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On Partial Differentiation and Integration of Triple Generalized Transformation

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Abstract. *In this paper, partial differential and integration of triple g-transformation are studied. Also by using gs-transformation of integration, the inverse of this transformation is found. We use partial differential of triple g-transormation for finding triple g-transformation of some function of the form* $x^k y^n z^m f(x, y, z)$.

Keywords: partial differential equation, integral transform, generalized transform, triple g-transformation, inverse integral transform.

Introduction

The problem of the inverse is one of the important problems in mathematics, as many analytical concepts depend on it to find a solution to differential and algebraic equations [ ].

Integral transformations are an important tool in solving differential equations by converting them into algebraic equations, then by using the inverse of the integral transformation, we can find the solution. In this paper, we introduce the inverse integral transformation by using derivative of function terms, and we also found a formula for triple g-transform the functions with the model $x^k y^n z^m f(x, y, z)$.

I- g- transformation :

1.1 Definition [9]:

g- transformation $g(f(x))$ for a function $f(x)$ where $x \in [\ 0, \infty\ [$ is defined by the following integral

$$g(f(x)) = p(s) \int_0^\infty e^{-q(s)x} \; f(x)dx \quad , p(s) \neq 0$$

Such that the integral is convergent for some q(s) , s is positive constant .

1.2 g- transformation of some selected function [9] :

ID	$f(x)$	$g(f(x)) = p(s) \int_0^\infty e^{-q(s)x} \; f(x)dx$
1	K , K constant	$K \frac{p(s)}{q(s)}$
2	$\sin(ax)$	$\frac{ap(s)}{(q(s))^2 + a^2}$
3	$\cos(ax)$	$\frac{p(s)q(s)}{(q(s))^2 + a^2}$
4	x^k	$\frac{k!\,p(s)}{(q(s))^{n+1}}$

1.3 Definition [13]:

Let f be a continuous function of three variables then the triple g-transformation of f(x, y, z) is defined as following :

$$T_{3g}(f(x,y,z)) = T_g^s\left(T_g^\alpha\left(T_g^\beta(f(x,y,z))\right)\right)$$

$$= P_1(s)P_2(\alpha)P_3(\beta) \int_0^\infty \int_0^\infty \int_0^\infty e^{q_1(s)x - q_2(\alpha)y - q_3(\beta)z} f(x,y,z)\, dxdydz$$

$$= p_{(s,\alpha,\beta)} \int_0^\infty \int_0^\infty \int_0^\infty e^{q_1(s)x - q_2(\alpha)y - q_3(\beta)z} f(x,y,z)\, dxdydz$$

where $p_{(s,\alpha,\beta)} = p_1(s) \cdot p_2(\alpha) \cdot p_3(\beta)$ and x,y,z > 0 and s, α and β are positive constants , and $\sup\left|\frac{f(x,y,z)}{e^{ax+by+cz}}\right| < 0$ for some $a, b, c \in R$. The inverse of T_{3g} – transform is defined as following :

$$f(x,y,z) = \frac{1}{2\pi i} \int_{\propto - i\infty}^{\propto - i\infty} \int_{\propto - i\infty}^{\propto + i\infty} \int_{\propto - i\infty}^{\propto + i\infty} P_1(s)P_2(\alpha)P_3(\beta)\, e^{q_1(s)x - q_2(\alpha)y - q_3(\beta)z} F(s,\alpha,\beta)\, dsd\alpha d\beta$$

2.4 Table of T_{3sg}-transformation of selected functions [13]

ID	$f(x)$	$T_{3g}(f(x,y,z)) = p_{(s,\alpha,\beta)} \int_0^\infty \int_0^\infty \int_0^\infty e^{q_1(s)x - q_2(\alpha)y - q_3(\beta)z}$

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1	K, K const	$K\frac{p}{q_1q_2q_3}$
2	$\sin(ax + by + cz)$	$p\left[\frac{aq_2q_3 - abc + bq_1q_3 + cq_1q_2}{(q_1^2 + a^{2)}(q_2^2 + b^{2)(}q_3^2 + c^{2)}}\right]$
3	$\cos(ax + by + cz)$	$p\left[\frac{q_1q_2q_3 - bcq_1 - abq_3 - acq_2}{(q_1^2 + a^{2)}(q_2^2 + b^{2)(}q_3^2 + c^{2)}}\right]$
4	$x^k y^m z^n$	$\frac{pk!\ m!\ n!}{(q_1)^{k+1}(q_2)^{m+1}(q_3)^{n+1}}$

