

Artin's characters table of the group $(Q_{2m} \times C_4)$ When $m=2^h \cdot p_1^{r_1} \cdot p_2^{r_2} \cdots p_n^{r_n}$

$h, r \in \mathbb{Z}^+$ and p is prime Number

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Abstract— The main purpose of this paper is to the general form of Artin's characters table of $Ar(Q_{2m} \times C_4)$ when $m=2^h \cdot p_1^{r_1} \cdot p_2^{r_2} \cdots p_n^{r_n}$, $h, r \in \mathbb{Z}^+$ and p is prime number, which is denoted by $Ar(Q_{2m} \times C_4)$ where Q_{2m} is the Quaternion group and C_4 is the cyclic group of order 4.

Keywords— Artin; the cyclic group; Artin's characters ;Artin's characters table .

1. INTRODUCTION

Let G be a finite group ,two elements of G are said to be Γ -conjugate if the cyclic subgroups they generate are conjugate in G and this defines an equivalence relation on G and its classes are called Γ -classes.let $R(G)$ denotes the abelian group generated by Z - valued characters of G under the operation of point wise addition. Inside this group there is a subgroup generated by Artin characters (The characters induced from the principal characters of cyclic subgroups of G). In 1967, T.Y. Lam [5] prove a sharp form of Artin theorem and he determine the least positive integer $A(G)$ such that $[R(G):(G)] = A(G)$.In 1976, I. M. Isaacs [3] studied Character Theory of Finite Groups. In 2008, A.H. Abdul-Mun' em[1] studied the Artin cokernel of the Quaternion Group Q_{2m} when m is an odd Number. The aim of this paper is to find the general form of the Artin's characters table of the group $(Q_{2m} \times C_p)$ when $m=2^h \cdot p_1^{r_1} \cdot p_2^{r_2} \cdots p_n^{r_n}$, $h, r \in \mathbb{Z}^+$ and p is prime number.

2. RELIMINARIES

This section introduce some important definitions and basic concepts the Artin characters, the Artin characters table.

2.1 Theorem : [3] Let H be a cyclic subgroup of G and h_1, h_2, \dots, h_m are chosen as representative for m -conjugate classes of H contained in $CL(g)$ in G , then :

$$1- \varphi'(g) = \frac{|C_G(g)|}{|C_H(g)|} \sum_{i=1}^m \varphi(h_i) \text{ if } h_i \in H \cap CL(g)$$

$$2- \varphi'(g) = 0 \text{ if } H \cap CL(g) = \emptyset.$$

2.2 Definition : [5] Let G be a finite group, all characters of G induced from a principal character of cyclic subgroups of G are called **Artin's characters of G**.

In theorem 2 , if φ is the principal character , then $\varphi(h_i) = \varphi(1) = 1$, where $h_i \in H$

2.3 Proposition : [2] The number of all distinct Artin's characters on a group G is equal to the number of Γ -classes on G . Furthermore, Artin's characters are constant on each Γ -classes.

2.4 Definition: [1] Artin's characters of finite group G can be displayed in a table **called Artin's characters table of G** which is denoted by $Ar(G)$.

The first row is the Γ - conjugate classes, the second row is the number of elements in each conjugate classes, the third row is the size of the centralize $|C_G(CL_\alpha)|$ and the rest rows contain the values of Artin's characters.

2.5 Proposition : [4] The Artin's characters table of the Quaternion group Q_{2m} when $m=2^h \cdot p_1^{r_1} \cdot p_2^{r_2} \cdots p_n^{r_n}$, $h, r \in \mathbb{Z}^+$ and p is prime number is given as follows:

