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The Numerical Method AI(MSU) For Calculating The Double Integrals

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Abstract: *The main goal of this paper is to calculate the values of double integrals with continuous integrals but improper partial and improper using the MSu method resulting from the mid-point rule on the outer dimension and the suggested method on the inner dimension. We have improved the results using Aitken acceleration, and two examples are discussed to illustrate different cases, in addition to the attached tables which show all the details.*

Keywords: Double integrals, Aitken acceleration, Improve results

1.Introduction:

Double integrals are crucial for calculating the volume under the double integral surface, determining the surface area, and calculating the intermediate centres and moments of inertia for flat surfaces.

In 2009, Dheyaa [3] offered four numerical approaches, RM(RS), RS(RS), RS(RM) and RM(RM), to calculate double integral values with continuing integrands and incorrect or incorrect derivatives. These methods combine Romberg's acceleration with the midpoint method and Romberg's acceleration with the Simpson method. Regarding the accuracy and quickness to approach the true values of integrations, RM(RM) was found to be the best method out of the four that were examined.

In 2011, [6] explored three numerical approaches made up of Romberg's acceleration and two formulations of Newton-Coats (Simpson and Midpoint) to calculate double integral values with integrands that have improper or merely improper derivatives. In tests, it was found that RSS offered the highest accuracy and speed of approach to the true value of integrations with a few incomplete periods. See [1,6] for further information on this topic.

In our study, we used the MSu method to estimate the values of double integrals, and then we used the Aitken acceleration to improve the results.

2.Mid-Point Method (M) [2]:-

Assume integral I is defined as follows:

$$I = \int_a^b f(x)dx = M(h) + \delta_M(h) + R_M \qquad \ldots(1)$$

Where $M(h)$ is the Mid-point method, $\delta_M(h)$ be the correction terms series of $M(h)$ and R_M is the remainder from truncation terms $\delta_M(h)$ after using certain terms of $\delta_M(h)$.

The general formula of the Mid-point method is: -

$$M(h) = h\sum_{i=1}^{n}(a + \frac{(2i-1)}{2}h) \qquad \ldots(2)$$

3. <u>Suggested method (Su)</u>

The Suggested technique proposed by Muhammad and colleagues [8] was adapted from the Trapezoidal and Mid-point methods used to compute single integrals and produced results in accuracy and speed of approach that were superior to the Trapezoidal and Mid-point methods.

The general formula of this method is:

$$Su(h) = \frac{h}{4}\left[f(a) + f(b) + 2f(a + (n-0.5)h) + 2\sum_{i=1}^{n-1}(f(a + (i-0.5)h) + f(a + ih))\right] \qquad \ldots(3)$$

4. Numerical method MSu

The general formula of the method MSu

$$MSu=\int_c^d\int_a^b f(x,y)dxdy=\frac{h^2}{4}\sum_{j=1}^{n}\Big[f(a,y_j)+f(b,y_j)+2(f(a+(n-0.5)h,y_j)+2\sum_{i=1}^{n-1}\big(f(a+(i-0.5)h,y_j)$$
$$f(a+ih,y_i)\Big] \quad \text{... (4)}$$

resulting from the mid-point rule on the outer dimension and the suggested method on the inner dimension.

5-Aitken's delta – Squared Process

In 1926, Alexander Aitken (1985-1926) found a new approach to accelerate the sequence convergence rate. To explain this method, we suppose the sequence $\{x_n\}$ such that $\{x_n\}=\{x_1,x_2,\dots x_k\dots\}$ linearly convergence to a certain final value β, so $\beta - x_{i+1} = C_i(\beta - x_i)$, Ralston [4], such that $|C_i|<1$ and $C_i \to C$.
We can see that C_i will be approximately steady and we can write

$$\beta - x_{i+1} \approx \bar{C}(\beta - x_i) \quad \text{…(5)}$$

Such that $|\bar{C}|=C$

We also can see that

$$\frac{\beta - x_{i+2}}{\beta - x_{i+1}} \approx \frac{\beta - x_{i+1}}{\beta - x_i} \quad \text{…(6)}$$

i.e, $$\beta \cong \frac{x_i x_{i+2}-x_{i+1}^2}{x_{i+2}-2x_{i+1}+x_i}=x_{i+2}-\frac{(\Delta x_{i+1})^2}{\Delta^2 x_i} \quad \text{…(7)}$$

such that $\Delta x_i=(x_{i+1}-x_i)$ and $\Delta^2 x_i = x_i - 2x_{i+1}-x_{i-2}$
when using u from elements of the sequence $\{x_u\}$, we can get u-2 of another sequence {S} Approaching faster than $\{x_u\}$

$$s_{i+2}=x_{i+2}-\frac{(\Delta x_{i+1})^2}{\Delta^2 x_i} \quad \text{…(8)}$$

where $i=1,2,\dots,u-2$
This process is accelerating the convergence to the final value β.

6. Examples:

6.1: The integral $\int_0^1\int_0^1 \sqrt{x+y}\,dxdy$ that its function had been shown in Fig:1, has analytic value 0.9751611332 which approximates to ten decimal digits.

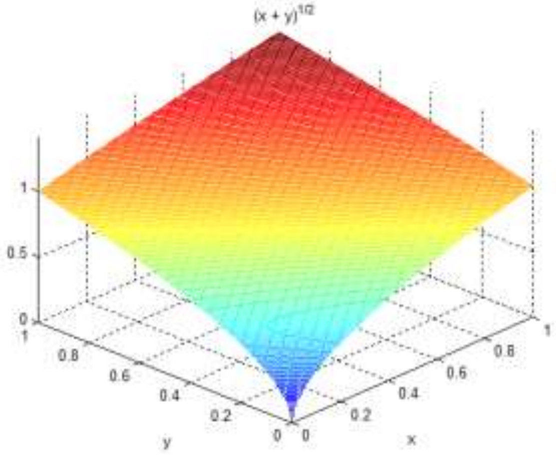

Fig:(1)

In this example, the integrand is continuous in integration region but the partial derivatives are improper at the point (x,y) = (0,0).Applying MSu method, we obtained six correct decimal digits at n=256 Moreover, when we used Aitken acceleration to improve these results we got ten correct decimal digits as shown in Table (1).