II. Differential and integration:

2.1 Theorem:

Let $q_1 = s$, $q_2 = \alpha$, $q_3 = \beta$, $p(s,\alpha,\beta) = p_1(s)\cdot p_2(\alpha)\cdot p_3(\beta)$ then

$$\frac{\partial^{k+n+m}T_{3g}\left(f(x,y,z)\right)}{\partial s^k\,\partial\alpha^n\,\partial\beta^m} = \sum_{r=0}^{m}\sum_{j=0}^{n}\sum_{i=0}^{k} c_i^k c_j^n c_r^m (-1)^{k+n+m-(i-j-r)} p_1{}^{(i)} p_2{}^{(j)} p_3{}^{(r)} T_{3g}\left(x^{k-i}\ y^{n-j}\ z^{m-r}\ f(x,y,z)\right)$$

Proof:

$$\frac{\partial^{k+n+m}T_{3g}\left(f(x,y,z)\right)}{\partial s^k\,\partial\alpha^n\,\partial\beta^m} = \int_0^\infty\int_0^\infty\int_0^\infty \frac{\partial^k}{\partial s^k}(p_1e^{-sx})\cdot\frac{\partial^n}{\partial\alpha^n}(p_2e^{-\alpha y})\cdot\frac{\partial^m}{\partial\beta^m}(p_3e^{-\beta z})f(x,y,z)dx\,dy\,dz$$

$$= \int_0^\infty\int_0^\infty\int_0^\infty\left[\sum_{i=0}^{k}c_i^k\left(p_1(s)\right)^{(i)}(e^{-sx})^{(k-i)}\right]\cdot\left[\sum_{j=0}^{n}c_j^n\left(p_2(\alpha)\right)^{(j)}(e^{-\alpha y})^{(n-j)}\right]\cdot\left[\sum_{r=0}^{m}c_r^m\left(p_3(\beta)\right)^{(r)}\left(e^{-\beta z}\right)^{(m-r)}\right]f(x,y,z)dx\,dy\,dz$$

$$= \int_0^\infty\int_0^\infty\int_0^\infty\left[\sum_{i=0}^{k}c_i^k\left(p_1(s)\right)^{(i)}(-1)^{(k-i)}x^{k-i}e^{-sx}\right]\cdot\left[\sum_{j=0}^{n}c_j^n\left(p_2(\alpha)\right)^{(j)}(-1)^{(n-j)}y^{n-j}e^{-\alpha y}\right]\cdot\left[\sum_{r=0}^{m}c_r^m\left(p_3(\beta)\right)^{(r)}(-1)^{(m-r)}z^{m-r}e^{-\beta z}\right]f(x,y,z)dx\,dy\,dz$$

$$= \sum_{r=0}^{m}\sum_{j=0}^{n}\sum_{i=0}^{k} c_i^k c_j^n c_r^m (-1)^{k+n+m-(i-j-r)} p_1{}^{(i)} p_2{}^{(j)} p_3{}^{(r)} \cdot\int_0^\infty\int_0^\infty\int_0^\infty x^{k-i}\ y^{n-j}\ z^{m-r}e^{-sx-\alpha y-\beta z}f(x,y,z)dx\,dy\,dz$$

$$= \sum_{r=0}^{m}\sum_{j=0}^{n}\sum_{i=0}^{k} c_i^k c_j^n c_r^m (-1)^{k+n+m-(i-j-r)} p_1{}^{(i)} p_2{}^{(j)} p_3{}^{(r)} T_{3g}\left(x^{k-i}\ y^{n-j}\ z^{m-r}\ f(x,y,z)\right)$$

2.2 Theorem:

Let $h(x), R(y), k(z)$ be a function work on $[0,\infty[$ then

$$h(x)\cdot R(y)\cdot k(z) = T_{3g}^{-1}\left(\frac{p}{q_1q_2q_3}\left[h(0) + \int_0^\infty \acute{h}(x)e^{-q_1x}\,dx\right]\cdot\left[R(0) + \int_0^\infty \acute{R}(y)e^{-q_2y}dy\right]\cdot\left[k(0) + \int_0^\infty \acute{k}(z)e^{-q_3z}dz\right]\right)$$