$$Ar(Q_{2^{h+1}} \cdot p_1^{r_1} \cdot p_2^{r_2} \cdots p_n^{r_n}) =$$

TABLE I

Γ - classes	Γ - classes of C_{2m}				[y]	[xy]
	[1]	$[x^{2^h p_1^{r_1} p_2^{r_2} \dots p_n^{r_n}}]$				
$ CL_\alpha $	1	1	2	2	2	$2^h \cdot p_1^{r_1} \cdot p_2^{r_2} \dots p_n^{r_n}$
$ C_{Q_{2^{h+1} p_1^{r_1} p_2^{r_2} \dots p_n^{r_n}}} (CL_\alpha) $	$2^{h+2} \cdot p_1^{r_1} \cdot p_2^{r_2} \dots p_n^{r_n}$	$2^{h+2} \cdot p_1^{r_1} \cdot p_2^{r_2} \dots p_n^{r_n}$	$2^{h+1} \cdot p_1^{r_1} \cdot p_2^{r_2} \dots p_n^{r_n}$	$2^{h+1} \cdot p_1^{r_1} \cdot p_2^{r_2} \dots p_n^{r_n}$	4	4
Φ_1	$2\text{Ar}(C 2^{h+1} \cdot p_1^{r_1} \cdot p_2^{r_2} \dots p_n^{r_n})$				0	0
Φ_2					0	0
\vdots					\vdots	\vdots
Φ_l					0	0
Φ_{l+1}	$2^h \cdot p_1^{r_1} \cdot p_2^{r_2} \dots p_n^{r_n}$	$2^h \cdot p_1^{r_1} \cdot p_2^{r_2} \dots p_n^{r_n}$	0 ... 0	0	2	0
Φ_{l+2}	$2^h \cdot p_1^{r_1} \cdot p_2^{r_2} \dots p_n^{r_n}$	$2^h \cdot p_1^{r_1} \cdot p_2^{r_2} \dots p_n^{r_n}$	0 ... 0	0	0	2

where l is the number of Γ - classes of C_{2m} and Φ_j ; $1 \leq j \leq l+2$ are the Artin characters of the Quaternion group Q_{2m} when $m =$

$$2^h \cdot p_1^{r_1} \cdot p_2^{r_2} \dots p_n^{r_n}, h, r \in \mathbb{Z}^+$$

and p is prime number .

3. THE MAIN RESULTS

In this section we find the general form of Artin's characters table of the group $(Q_{2m} \times C_4)$ when $m = 2^h \cdot p_1^{r_1} \cdot p_2^{r_2} \cdots p_n^{r_n}$, $h, r \in \mathbb{Z}^+$ and p is prime number.

3.1 Proposition : The general form of the Artin's characters table of the group $(Q_{2m} \times C_4)$ when $m = 2^h \cdot p_1^{r_1} \cdot p_2^{r_2} \cdots p_n^{r_n}$, $h, r \in \mathbb{Z}^+$ and p is prime number is given as follows:

$$Ar(Q_2^{h+1} \cdot p_1^{r_1} \cdot p_2^{r_2} \cdots p_n^{r_n} \times C_4) =$$

TABLE II

Γ - classes of $(Q_{2m}) \times \{I\}$						Γ - classes of $(Q_{2m}) \times \{z^2\}$						Γ - classes of $(Q_{2m}) \times \{z\}$							
Γ - classes	[1, I]	[x^m , I]	.	[x, I]	[y, I]	[xy, I]	[I, z^4]	[x^m , z^2]	...	[x, z^2]	[y, z^2]	[xy, z^2]	[I, z]	[x^m , z]	...	[x, z]	[y, z]	[xy, z]	
$ CL_\alpha $	1	1	.	2	m	m	1	1	.	2	m	m	1	1	...	2	m	m	
$ C_{Q_{2m} \times C_4}(CL_\alpha) $	16	16	.	8	16	16	16	16	m	16	16	16	16m	16	m	...	8m	16	16
$\Phi_{(I,I)}$	4Ar(Q_{2m})						0						0						
$\Phi_{(2,I)}$																			
\vdots																			
$\Phi_{(l,I)}$																			
$\Phi_{(l+1,I)}$																			
$\Phi_{(l+2,I)}$																			
$\Phi_{(I,2)}$	2Ar(Q_{2m})						2Ar(Q_{2m})						0						
$\Phi_{(2,2)}$																			
\vdots																			
$\Phi_{(l,2)}$																			
$\Phi_{(l+1,2)}$																			
$\Phi_{(l+2,2)}$																			
$\Phi_{(I,3)}$	Ar(Q_{2m})						Ar(Q_{2m})						Ar(Q_{2m})						
$\Phi_{(2,3)}$																			
\vdots																			
$\Phi_{(l,3)}$																			
$\Phi_{(l+1,3)}$																			
$\Phi_{(l+2,3)}$																			

Proof : Let $g \in (Q_{2m} \times C_4)$; $g=(q, I)$ or $g=(q, z)$

or $g=(q, z^2)$ or $g=(q, z^3)$ $q \in Q_{2m}, I, z, z^2, z^3 \in C_4$

Case (I):

If H is a cyclic subgroup of $Q_{2m} \times \{I\}$, then:

1. $H=\langle(x, I)\rangle$
2. $H=\langle(y, I)\rangle$
3. $H=\langle(xy, I)\rangle$

