4] If A and B are two matrices of degree m and t respectively, then:

$\det(A\otimes B) = (\det(A))^{t} . (\det(B))^{m}$.

2.13 Lemma:[4] Let A and B be two non-singular matrices of rank l and m respectively, over a principal domain R and let:

$P_1 A W_1 = D(A) = \mathrm{diag}\{d_1(A), d_2(A), \ldots, d_l(A)\}$ and $P_2 A W_2 = D(B) = \mathrm{diag}\{d_1(B), d_2(B), \ldots, d_m(B)\}$

The invariant factor matrices of A and B then:

$(P_1 \otimes P_2)(A \otimes B)(W_1 \otimes W_2) = D(A) \otimes D(B)$

and from this the invariant factor matrices of A ⊗ B can be obtained.

2.14 Proposition :[2] Let H_1 and H_2 be $p_1$and p_2 - groups respectively where p_1 and p_2 are distinct primes and if M_1 is the matrix from all cyclic subgroups of $\overline{R}$ (H_1) basis and M_2 is the matrix which expresses the $T(H_2)$ basis terms of $\overline{R}$ (H_2) basis then the matrix which expresses the $T(H_1 \times H_2)$ basis of $\overline{R}$ $(H_1 \times H_2)$ basis is $M_1 \otimes M_2$.

2.15 Remarks: [1] In general if m= $2^h.p_1^{r_1}.p_2^{r_2}\ldots..p_n^{r_n}$ such that p_i are prime numbers $p_i \neq 2$ and g.c.d (p_i , p_j)=1 ,h and r_i are any positive integer numbers for all i=1,2,…,n then we can write C_m as the from :

$$C_m = C_{2^h} \times C_{p_1^{r_1}} \times C_{p_2^{r_2}} \times \cdots \times C_{p_n^{r_n}}$$

(i) By the proposition 5 we get

$M(C_m)= M(C_{2^h}) \otimes M(C_{p_1^{r_1}}) \otimes M(C_{p_2^{r_2}}) \otimes \cdots \otimes M(C_{p_n^{r_n}})$

We can write $M(C_m)$ in the form:

$$M(C_m)=\left[\begin{array}{cccccc}
 & & h.times & \begin{cases}1 \\ \vdots \\ 1\end{cases} & \begin{array}{c}1\\1\\ \\ \end{array} \\
 & & & 0 & \\
 & R_2(C_m) & h.times & \begin{cases}1 \\ \vdots \\ 1\end{cases} & \vdots \\
 & & & 0 & \\
 & & & \vdots & \\
 & & h.times & \begin{cases}1 \\ \vdots \\ 1\end{cases} & \\
0 & 0 & & 1 & 1 \\
0 & 0 & \cdots & 0 & 1
\end{array}\right]$$

which is $(r_1+1)\ldots\ldots(r_n+1)(h+1)\times(r_1+1)\ldots\ldots(r_n+1)(h+1)$ square matrix,

$R_2(C_m)$ is the matrix obtained by omitting the last two rows $\{0,0,\cdots,1,1\}$ and $\{0,0,\cdots,0,1\}$ and the last two columns $\{1,\cdots,1,0,1,\cdots,1,0,\cdots,1,0\}$ and $\{1,1,\cdots,1\}$ from the tensor product.

$$M(C_{2^h}) \otimes M(C_{p_1^{r_1}}) \otimes M(C_{p_2^{r_2}}) \otimes \cdots \otimes M(C_{p_n^{r_n}})$$

(ii) By lemma 3 we have :

1- $P(C_m)= P(C_{2^h}) \otimes P(C_{p_1^{r_1}}) P \otimes (C_{p_2^{r_2}}) \otimes \cdots \otimes P(C_{p_n^{r_n}})$

2- $W(C_m)= W(C_{2^h}) \otimes W(C_{p_1^{r_1}}) \otimes W(C_{p_2^{r_2}}) \otimes \cdots \otimes W(C_{p_n^{r_n}})$