Proof:

$$T_{3g}\left(f(x,y,z)\right) = p\int_0^\infty\int_0^\infty\int_0^\infty e^{-q_1x-q_2y-q_3z}f(x,y,z)dx\,dy\,dz$$

$$f(x,y,z) = h(x)\cdot R(y)\cdot k(z)$$

thus

$$T_{3g}\left(f(x,y,z)\right) = p\int_0^\infty\int_0^\infty\int_0^\infty e^{-q_1x-q_2y-q_3z}h(x)\cdot R(y)\cdot k(z)dx\,dy\,dz$$

$$= p\left[\int_0^\infty h(x)e^{-q_1x}dx \cdot \int_0^\infty R(y)e^{-q_2y}dy \cdot \int_0^\infty k(z)e^{-q_3z}\,dz\right]$$

By using partition integral meth

We get:

$$T_{3g}\big(f(x,y,z)\big) = p\left[\frac{-1}{q_1}h(x)e^{-q_1x}\Big|_0^\infty + \frac{1}{q_1}\int_0^\infty \acute{h}(x)e^{-q_1x}dx\right] \cdot \left[\frac{-1}{q_2}R(y)e^{-q_2y}\Big|_0^\infty + \frac{1}{q_2}\int_0^\infty \acute{R}(y)e^{-q_2y}dy\right] \cdot \left[\frac{-1}{q_3}k(z)e^{-q_3z}\Big|_0^\infty + \frac{1}{q_3}\int_0^\infty \acute{k}(z)e^{-q_3z}dz\right]$$

$$= p\left[\frac{1}{q_1}h(0) + \frac{1}{q_1}\int_0^\infty \acute{h}(x)e^{-q_1x}dx\right] \cdot \left[\frac{1}{q_2}R(0) + \frac{1}{q_2}\int_0^\infty \acute{R}(y)e^{-q_2y}dy\right] \cdot \left[\frac{1}{q_3}k(0) + \frac{1}{q_3}\int_0^\infty \acute{k}(z)e^{-q_3z}dz\right]$$

$$= \frac{p}{q_1q_2q_3}\left[h(0) + \int_0^\infty \acute{h}(x)e^{-q_1x}\,dx\right] \cdot \left[R(0) + \int_0^\infty \acute{R}(y)e^{-q_2y}dy\right] \cdot \left[k(0) + \int_0^\infty \acute{k}(z)e^{-q_3z}dz\right]$$

Therefore

$$h(x) \cdot R(y) \cdot k(z) = T_{3g}^{-1}\left(\frac{p}{q_1q_2q_3}\left[h(0) + \int_0^\infty \acute{h}(x)e^{-q_1x}\,dx\right] \cdot \left[R(0) + \int_0^\infty \acute{R}(y)e^{-q_2y}dy\right] \cdot \left[k(0) + \int_0^\infty \acute{k}(z)e^{-q_3z}dz\right]\right)$$

2.3 Note:

In Theorem() we note that if the require is $T_{3g}\left(x^ky^nz^mf(x)\right)$ then we have 2^{k+n+m} stages for construction the final result

2.4 Example:

Find $T_{3g}(xyze^{x+y+z})$ $k = 1$, $n = 1$, $m = 1$

Solution:

By note ( ) we have

o	k_o	n_o	m_o	$s^{(i)}\alpha^{(j)}\beta^{(r)}T_{3g}(x^{1-i}y^{1-j}z^{1-r}e^{x+y+z})$
1	0	0	1	$s\alpha(xye^{x+y+z})$
2	0	1	0	$s\beta T_{3g}(xze^{x+y+z})$
3	0	1	1	$sT_{3g}(xe^{x+y+z})$
4	1	0	0	$\alpha\beta T_{3g}(yze^{x+y+z})$
5	1	0	1	$\alpha T_{3g}(ye^{x+y+z})$
6	1	1	0	$\beta T_{3g}(ze^{x+y+z})$
7	1	1	1	$T_{3g}(e^{x+y+z})$