And Φ the principal character of H , Φ_j Artin characters of Q_{2m} where $1 \leq j \leq l+2$ then by using Theorem 2.1

1. $H=\langle(x, I)\rangle$

(i) If $g=(1, I)$ and $g \in H$

$$\Phi_{(j,1)}((1,I)) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{16m}{|C_H(I,1)|} \cdot 1 = \frac{4.4m}{|C_H(I,1)|} \cdot 1 = \frac{4|C_{Q_{2m}}(1)|}{|C_{\langle x \rangle}(1)|} \cdot \varphi(1) = 4 \cdot \Phi_j(1)$$

since $H \cap CL(1,I) = \{(1,I)\}$

(ii) if $g = (x^m, I)$ and $g \in H$

$$\Phi_{(j,1)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot \varphi(g)$$

$$= \frac{16m}{|C_H(g)|} \cdot 1 = \frac{4.4m}{|C_H(g)|} \cdot 1 = \frac{4|C_{Q_{2m}}(x^m)|}{|C_{\langle x \rangle}(x^m)|} \cdot \varphi(g) = 4 \cdot \Phi_j(x^m)$$

since $H \cap CL(g) = \{g\}, \varphi(g) = 1$

(iii) if $g = (x^i, I), i \neq m$ and $i \neq 2m$ and $g \in H$

$$\Phi_{(j,1)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot (\varphi(g) + \varphi(g^{-1})) = \frac{8m}{|C_H(g)|} \cdot (1+1) = \frac{4.2m}{|C_H(g)|} \cdot (1+1) = \frac{4|C_{Q_{2m}}(q)|}{|C_{\langle x \rangle}(q)|} \cdot (\varphi(g) + \varphi(g^{-1})) = 4 \cdot \Phi_j(q)$$

since $H \cap CL(g) = \{g, g^{-1}\}$ and $\Phi(g) = \Phi(g^{-1}) = 1, g = (q, I), q \in Q_{2m}$ and $q \neq x^m, q \neq 1$.

$$\Phi_{(j,1)}(g) = 4.0 = 4 \cdot \Phi_j(q) \quad (iv) \quad \text{if } g \notin H \quad \text{Since } H \cap CL(g) = \emptyset$$

2. $H = \{(y, I)\} = \{(1,I), (y,I), (y^2,I), (y^3,I)\}$

(i) If $g = (1,I)$ $H \cap CL(1,I) = \{(1,I)\}$

$$\Phi_{(l+1,1)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{16m}{4} \cdot 1 = 4m = 4 \cdot \Phi_{l+1}(1)$$

(ii) If $g = (x^m, I) = (y^2, I)$ and $g \in H$

$$\Phi_{(l+1,1)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{16m}{4} \cdot 1 = 4m = 4 \cdot \Phi_{l+1}(x^m)$$

Since $H \cap CL(g) = \{g\}, \varphi(g) = 1$

(iii) $g = (y, I)$ or $g = (y^3, I)$ and $g \in H$

$$\Phi_{(l+1,1)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot (\varphi(g) + \varphi(g^{-1})) = \frac{16}{4} \cdot (1+1) = 4.2 = 4 \cdot \Phi_{l+1}(y)$$

since $H \cap CL(g) = \{g, g^{-1}\}$ and $\Phi(g) = \Phi(g^{-1}) = 1$

Otherwise

$$\Phi_{(l+1,1)}(g) = 0 \quad \text{since } H \cap CL(g) = \emptyset$$

3- $H = \{(xy, I)\} = \{(1,I), (xy,I), ((xy)^2,I), ((xy)^3,I)\}$

(i) If $g = (1,I)$ $H \cap CL(1,I) = \{(1,I)\}$

$$\Phi_{(l+2,1)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{16m}{4} \cdot 1 = 4m = 4 \cdot \Phi_{l+2}(1)$$

(ii) If $g = (x^m, I) = ((xy)^2, I) = (y^2, I)$ and $g \in H$

$$\Phi_{(l+2,1)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{16m}{4} \cdot 1 = 4m = 4 \cdot \Phi_{l+2}(x^m)$$

Since $H \cap CL(g) = \{g\}$, $\varphi(g) = 1$

(iii) If $g = (xy, I)$ or $g = ((xy)^3, I) = (xy^3, I)$ and $g \in H$

$$\Phi_{(l+2,1)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot (\varphi(g) + \varphi(g^{-1})) = \frac{16}{4} \cdot (1+1) = 4 \cdot 2 = 4 \cdot \Phi_{l+2}(xy)$$

since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$

Otherwise

$$\Phi_{(l+2,1)}(g) = 0 \quad \text{since } H \cap CL(g) = \emptyset$$

Case (II):