2.16 Proposition:[6] If $m = 2^h \cdot p_1^{r_1} \cdot p_2^{r_2} \cdot \cdots \cdot p_n^{r_n}$, such that p_i's are all distinct primes, $p_i \neq 2$ and g.c.d $(p_i, p_j) = 1$ for all i=1,2,⋯,n , h and r_i any positive integers then

$$M(Q_{2m}) = \left[\begin{array}{cccccccc|cccc l}
 & & & & & & & & 1 & 1 & 1 & 1 & \\
 & & & & & & & & \vdots & \vdots & \vdots & \vdots & h+1.times \\
 & & & 2.R_2(C_{2m}) & & & & & 1 & 1 & 1 & 1 & \\
 & & & & & & & & 0 & 1 & 0 & 1 & \\
 & & & & & & & & 1 & 1 & 1 & 1 & \\
 & & & & & & & & \vdots & \vdots & \vdots & \vdots & h+1.times \\
 & & & & & & & & 1 & 1 & 1 & 1 & \\
 & & & & & & & & 0 & 1 & 0 & 1 & \\
 & & & & & & & & \vdots & \vdots & \vdots & \vdots & \\
 & & & & & & & & 1 & 1 & 1 & 1 & \\
 & & & & & & & & \vdots & \vdots & \vdots & \vdots & h+1.times \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 1 & 1 & 1 & \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & 0 & 1 & \\
0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 & 0 & 0 & 1 & 1 & \\
0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 & 1 & 0 & 0 & 1 &
\end{array}\right]$$

which is [(r1+1)(r2+1)⋯(rn+1)(h+2)+2]×[(r1+1)(r2+1)⋯(rn+1)(h+2)+2] square matrix .

$R_2(C_{2m})$ is similar to the matrix in the remark 1

2.17 Proposition:[6] If $m = 2^h \cdot p_1^{r_1} \cdot p_2^{r_2} \cdot \cdots \cdot p_n^{r_n}$, such that p_i's are all distinct primes, $p_i \neq 2$ and g.c.d $(p_i, p_j) = 1$ for all *i=1,2,⋯,n* , h and r_i any positive integers then the matrices $P(Q_{2m})$ and $W(Q_{2m})$ are taking the forms :

$$P(Q_{2m}) = \begin{bmatrix}
 & & & & & 0 & 0 \\
 & & & & & 0 & 0 \\
 & & P(C_{2m}) & & & \vdots & \vdots \\
 & & & & & \vdots & \vdots \\
 & & & & & 0 & 0 \\
 & & & & & -1 & 1 \\
 & & & & 0 & 0 & -1 \\
0 & 0 & \cdots & \cdots & 0 & 1 & -1 \\
0 & 0 & \cdots & \cdots & 0 & 0 & 1
\end{bmatrix} \text{ And } W(Q_{2m}) = \begin{bmatrix}
 & & & & & & & & & 0 & 0 & 0 \\
 & & & & & & & & & 0 & 0 & 0 \\
 & & & & I_k & & & & & \vdots & \vdots & \vdots \\
 & & & & & & & & & \vdots & \vdots & \vdots \\
 & & & & & & & & & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & \cdots & 0 & -1 & -1 & \cdots & -1 & -1 & 0 & 0 & 1
\end{bmatrix}$$

$k = [(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)]-1$ and I_k is the identity matrix of the order k ,they are $[(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)+2] \times [(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)+2]$ square matrix .

2.18 Theorem:[6] If $m = 2^h \cdot p_1^{r_1} \cdot p_2^{r_2} \cdot \cdots \cdot p_n^{r_n}$, such that p_i's are all distinct primes, $p_i \neq 2$ and g.c.d $(p_i, p_j) = 1$ for all *i=1,2,⋯,n* , h and r_i any positive integers then the cyclic decomposition of $AC(Q_{2m})$ is :

$$AC(Q_{2m}) = \bigoplus_{i=1}^{(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)-1} C_2 .$$

3. THE MAIN RESULTS

In this section we find the general form of The Cyclic decomposition the group ($Q_{2m} \times C_4$) when m= $2^h.p_1^{r_1}.p_2^{r_2}.....p_n^{r_n}$,h,r ∈ Z^+ and p is prime number.