Stage -1: $k_0 = 0$, $n_1 = 1$, $r_1 = 1$

$$T_{3g}(x\ \ e^{x+y+z})$$

Where $q_1 = s$, $q_2 = \alpha$, $q_3 = \beta$

$$p_1 = s,\ \ p_2 = 1,\ \ p_3 = \beta$$

$$\frac{\partial T_{3g}e^{x+y+z}}{\partial s} = \sum_{k=0}^{0}\sum_{j=0}^{0}\sum_{i=0}^{1} c_i^1c_j^0c_r^0 -1^{1+0+0-(i+j+1)}s^{(i)}\alpha^{(j)}\beta^{(r)}T_{3g}(x^{1-i}y^{0-j}z^{0-r}e^{x+y+z})$$

Since,

$$T_{3g}e^{x+y+z} = \frac{s\beta}{(s-1)(\alpha-1)(\beta-1)}$$

Thus

$$\frac{\partial T_{3g}(e^{x+y+z})}{\partial s} = \frac{\beta}{(\alpha-1)(\beta-1)}\left[\frac{s-1-s}{(s-1)^2}\right] = \frac{-\beta}{(\alpha-1)(\beta-1)(s-1)^2}$$

Therefore:

$$\frac{-\beta}{(\alpha-1)(\beta-1)(s-1)^2} = \sum_{i=0}^{1} c_i^1(-1)^{1-i}s^{(i)}\alpha\beta T_{3g}(x^{1-i}e^{x+y+z})$$

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$$= -s\alpha\beta \ \ T_{3g}(x \ \ e^{x+y+z}) + \alpha\beta \ \ T_{3g}(e^{x+y+z})$$

$$= -s\alpha\beta \ T_{3g}(x \ \ e^{x+y+z}) + \alpha\beta \ T_{3g}(e^{x+y+z})$$

$$= -s\alpha\beta \ \ T_{3g}(x \ \ e^{x+y+z}) = \frac{-\beta}{(\alpha-1)(\beta-1)(s-1)^2} - \frac{s\alpha\beta^2}{(s-1)(\alpha-1)(\beta-1)}$$

$$= \frac{-\beta - s\alpha\beta^2(s-1)}{(\alpha-1)(\beta-1)(s-1)^2}$$

Thus

$$T_{3g}(x \ \ e^{x+y+z}) = \frac{1+s\alpha\beta(s-1)}{s\alpha(\alpha-1)(\beta-1)(s-1)^2}$$

Stage-2 : $T_{3g}(y \ \ e^{x+y+z})$ $k_2 = 1$, $n_2 = 0$, $m_2 = 1$

$$\frac{\partial T_{3g} e^{x+y+z}}{\partial \alpha} = \sum_{k=0}^{0}\sum_{j=0}^{1}\sum_{i=0}^{0} c_i^0 c_j^1 c_r^0 (-1)^{0+1+0-(i+1+r)} s^{(i)} \alpha^{(j)} \beta^{(r)} T_{3g}(x^{0-i} y^{1-j} z^{0-r} e^{x+y+z})$$

Since,

$$T_{3g} e^{x+y+z} = \frac{s\beta}{(s-1)(\alpha-1)(\beta-1)}$$

Thus

$$\frac{\partial T_{3g}(e^{x+y+z})}{\partial \alpha} = \frac{s\beta}{(s-1)(\beta-1)}\left[\frac{\alpha-1-\alpha}{(\alpha-1)^2}\right] = \frac{-s\beta}{(s-1)(\beta-1)(\alpha-1)^2}$$

Therefore:

$$\frac{-s\beta}{(\alpha-1)(\beta-1)(s-1)^2} = \sum_{j=0}^{1} c_j^1 (-1)^{1-j} s\alpha^{(j)} \beta \ T_{3g}(y^{1-j} e^{x+y+z})$$

$$= -s\alpha\beta \ \ T_{3g}(y \ \ e^{x+y+z}) + s\beta \ \ T_{3g}(e^{x+y+z})$$

$$= -s\alpha\beta \ T_{3g}(y \ \ e^{x+y+z}) + \frac{s^2\beta^2}{(s-1)(\alpha-1)(\beta-1)}$$