If H is a cyclic subgroup of $Q_{2m} \times \{z^2\}$, then:

$$1. H = \langle (x, I) \rangle = \langle (x, z^2) \rangle \quad 2. H = \langle (y, I) \rangle = \langle (y, z^2) \rangle$$

$$3. H = \langle (xy, I) \rangle = \langle (xy, z^2) \rangle$$

And φ the principal character of H , Φ_j Artin characters of Q_{2m} where $1 \leq j \leq l+2$ then by using Theorem 2.1

$$1. H = \langle (x, I) \rangle = \langle (x, z^2) \rangle$$

(i) If $g = (1, I)$ or $g = (1, z^2)$ and $g \in H$

$$\Phi_{(j,2)}((1, I)) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{16m}{|C_H(I, I)|} \cdot 1 = \frac{4.4m}{|C_H(I, I)|} \cdot 1 = \frac{4|C_{Q_{2m}}(1)|}{2|C_{\langle x \rangle}(1)|} \cdot \varphi(1) = 2 \cdot \Phi_j(1)$$

since $H \cap CL(1, I) = \{(1, I), (1, z^2)\}$

(ii) if $g = (x^m, I)$ and $g \in H$

$$\Phi_{(j,2)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{16m}{|C_H(g)|} \cdot 1 = \frac{4.4m}{|C_H(g)|} \cdot 1 = \frac{4|C_{Q_{2m}}(x^m)|}{2|C_{\langle x \rangle}(x^m)|} \cdot \varphi(g) = 2 \cdot \Phi_j(x^m)$$

since $H \cap CL(g) = \{g\}$, $\varphi(g) = 1$

(iii) if $g = (x^i, I)$, $i \neq m$ and $i \neq 2m$ and $g \in H$

$$\Phi_{(j,2)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot (\varphi(g) + \varphi(g^{-1})) = \frac{8m}{|C_H(g)|} \cdot (1+1) = \frac{4.2m}{|C_H(g)|} \cdot (1+1) = \frac{4|C_{Q_{2m}}(q)|}{2|C_{\langle x \rangle}(q)|} \cdot (\varphi(g) + \varphi(g^{-1})) = 2 \cdot \Phi_j(q)$$

since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$, $g = (q, I)$, $q \in Q_{2m}$

and $q \neq x^m$, $q \neq 1$

(iv) if $g \notin H$ Since $H \cap CL(g) = \emptyset$

$$2. H = \langle (y, I) \rangle = \{(1, I), (y, I), (y^2, I), (y^3, I), (1, z^2), (y, z^2), (y^2, z^2), (y^3, z^2)\}$$

(i) If $g = (1, I)$ or $g = (1, z^2)$ $H \cap CL(1, I) = \{(1, I), (1, z^2)\}$

$$\Phi_{(j,2)}(g) = 2.0 = 2 \cdot \Phi_j(q)$$

$$\Phi_{(l+1,2)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{16m}{8} \cdot 1 = 2m = 2 \cdot \Phi_{l+1}(1)$$

(ii) If $g = (x^m, I) = (y^2, I)$ or $g = (y^2, z^2)$ and $g \in H$

$$\Phi_{(l+1,2)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{16m}{8} \cdot 1 = 2m = 2 \cdot \Phi_{l+1}(x^m)$$

Since $H \cap CL(g) = \{g\}$, $\varphi(g) = 1$

(iii) $g = (y, I)$ or $g = (y^3, I)$ or $g = (y, z^2)$ or $g = (y^3, z^2)$ and $g \in H$

$$\Phi_{(l+1,2)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot (\varphi(g) + \varphi(g^{-1})) = \frac{16}{8} \cdot (1+1) = 2 \cdot 2 = 2 \cdot \Phi_{l+1}(y)$$

since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$

Otherwise

$$\Phi_{(l+1,2)}(g) = 0 \quad \text{since } H \cap CL(g) = \emptyset$$

3- $H = \langle (xy, I) \rangle = \{(1, I), (xy, I), ((xy)^2, I), ((xy)^3, I), (1, z^2), (xy, z^2), ((xy)^2, z^2), ((xy)^3, z^2)\}$

