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The Rational characters table of Group $(Q_{2m} \times C_4)$ When m is an Odd Number

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***Abstract*:** The main purpose of this paper is to find the Rational characters table when m is an odd Number, which is denoted by $\equiv^*(Q_{2m} \times C_4)$ where Q_{2m} is the Quaternion group and C_4 is the cyclic group of order 4 .

***Keywords*:** Quaternion group; the cyclic group; the Rational characters; the Rational characters table .

1. INTRODUCTION

Let *G* be a finite group, two elements of *G* are said to be r-conjugate if the cyclic subgroups they generate are conjugate in G. This process defines an equivalence relation on G; its classes are calledr-classes.

Let $\equiv^*(G)$ denotes the r×r matrix which the rows corresponds to the θ_i 's and the columns correspond to the r-classes of *G*. The matrix expressing $\overline{R}(G)$ basis in terms of the *cf(G,Z)* basis is $\equiv^*(G)$.In 1959, M.J.Hall[3] is found" The rational valued characters table of finite group". In 1982, M.S. Kirdar [4] is found "The rational valued characters table of the cyclic group Cn".In this work we have found the rational valued characters table of the group$(Q_{2m} \times C_4)$.

2. PRELIMINARIES

This section introduce some important definitions and basic concepts of the Rational characters table .

2.1 Theorem:[1]Let $T_1 : G_1 \to GL(n,F)$ and $T_2 : G_2 \to GL(m,F)$ be two irreducible representations of the groups G_1 and G_2 with characters χ_1 and χ_2 respectively then :

$T_1 \otimes T_2$ is irreducible representation of the group $G_1 \times G_2$ with the character $\chi_1 \cdot \chi_2$.

2.2 Definition :[2]A rational valued character θ of G is a character whose values are in Z, which is $\theta(g) \in Z$, for all $g \in G$.

2.3 Corollary:[3]The rational valued characters $\theta_i = \sum_{\sigma \in Gal(Q(\chi_i)/Q)} \sigma(\chi_i)$ form the basis for $\overline{R}$ (G), where χ_i are the irreducible characters of G, and their numbers are equal to the number of conjugacy classes of cyclic subgroup of G .

2.4 Definition : [3] The complete information about rational valued characters of a finite group G is displayed in a table called **rational valued characters table of G**. We refer to it by $\equiv^*(G)$ which is n×n matrix whose columns are Γ-classes and rows which are the values of all rational valued characters of G, where n is the number of Γ-classes.

3. THE MAIN RESULTS

In this section we find the general form of The Rational characters of the group $(Q_{2m} \times C_4)$ when m is an odd number .

Proposition 6: The rational valued characters table of the group $(Q_{2m} \times C_4)$ when m is an odd number is equal to the tensor product of the rational valued characters table of Q_{2m} when m is an odd number and the rational valued characters table of C_4 that is:

$$\overset{*}{\equiv}(Q_{2m} \times C_4) = \overset{*}{\equiv}(Q_{2m}) \otimes \overset{*}{\equiv}(C_4) \ .$$

Proof :-

$C_4 = \{I, c, c^2, c^3\}$

Since $\overset{*}{\equiv}(C_4) =$

Table 1 The rational characters table of C_4

	h_1'	h_2'	h_3'
χ_1'	2	-2	**0**
χ_2'	1	1	**-1**
χ_3'	**1**	**1**	**1**

$h_1' = \{(I)\}$, $h_2' = \{c^2\}$, $h_3' = \{c, c^3\}$ then,

$\chi_1'(h_1') = \theta_1'(h_1') = 2$

$\chi_1'(h_2') = \theta_1'(h_2') = -2$

$\chi_1'(h_3') = \theta_1'(h_3') = 0$

$\chi_2'(h_1') = \chi_2'(h_2') = \theta_2'(h_1') = \theta_2'(h_2') = 1$

$\chi_2'(h_3') = \theta_2'(h_3') = -1$

$\chi_3'(h_1') = \chi_3'(h_2') = \chi_3'(h_3') = \theta_3'(h_1') = \theta_3'(h_2') = \theta_3'(h_3') = 1$