$$= -s\alpha\beta \ \ T_{3g}(y \ \ e^{x+y+z}) = \frac{-s\beta}{(s-1)(\beta-1)(\alpha-1)^2} + \frac{s^2\beta^2}{(s-1)(\alpha-1)(\beta-1)}$$

$$= \frac{\alpha-2}{(s-1)(\beta-1)(\alpha-1)^2}$$

Thus

$$T_{3g}(y \ \ e^{x+y+z}) = \frac{2-\alpha}{\alpha(s-1)(\beta-1)(\alpha-1)^2}$$

Stage -3: $T_{3g}(z \ \ e^{x+y+z})$ $k_3 = 1$, $n_3 = 1$, $m_3 = 0$

$$\frac{\partial T_{3g} e^{x+y+z}}{\partial \beta} = \sum_{k=0}^{0}\sum_{j=0}^{0}\sum_{i=0}^{1} c_i^0 c_j^0 c_r^1 (-1)^{0+0+1-(i+j+1)} s^{(i)} \alpha^{(j)} \beta^{(r)} T_{3g}(x^{0-i} y^{0-j} z^{1-r} e^{x+y+z})$$

Since,

$$T_{3g} e^{x+y+z} = \frac{s\beta}{(s-1)(\alpha-1)(\beta-1)}$$

Thus

$$\frac{\partial T_{3g}(e^{x+y+z})}{\partial \beta} = \frac{s\beta}{(s-1)(\alpha-1)}\left[\frac{\beta-1-\beta}{(\beta-1)^2}\right] = \frac{-s}{(s-1)(\alpha-1)(\beta-1)^2}$$

Therefore

$$\frac{-s}{(\alpha-1)(\beta-1)(s-1)^2} = \sum_{r=0}^{1} c_r^1 (-1)^{1-r} \beta^r \ s\alpha \ \ T_{3g}(z^{1-r} \ \ e^{x+y+z})$$

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$$= -s\alpha\beta\ T_{3g}(z\ \ e^{x+y+z}) + s\beta\ T_{3g}(e^{x+y+z}) - s\alpha\beta\ T_{3g}(z\ \ e^{x+y+z}) + \frac{s^2\beta}{(s-1)(\alpha-1)(\beta-1)}$$

$$= -s\alpha\beta\ \ T_{3g}(z\ \ e^{x+y+z}) = \frac{-s}{(s-1)(\beta-1)(\alpha-1)^2} - \frac{s^2\beta}{(s-1)(\alpha-1)(\beta-1)}$$

$$= \frac{-s - s^2\beta^2(\beta-1)}{-s\alpha\beta(s-1)(\alpha-1)(\beta-1)^2}$$

Thus

$$T_{3g}(z\ \ e^{x+y+z}) = \frac{-1 - s\beta(\beta-1)}{\alpha\beta(s-1)(\alpha-1)(\beta-1)^2}$$

Stage-4

$T_{3g}(xy\ \ e^{x+y+z})$ $\quad k_4 = 0 \quad , n_4 = 0 \quad , m_4 = 1$

$$\frac{\partial T_{3g} e^{x+y+z}}{\partial s \partial \alpha} = \sum_{k=0}^{1}\sum_{j=0}^{1}\sum_{i=0}^{0} c_i^1 c_j^1 c_r^0 (-1)^{1+1+0-(i+j+1)} s^{(i)} \alpha^{(j)} \beta^{(r)} T_{3g}(x^{1-i} y^{1-j} z^{0-r} e^{x+y+z})$$

Since,

$$T_{3g} e^{x+y+z} = \frac{s\beta}{(s-1)(\alpha-1)(\beta-1)}$$

Thus

$$\frac{\partial^2 T_{3g}(e^{x+y+z})}{\partial\alpha\,\partial s} = \frac{-\beta}{(s-1)^2(\beta-1)}\ \frac{\partial}{\partial\alpha}\left(\frac{1}{\alpha-1}\right) = \frac{\beta}{(\beta-1)(s-1)^2(\alpha-1)^2}$$