3.1 Proposition: If $m = 2^h \cdot p_1^{r_1} \cdot p_2^{r_2} \cdot \cdots \cdot p_n^{r_n}$, such that p_i's are all distinct primes, $p_i \neq 2$ and g.c.d $(p_i, p_j) = 1$ for all i=1,2,⋯,n , h and r_i any positive integers then:

$$M(Q_{2m} \times C_4) = \left[\begin{array}{c|c|c}
M(Q_{2m}) & M(Q_{2m}) & M(Q_{2m}) \\ \hline
0 & M(Q_{2m}) & M(Q_{2m}) \\ \hline
0 & 0 & M(Q_{2m})
\end{array}\right]$$

which is [3(r1+1)(r2+1)⋯(rn+1)(h+2)+6]×[3(r1+1)(r2+1)⋯(rn+1)(h+2)+6] square matrix M (Q_{2m}) is similar to the matrix of the proposition 6 .

International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 6 Issue 8, August - 2022, Pages: 195-200

Proof : By Proposition 2.18 and the Artin's characters Table Ar(Q_{2m}×C_4) of the group (Q_{2m}×C_4) when m= $2^h.p_1^{r_1}.p_2^{r_2}\ldots..p_n^{r_n}$,$h,r \in Z^+$ and p is prime number and from the Proposition 2.18 we get the rational valued characters table $(\overset{*}{\equiv}(Q_{2m} \times C_4))$ of the group (Q_{2m}×C_4) when m= $2^h.p_1^{r_1}.p_2^{r_2}\ldots..p_n^{r_n}$,$h,r \in Z^+$ and p is prime number.

Thus, by definition of M(G) we can find the matrix M(Q_{2m}×C_4) when $2^h.p_1^{r_1}.p_2^{r_2}\ldots..p_n^{r_n}$,$h,r \in Z^+$ and p is prime number.

$$M(Q_{2m} \times C_4) = Ar(Q_{2m} \times C_4) \cdot (\overset{*}{\equiv}(Q_{2m} \times C_4))^{-1} = \left[\begin{array}{c|c|c} M(Q_{2m}) & M(Q_{2m}) & M(Q_{2m}) \\ \hline 0 & M(Q_{2m}) & M(Q_{2m}) \\ \hline 0 & 0 & M(Q_{2m}) \end{array}\right] = M(Q_{2m} \times C_4)$$

3.2 Example :

Consider the group (Q_{48}×C_4), we can find the matrix M(Q_{48}×C_4)

by using: $M(Q_{48} \times C_4) = M(Q_{3.2^4} \times C_4) = Ar(Q_{3.2^4} \times C_4) \cdot (\overset{*}{\equiv}(Q_{3.2^4} \times C_4))^{-1}$