The integral $\int_0^1\int_0^1 y\sqrt{1-xy}\,dxdy$ that its function had been shown in Fig:2, has exact value04.

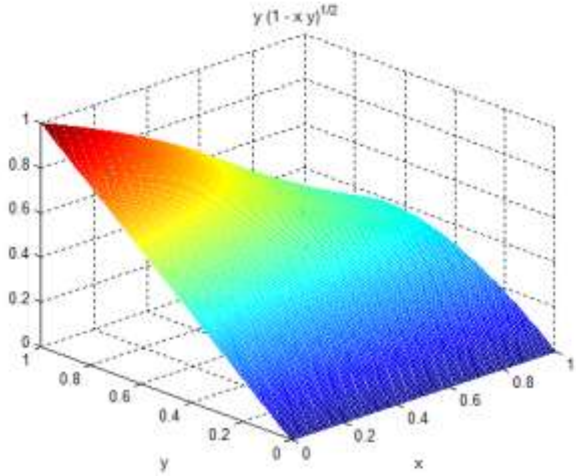

Fig:(2)

In this example, the integrand is continuous in integration region but the partial derivatives are improper at the point (x,y) = (1,1).Applying MSu method, we obtained five correct decimal digits at n=256 Moreover, when we used Aitken acceleration to improve these results we got eight correct decimal digits as shown in Table (2).

6.3: The integral $J=\int_{-1}^{0}\int_{0}^{1}\frac{(1+y)}{(1+x+y)^2}dxdy$ that its function had been shown in Fig:3, has analytic value 0.69314718055995 which approximates to fourteen decimal digits.

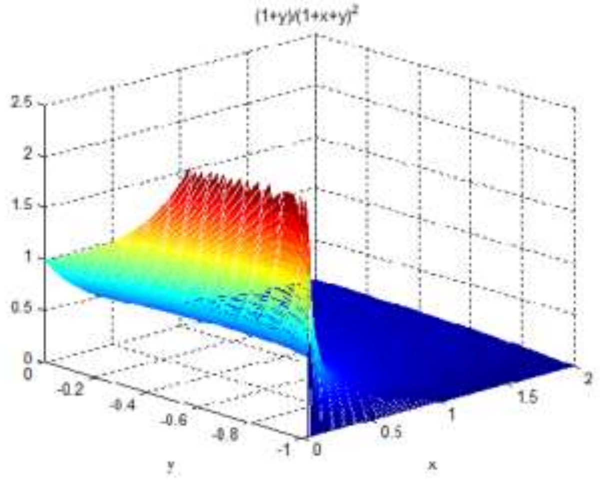

Fig (3)

In this example, the integrand is continuous in integration region except at the point *(x,y) = (-1,0)*, so it is improper ,Applying MSu method, we obtained three correct decimal digits at n=256 Moreover, when we used Aitken acceleration to improve these results we got ten correct decimal digits as shown in Table (3).

7. Conclusion:

When we calculated the approximate value of a triple integral with a continuous integrand using the composite rule, MSu gave us the correct value (to a few decimal places) when compared to the actual value for integrals using multiple subintervals without using any acceleration method, whereas using Aitken acceleration gave us the correct value to 10 decimal places for all examples.

Table(1): The value of $\int_0^1\int_0^1 \sqrt{x+y}\,dxdy = 0.9751611332$					
n	MSu	AI(MSu)	AI(MSu)	AI(MSu)	AI(MSu)
1	0.9829629131				
2	0.9774223211				
4	0.9757835064	0.9750951755			
8	0.9753269571	0.9751506540			
16	0.9752044067	0.9751594405	0.9751610940		
32	0.9751722734	0.9751608536	0.9751611244		
64	0.9751639752	0.9751610861	0.9751611319	0.9751611344	
128	0.9751618538	0.9751611252	0.9751611330	0.9751611426	
256	0.9751613151	0.9751611318	0.9751611350	0.9751610330	0.9751611332

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Table(2): The value of $I = \int_0^1\int_0^1 y\sqrt{1-xy}\,dxdy = 0.4$				
n	MSu	AI(MSu)	AI(MSu)	AI(MSu)
1	0.4298946986			
2	0.4077576840			
4	0.4019908064	0.3999592487		
8	0.4005072579	0.3999934248		
16	0.4001285637	0.3999987645	0.3999997532	
32	0.4000324563	0.3999997703	0.4000000038	
64	0.4000081704	0.3999999583	0.4000000016	0.4000000015

Table(3): The value of $I = \int_{-1}^{0}\int_{0}^{1} \frac{(1+y)}{(1+x+y)^2}dxdy = 0.6931471806$					
n	MSu	AI(MSu)	AI(MSu)	AI(MSu)	AI(MSu)
1	0.8055555556				
2	0.7658616780				
4	0.7341560709	0.6083158743			
8	0.7148568713	0.6848353600			
16	0.7043062172	0.6915822391	0.6922346495		
32	0.6988029310	0.6928025443	0.6930719960		
64	0.6959941254	0.6930661441	0.6931387734	0.6931445603	
128	0.6945754211	0.6931275253	0.6931461569	0.6931471748	
256	0.6938624929	0.6931423401	0.6931470534	0.6931471873	0.6931471806

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