Therefore

$$\frac{\beta}{(\beta-1)(s-1)^2(\alpha-1)^2} = \sum_{i=0}^{1}\sum_{j=0}^{1} c_i^1\ c_j^1 (-1)^{1-i+j} s^{(i)}\ \ \alpha^{(j)}\ \beta\ \ T_{3g}(x^{1-i} y^{1-j}\ \ e^{x+y+z})$$

$$= s\alpha\beta\ \ T_{3g}(xy\ \ e^{x+y+z}) - s\beta\ \ T_{3g}(x\, e^{x+y+z}) - \alpha\beta T_{3g}(y e^{x+y+z}) + \beta T_{3g}(e^{x+y+z})$$

$$= s\alpha\beta\ \ T_{3g}(xy\ \ \ e^{x+y+z}) - \frac{\beta}{(s-1)^2(\alpha-1)^2(\beta-1)} + \frac{s\beta\big(1 + s\alpha\beta(s-1)\big)}{s\alpha(\alpha-1)(\beta-1)(s-1)^2} + \frac{\alpha\beta(2-\alpha)}{\alpha(s-1)(\beta-1)(\alpha-1)^2} + \frac{s\beta^2}{(s-1)(\alpha-1)(\beta-1)}$$

$$= \frac{\alpha\beta + \beta\big(1 + s\alpha\beta(s-1)\big)(\alpha-1) + \alpha\beta(2-\alpha)(s-1) + s\alpha\beta^2(s-1)(\alpha-1)}{\alpha(\beta-1)(s-1)^2(\alpha-1)^2}$$

$$T_{3g}(xy\ e^{x+y+z}) = \frac{\alpha + \big(1 - s\alpha\beta(s-1)\big)(\alpha-1) + \alpha(2-\alpha)(s-1) + s\alpha\beta(s-1)(\alpha-1)}{s\alpha^2(\beta-1)(s-1)^2(\alpha-1)^2}$$

Stage-5

$T_{3g}(yz\ \ e^{x+y+z})$ $\quad k_5 = 1 \quad , n_5 = 0 \quad , m_5 = 0$

$$\frac{\partial^2 T_{3g} e^{x+y+z}}{\partial\alpha\partial\beta} = \sum_{k=0}^{1}\sum_{j=0}^{1}\sum_{i=0}^{0} c_i^1 c_j^1 c_r^0 (-1)^{1+1+0-(i+j+1)} s^{(i)} \alpha^{(j)} \beta^{(r)} T_{3g}(x^{1-i} y^{0-j} z^{0-r} e^{x+y+z})$$

Since,

$$T_{3g} e^{x+y+z} = \frac{s\beta}{(s-1)(\alpha-1)(\beta-1)}$$

Thus

$$\frac{\partial T_{3g}(e^{x+y+z})}{\partial\alpha} = \frac{s\beta}{(s-1)(\beta-1)}\ \frac{\partial}{\partial\alpha}\left(\frac{-1}{(\alpha-1)^2}\right) = \frac{-s\beta}{(s-1)(\beta-1)(\alpha-1)^2}$$

$$\frac{\partial^2 T_{3g}(e^{x+y+z})}{\partial\alpha\,\partial\beta} = \frac{-s}{(s-1)(\alpha-1)^2}\ \frac{\partial}{\partial\beta}\left(\frac{-1}{(\beta-1)^2}\right) = \frac{s}{(s-1)(\alpha-1)^2(\beta-1)^2}$$

Therefore

$$\frac{s}{(s-1)(\alpha-1)^2(\beta-1)^2} = \sum_{i=0}^{1}\sum_{j=0}^{1} c_i^1\ c_j^1 (-1)^{1-i+j} s^{(i)}\ \ \alpha^{(j)}\ \beta\ \ T_{3g}(x^{1-i} y^{1-j}\ \ e^{x+y+z})$$

$$= s\alpha\beta\ \ T_{3g}(yz\ \ e^{x+y+z}) - s\alpha\ \ T_{3g}(y\, e^{x+y+z}) - s\beta T_{3g}(z\, e^{x+y+z}) + s T_{3g}(e^{x+y+z})$$

$$= s\alpha\beta \;\; T_{3g}(yz \quad e^{x+y+z}) - \frac{s}{(\beta-1)^2(\alpha-1)^2(s-1)} + \frac{s\alpha(2-\alpha)}{\alpha(s-1)(\beta-1)(\alpha-1)^2} + \frac{s\beta\big(1+s\beta(\beta-1)\big)}{\alpha\beta(s-1)(\alpha-1)(\beta-1)^2} - \frac{s^2\beta}{(s-1)(\beta-1)(\alpha-1)}$$