(i) If $g = (1, I)$ or $g = (1, z^2)$ $H \cap CL(1, I) = \{(1, I), (1, z^2)\}$

$$\Phi_{(l+2,2)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{16m}{8} \cdot 1 = 2m = 2 \cdot \Phi_{l+2}(1)$$

(ii) If $g = (x^m, I) = ((xy)^2, I) = (y^2, I)$ or

$$g = (x^m, z^2) = ((xy)^2, z^2) = (y^2, z^2) \text{ and } g \in H$$

$$\Phi_{(l+2,2)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{16m}{8} \cdot 1 = 2m = 2 \cdot \Phi_{l+2}(x^m)$$

Since $H \cap CL(g) = \{g\}$, $\varphi(g) = 1$

(iii) If $g = (xy, I)$ or $g = ((xy)^3, I) = (xy^3, I)$ or $g = (xy, z^2)$

or $g = ((xy)^3, z^2) = (xy^3, z^2)$ and $g \in H$

$$\Phi_{(l+2,2)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot (\varphi(g) + \varphi(g^{-1})) = \frac{16}{8} \cdot (1+1) = 2 \cdot 2 = 2 \cdot \Phi_{l+2}(xy)$$

since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$

Otherwise

$$\Phi_{(l+2,2)}(g) = 0 \text{ since } H \cap CL(g) = \emptyset$$

Case (III):

If H is a cyclic subgroup of $(Q_{2m} \times \{z\})$, then:

$$1. H = \langle (x, z) \rangle = \langle (x, z^2) \rangle = \langle (x, z^3) \rangle$$

$$2. H = \langle (y, z) \rangle = \langle (y, z^2) \rangle = \langle (y, z^3) \rangle$$

$$3. H = \langle (xy, z) \rangle = \langle (xy, z^2) \rangle = \langle (xy, z^3) \rangle$$

And φ the principal character of H , Φ_j Artin characters of Q_{2m} where $1 \leq j \leq l+2$ then by using Theorem 2.1

$$1. H = \langle (x, z) \rangle$$

(i) If $g = (1, I)$ or $g = (1, z)$ or $g = (1, z^2)$ or $g = (1, z^3)$ and $g \in H$

$$\Phi_{(j,3)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{16m}{|C_H(g)|} \cdot 1 = \frac{4.4m}{|C_{\langle(x,z)\rangle}(g)|} \cdot 1 = \frac{4|C_{Q_{2m}}(1)|}{4|C_{\langle x \rangle}(1)|} \cdot \varphi(1) = \Phi_j(1)$$

since $H \cap CL(g) = \{(1,I), (1,z), (1,z^2), (1,z^3)\}$

(ii) If $g = (1,I)$ or $g = (x^m, I)$ or $g = (x^m, z)$ or $g = (1, z)$ or $g = (x^m, z^2)$ or $g = (1, z^2)$ or $g = (1, z^3)$ or $g = (x^m, z^3)$ and $g \in H$

(a) if $g = (1,I)$ or $g = (1,z)$ or $g = (1,z^2)$ or $g = (1,z^3)$ and $g \in H$.

$$\Phi_{(j,3)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{16m}{|C_H(g)|} \cdot 1 = \frac{4.4m}{|C_{\langle(x,z)\rangle}(g)|} \cdot 1 = \frac{4|C_{Q_{2m}}(1)|}{4|C_{\langle x \rangle}(1)|} \cdot \varphi(1) = \Phi_j(1)$$

since $H \cap CL(g) = \{g\}, \varphi(g) = 1$

(b) If $g = (x^m, I)$ or $g = (x^m, z)$ or $g = (x^m, z^2)$ or $g = (x^m, z^3)$ and $g \in H$

$$\Phi_{(j,3)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{16m}{|C_H(g)|} \cdot 1 = \frac{4.4m}{|C_H(g)|} \cdot 1 = \frac{4|C_{Q_{2m}}(x^m)|}{4|C_{\langle x \rangle}(x^m)|} \cdot \varphi(x^m) = \Phi_j(x^m)$$

since $H \cap CL(g) = \{g\}, \varphi(g) = 1$

(iii) If $g = \{(x^i, I), (x^i, z), (x^i, z^2), (x^i, z^3)\}, i \neq m, i \neq 2m$ and $g \in H$

$$\Phi_{(j,3)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot (\varphi(g) + \varphi(g^{-1})) = \frac{8m}{|C_H(g)|} \cdot (1+1) = \frac{4.2m}{|C_H(g)|} \cdot (1+1) = \frac{4|C_{Q_{2m}}(q)|}{4|C_{\langle x \rangle}(q)|} \cdot (\varphi(g) + \varphi(g^{-1})) = \Phi_j(q)$$

since $H \cap CL(g) = \{g, g^{-1}\}$ and $\Phi(g) = \Phi(g^{-1}) = 1, g = (q, z) = (q, z^3), q \in Q_{2m}$ and $q \neq x^m, q \neq 1$