From the definition of $Q_{2m} \times C_4$,
and Theorem(2.1) we have

$\equiv(Q_{2m} \times C_4) = (\equiv Q_{2m}) \otimes (\equiv C_4)$

Each element in $Q_{2m} \times C_4$

$h_{ng} = h_n \cdot h_g' \quad \forall h_n \in Q_{2m}$, $h_g' \in C_4$,

$n = 1,2,3,\ldots,4m$, $\mathbf{g} \in \{I, c, c^2, c^3\}$,

each irreducible character of $Q_{2m} \times C_4$ is

$\chi_{(i,j)} = \chi_i \cdot \chi_j'$ where χ_i is an irreducible character of Q_{2m}

and χ_j' is the irreducible character of C_4, then

$$\chi_{(i,j)}(h_{ng}) = \begin{cases} 2\chi_i(h_n) & if \quad j=1 \ and \ g \in \{I\} \\ -2\chi_i(h_n) & if \quad j=1 \ and \ g \in \{c^2\} \\ 0\chi_i(h_n) & if \quad j=1 \ and \ g \in \{c,c^3\} \\ \chi_i(h_n) & if \quad j=2 \ and \ g \in \{I,c^2\} \\ -\chi_i(h_n) & if \quad j=2 \ and \ g \in \{c,c^3\} \\ \chi_i(h_n) & if \quad j=3 \ and \ g \in C_4 \end{cases}$$

From Corollary (2.3)

$\theta_{(i,j)}$ =where $\theta_{(i,j)}$ is the rational valued character of $Q_2 \sum_{\sigma \in Gal(Q(\chi_{(i,j)})/Q)} \sigma(\chi_{(i,j)})$ $_m \times C_4$ Then,

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_{(i,j)}(h_{ng}))/Q)} \sigma(\chi_{(i,j)}(h_{ng}))$$

(I) (a) If j=1 and $g \in \{I\}$

$$\theta_{(i,j)}(h_{ng}) = \theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(2\chi_i(h_n)) = 2 \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n)) = \theta_i(h_n) \,.\, 2$$

$= \theta_i(h_n) \,.\, \theta_j'(h_g')$

(b) If j=1 and $g \in \{c^2\}$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_I(h_n))/Q)} \sigma(-2\chi_i(h_n)) = -2 \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n))$$

$= \theta_i(h_n) \,.\, -2 = \theta_i(h_n) \,.\, \theta_j'(h_g')$

(c) if j=1 and $g \in \{c,^3\}$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_I(h_n))/Q)} \sigma(0\chi_i(h_n)) = 0 \sum_{\sigma \in Gal(Q(\chi_I(h_n))/Q)} \sigma(\chi_i(h_n)) = \theta_i(h_n) \,.\, 0 = \theta_i(h_n) \,.\, \theta_j'(h_g')$$

where θ_i is the rational valued character of Q_{2m}.

(II) (a) If j=2 and $g \in \{I, c^2\}$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n)) = \theta_i(h_n) \,.\, 1 = \theta_i(h_n) \,.\, \theta_j'(h_g')$$

(b) If j=2 and $g \in \{c, c^3\}$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(-\chi_i(h_n)) = -\sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n))$$

$$= \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n)) \, .\text{-}1 = \theta_i(h_n) \, .\text{-}1 =$$

$\theta_i(h_n) \, . \, \theta'_j(h'_g)$.

(III) If j=3 and $g \in C_4$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n)) = \theta_i(h_n) \, . \, 1 = \theta_i(h_n) \, . \, \theta'_j(h'_g)$$

From [I], [II] and [III] we have

$\theta_{(i,j)} = \theta_i \, . \, \theta'_j$.

Then $\equiv^*(Q_{2m} \times C_4) = \equiv^*(Q_{2m}) \otimes \equiv^*(C_4)$.

3.2 Example : To find $\equiv^*(Q_{10} \times C_4)$by using the Proposition 3.1 we get the following table:

$$\begin{bmatrix}
8 & -2 & 8 & -2 & 0 & -8 & 2 & -8 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 2 & 2 & 2 & 2 & -2 & -2 & -2 & -2 & -2 & 0 & 0 & 0 & 0 & 0 \\
8 & -2 & -8 & 2 & 0 & -8 & 2 & 8 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 2 & 2 & 2 & -2 & -2 & -2 & -2 & -2 & 2 & 0 & 0 & 0 & 0 & 0 \\
4 & 4 & -4 & -4 & 0 & -4 & -4 & 4 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & -1 & 4 & -1 & 0 & 4 & -1 & 4 & -1 & 0 & -4 & 1 & -4 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 \\
4 & -1 & -4 & 1 & 0 & 4 & -1 & -4 & 1 & 0 & -4 & 1 & 4 & -1 & 0 \\
1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 \\
2 & 2 & -2 & -2 & 0 & 2 & 2 & -2 & -2 & 0 & -2 & -2 & 2 & 2 & 0 \\
4 & -1 & 4 & -1 & 0 & 4 & -1 & 4 & -1 & 0 & 4 & -1 & 4 & -1 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
4 & -1 & -4 & 1 & 0 & 4 & -1 & -4 & 1 & 0 & 4 & -1 & -4 & 1 & 0 \\
1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 \\
2 & 2 & -2 & -2 & 0 & 2 & 2 & -2 & -2 & 0 & 2 & 2 & -2 & -2 & 0
\end{bmatrix}$$

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