$$= \frac{\alpha s + \alpha s(2-\alpha)(\beta-1) + s(-1\alpha)\big(1+s\beta(\beta-1)\big) - s^2\alpha\beta(\alpha-1)(\beta-1)}{\alpha(s-1)(\alpha-1)^2(\beta-1)^2}$$

$$T_{3g}(yz \;\; e^{x+y+z}) = \frac{\alpha + \alpha(2-\alpha)(\beta-1) + (\alpha-1)\big(1-s\beta(\beta-1)\big) - s\alpha\beta(\alpha-1)(\beta-1)}{\alpha^2\beta(s-1)(\alpha-1)^2(\beta-1)^2}$$

Stage-6

$T_{3g}(xz \;\; e^{x+y+z}) \qquad k_6 = 0 \qquad , n_6 = 1 \qquad , m_6 = 0$

$$\frac{\partial^2 T_{3g} e^{x+y+z}}{\partial s \partial \beta} = \sum_{k=0}^{1}\sum_{j=0}^{1}\sum_{i=0}^{0} c_i^1 c_j^1 c_r^0 (-1)^{1+1+0-(i+j+1)} s^{(i)} \alpha^{(j)} \beta^{(r)} T_{3g}(x^{1-i} y^{0-j} z^{0-r} e^{x+y+z})$$

Since,

$$T_{3g}e^{x+y+z} = \frac{s\beta}{(s-1)(\alpha-1)(\beta-1)}$$

Thus

$$\frac{\partial T_{3g}(e^{x+y+z})}{\partial s} = \frac{-\beta}{(\alpha-1)(\beta-1)} \; \frac{\partial}{\partial \alpha}\left(\frac{-1}{(s-1)^2}\right) = \frac{-\beta}{(\alpha-1)(\beta-1)(s-1)^2}$$

$$\frac{\partial^2 T_{3g}(e^{x+y+z})}{\partial \beta \, \partial s} = \frac{-s}{(s-1)(\alpha-1)(\beta-1)^2} = \frac{s\beta}{(\alpha-1)(s-1)^2(\beta-1)^2}$$

Therefore

$$\frac{s\beta}{(\alpha-1)(s-1)^2(\beta-1)^2} = \sum_{i=0}^{1}\sum_{j=0}^{1} c_i^1 \, c_j^1 (-1)^{1-i+j} s^{(i)} \;\; \alpha^{(j)} \, \beta \;\; T_{3g}(x^{1-i} y^{1-j} \;\; e^{x+y+z})$$

$$= s\alpha\beta \;\; T_{3g}(xz \;\; e^{x+y+z}) - s\alpha \;\; T_{3g}(x\, e^{x+y+z}) - \beta\alpha T_{3g}(z\, e^{x+y+z}) + \alpha T_{3g}(e^{x+y+z})$$

$$= s\alpha\beta \;\; T_{3g}(xz \quad e^{x+y+z}) - \frac{s\beta}{(\beta-1)^2(s-1)^2(\alpha-1)} + \frac{s\alpha\big(1+s\alpha\beta(s-1)\big)}{s\alpha(\alpha-1)(\beta-1)(s-1)^2} + \frac{\beta\alpha\big(1+s\beta(\beta-1)\big)}{\alpha\beta(s-1)(\alpha-1)(\beta-1)^2} - \frac{s\alpha\beta}{(\alpha-1)(s-1)^2(\beta-1)^2}$$

$$= \frac{s\beta + (\beta-1)\big(1+s\alpha\beta(s-1)\big) + (s-1)\big(1+s\beta(\beta-1)\big) - s\alpha\beta}{(\alpha-1)(s-1)^2(\beta-1)^2}$$

$$T_{3g}(xz \;\; e^{x+y+z}) = \frac{s\beta + (\beta-1)\big(1+s\alpha\beta(s-1)\big) + (s-1)\big(1+s\beta(\beta-1)\big) - s\alpha\beta}{(\alpha-1)(s-1)^2(\beta-1)^2}$$

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