$$M(Q_{3.2^4} \times C_4) = \left[\begin{array}{cccccccccccccccccccccccccccccccccccc}
2&2&2&2&2&2&2&2&1&1&1&1&2&2&2&2&2&2&2&2&1&1&1&1&2&2&2&2&2&2&2&2&1&1&1&1\\
0&2&2&2&2&0&2&2&1&1&1&1&0&2&2&2&2&0&2&2&1&1&1&1&0&2&2&2&2&0&2&2&1&1&1&1\\
0&0&2&2&2&0&0&2&1&1&1&1&0&0&2&2&2&0&0&2&1&1&1&1&0&0&2&2&2&0&0&2&1&1&1&1\\
0&0&0&2&2&0&0&0&1&1&1&1&0&0&0&2&2&0&0&0&1&1&1&1&0&0&0&2&2&0&0&0&1&1&1&1\\
0&0&0&0&2&0&0&0&0&1&0&1&0&0&0&0&2&0&0&0&0&1&0&1&0&0&0&0&2&0&0&0&0&1&0&1\\
0&0&0&0&0&2&2&2&1&1&1&1&0&0&0&0&0&2&2&2&1&1&1&1&0&0&0&0&0&2&2&2&1&1&1&1\\
0&0&0&0&0&0&2&2&1&1&1&1&0&0&0&0&0&0&2&2&1&1&1&1&0&0&0&0&0&0&2&2&1&1&1&1\\
0&0&0&0&0&0&0&2&1&1&1&1&0&0&0&0&0&0&0&2&1&1&1&1&0&0&0&0&0&0&0&2&1&1&1&1\\
0&0&0&0&0&0&0&0&1&1&1&1&0&0&0&0&0&0&0&0&1&1&1&1&0&0&0&0&0&0&0&0&1&1&1&1\\
0&0&0&0&0&0&0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&1&0&1\\
0&0&1&1&1&1&1&1&0&0&1&1&0&0&1&1&1&1&1&1&0&0&1&1&0&0&1&1&1&1&1&1&0&0&1&1\\
0&0&1&1&1&1&1&1&1&0&0&1&0&0&1&1&1&1&1&1&1&0&0&1&0&0&1&1&1&1&1&1&1&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&2&2&2&2&2&1&1&1&1&2&2&2&2&2&2&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&2&0&2&2&1&1&1&1&0&2&2&2&2&0&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&0&0&2&1&1&1&1&0&0&2&2&2&0&0&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&0&0&0&1&1&1&1&0&0&0&2&2&0&0&0&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&0&0&0&1&0&1&0&0&0&0&2&0&0&0&0&1&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&1&1&1&1&0&0&0&0&0&2&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&1&1&1&1&0&0&0&0&0&0&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&1&1&1&1&0&0&0&0&0&0&0&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&0&0&0&0&0&0&0&0&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&1&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1&1&0&0&1&1&0&0&1&1&1&1&1&1&0&0&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1&1&1&0&0&1&0&0&1&1&1&1&1&1&1&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&2&2&2&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&2&0&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&0&0&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&0&0&0&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&0&0&0&1&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1&1&0&0&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1&1&1&0&0&1
\end{array}\right]$$

3.3 Proposition : If $m = 2^h \cdot p_1^{r_1} \cdot p_2^{r_2} \cdot\cdot\cdot\cdot\cdot p_n^{r_n}$, such that p_i's are all distinct primes, $p_i \neq 2$ and g.c.d $(p_i, p_j) = 1$ for all i=1,2,⋯,n , h and r_i any positive integers then the matrices P(Q_{2m}×C_4) and W(Q_{2m}×C_4) are taking the forms :

$$P(Q_{2m} \times C_4) = \left[\begin{array}{c|c|c} P(Q_{2m}) & -P(Q_{2m}) & 0 \\ \hline 0 & P(Q_{2m}) & -P(Q_{2m}) \\ \hline 0 & 0 & P(Q_{2m}) \end{array}\right]$$

Which is $[3(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)+6]\times[3(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)+6]$ square matrix .

And

$$W(Q_{2m}\times C_4)=\left[\begin{array}{c|c|c} W(Q_{2m}) & 0 & 0 \\ \hline 0 & W(Q_{2m}) & 0 \\ \hline 0 & 0 & W(Q_{2m}) \end{array}\right]$$

which is $[3(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)+6]\times[3(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)+6]$ square matrix .