(iv) if $g \notin H$ Since $H \cap CL(g) = \emptyset$

$$\Phi_{(j,3)}(g) = 0$$

2. $H = \langle(y, z) \rangle = \{(1,I), (y,I), (y^2,I), (y^3,I), (1,z), (y,z), (y^2,z), (y^3,z), (1,z^2), (y,z^2), (y^2,z^2), (y^3,z^2), (1,z^3), (y,z^3), (y^2,z^3), (y^3,z^3)\}$

(i) If $g = (1,I)$ or $g = (1,z)$ or $g = (1,z^2)$ or $g = (1,z^3)$ and $g \in H$ $H \cap CL(g) = \{(1,I), (1,z), (1,z^2), (1,z^3)\}$

$$\Phi_{(l+1,3)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{16m}{16} \cdot 1 = m = \Phi_{l+1}(1)$$

(ii) If $g = (x^m, I) = (y^2, I)$ or $g = (y^2, z)$ or $g = (y^2, z^2)$ or $g = (y^2, z^3)$ and $g \in H$

$$\Phi_{(l+1,3)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{16m}{16} \cdot 1 = m = \Phi_{l+1}(x^m)$$

Since $H \cap CL(g) = \{g\}, \varphi(g) = 1$

(iii) $g = (y, I)$ or $g = (y, z)$ or $g = (y, z^2)$ or $g = (y, z^3)$ or $g = (y^3, I)$ or $g = (y^3, z)$ or $g = (y^3, z^2)$ or $g = (y^3, z^3)$ and $g \in H$

$$\Phi_{(l+1,3)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot (\varphi(g) + \varphi(g^{-1})) = \frac{16}{16} \cdot (1+1) = 2 = \Phi_{l+1}(y)$$

since $H \cap CL(g) = \{g, g^{-1}\}$ and $\Phi(g) = \Phi(g^{-1}) = 1$

Otherwise

$$\Phi_{(l+1,3)}(g) = 0 \quad \text{since } H \cap CL(g) = \emptyset$$

3. $H = \langle(xy, z) \rangle = \{(1,I), (xy, I), ((xy)^2, I) = (y^2, I), ((xy)^3, I) = (xy^3, I), (1, z), (xy, z),$

$((xy)^2, z), ((xy)^3, z), (1, z^2), (xy, z^2), ((xy)^2, z^2), ((xy)^3, z^2), (1, z^3)(xy, z^3), ((xy)^2, z^3), ((xy)^3, z^3)\}$

(i) If $g=(1,I)$ or $g=(1,z)$ or $g=(1,z^2)$ or $g=(1,z^3)$ $H \cap CL(g) = \{g\}$

$$\Phi_{(l+2,3)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{16m}{16} \cdot 1 = m = \Phi_{l+2}(1)$$

(ii) If $g = (x^m, I) = ((xy)^2, I) = (y^2, I)$

or $g = ((xy)^2, z) = (y^2, z)$ or $g = ((xy)^2, z^2) = (y^2, z^2)$

or $g = ((xy)^2, z^3) = (y^2, z^3)$ and $g \in H$

$$\Phi_{(l+2,3)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{16m}{16} \cdot 1 = m = \Phi_{l+2}(x^m)$$

Since $H \cap CL(g) = \{g\}$, $\varphi(g) = 1$

(iii) If $g = (xy, I)$ or $g = ((xy)^3, I)$ or $g = (xy, z)$ or $g = ((xy)^3, z)$ or $g = (xy, z^2)$ or

(iv) $g = ((xy)^3, z^2)$ or $g = (xy, z^3)$ or $g = ((xy)^3, z^3)$ and $g \in H$

$$\Phi_{(l+2,3)}(g) = \frac{|C_{Q_{2m} \times C_4}(g)|}{|C_H(g)|} \cdot (\varphi(g) + \varphi(g^{-1})) = \frac{16}{16} \cdot (1 + 1) = 2 = \Phi_{l+2}(xy)$$

since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$

Otherwise

$$\Phi_{(l+2,3)}(g) = 0 \quad \text{since } H \cap CL(g) = \emptyset$$

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