<u>Proof :</u>

By using the proposition 3.1 taking the matrix $M(Q_{2m}\times C_4)$ and the above forms $P(Q_{2m}\times C_4)$ and $W(Q_{2m}\times C_4)$ then we have :
$P(Q_{2m}\times C_4).\ M(Q_{2m}\times C_4).\ W(Q_{2m}\times C_4)=$

$$\text{diag}\ \{\underbrace{2,2,2,2,\cdots,2}_{[3(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)-3]-time},1,1,1,1,1,1,1,1,1\}$$

$$=D(Q_{2m}\times C_4)$$

which is $[3(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)+6]\times[3(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)+6]$ square matrix .

3.4 Example :

To find the matrices $P(Q_{48}\times C_4)$ and $W(Q_{48}\times C_4)$ by the proposition 2.17 to find $P(Q_{48})$ and $W(Q_{48})$:

$$P(Q_{3.2^4})=\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

And

$$W(Q_{3.2^4})=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & 1 \end{bmatrix}$$

And by the proposition 9 then:

$$P(Q_{48}\times C_4)=\begin{bmatrix} P(Q_{48}) & -P(Q_{48}) & 0 \\ 0 & P(Q_{48}) & -P(Q_{48}) \\ 0 & 0 & P(Q_{48}) \end{bmatrix} \quad and \quad W(Q_{48}\times C_4)=\begin{bmatrix} W(Q_{48}) & 0 & 0 \\ 0 & W(Q_{48}) & 0 \\ 0 & 0 & W(Q_{48}) \end{bmatrix}$$

3.5 Example : To find $D(Q_{48}\times C_4)$ and the cyclic decomposition of the factor group
We find the matrices $P(Q_{48}\times C_4)$ and $W(Q_{48}\times C_4)$ as in example 3.4 and $M(Q_{48}\times C_4)$ as in example 3.2,then :
$P(Q_{48}\times C_4).M(Q_{48}\times C_4).W(Q_{48}\times C_4)=$
diag{2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,1,1,1,1,1,1,1,1,1}=$D(Q_{48}\times C_4)$
Then by Theorem 5 we have

$$AC(D(Q_{48}\times C_4))=\bigoplus_{i=1}^{27} C_2$$

The following theorem gives the cyclic decomposition of the factor group $AC(D(Q_{2m}\times C_4))$ when m= $2^h.p_1^{r_1}.p_2^{r_2}.....p_n^{r_n}$,$h,r\in Z^+$ and p is prime number .

3.6 Theorem: If m= $2^h.p_1^{r_1}.p_2^{r_2}.....p_n^{r_n}$,$h,r\in Z^+$ and p is prime number then the cyclic decomposition of $AC(Q_{2m}\times C_4)$ is :

$$AC(D(Q_{2m}\times C_4))=\bigoplus_{i=1}^{3(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)-3} C_2$$

Proof : By using the proposition 3.1,we can find matrix $M(Q_{2m}\times C_4)$ and by the proposition 3.2,we find $P(Q_{2m}\times C_4)$ and $W(Q_{2m}\times C_4)$ when m= $2^h.p_1^{r_1}.p_2^{r_2}.....p_n^{r_n}$,$h,r\in Z^+$ and p is prime number:
$P(Q_{2m}\times C_4).\ M(Q_{2m}\times C_4).\ W(Q_{2m}\times C_4)=$
diag{2,2,2,2,2,2,…,2,2,2,1,1,1,1,1,1,1,1,1}
Then ,by the theorem 2.18 we have :

$$AC(D(Q_{2m}\times C_4))=\bigoplus_{i=1}^{3(r_1+1)(r_2+1)\cdots(r_n+1)(h+2)-3} C_2$$

3.7 Example : Consider the groups $(Q_{7087500}\times C_4)$,$(Q_{98000}\times C_4)$,then :

1. $AC(Q_{7087500}\times C_4)=AC(Q_{2^2.3^4.7.5^5}\times C_4)=\bigoplus_{i=1}^{537} C_2$
2. $AC(Q_{98000}\times C_4)=AC(Q_{2^4.7^2.5^3}\times C_4)=\bigoplus_{i=1}^{141} C_2$

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