International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 7 Issue 1, January - 2023, Pages: 60-66

Second Accuracy Approximation Using Combined Edges Systems

Hussein R. Jaffer[1] and Khalid Sh. Al'Dzhabri[2]

1Department of Mathematics, University of Al-Qadisiyah , College of Education, Iraq , Al Diwaniyah

edu-math.post31@qu.edu.iq

2Department of Mathematics, University of Al-Qadisiyah , College of Education, Iraq , Al Diwaniyah

khalid.aljabrimath@qu.edu.iq

Abstract: *The main concept of this work is to generate and investigate second lower and upper approximations of* ƕ *combined edges systems as well as second lower and upper approximations operators of* ƕ *using incident edges systems (as opposed to second lower and upper approximations of* ƕ *using non incident edges systems). In addition, we utilized Using (incident, non-incident, and combined) edges systems, the second accuracy of the approximation of a subund. g.* $ƕ \subseteq \Omega$ *is explored, along with some of its properties.*

Keywords— graph, topology, accuracy and lower and upper approximations.

1. Introduction:

The field of graph theory that deals with combinatorics has close ties to topology, group theory, and matrix theory. The second argument is that when many concepts are experimentally represented by graphs, they will be highly helpful in practice. Mathematics' topological graph theory has a wide range of theoretical and practical applications [1, 2, 3, 4, 5, 8, and 9]. We predicted that topological graph structure will play a crucial role in bridging the gap between topology and applications. For all concepts and notation relating to graph theory, we cite Harary [6], and for all terms and notation relating to topology, we cite Moller [7]. These graph theory essential concepts are listed in [10]. A undirected graph or graph is pair $\Omega = (Ʊ(\Omega), \mathcal{E}(\Omega))$ where $Ʊ(\Omega)$ is a non-empty set whose elements are called points or vertices (called vertex set) and $\mathcal{E}(\Omega)$ is the set of unordered pairs of elements of $Ʊ(\Omega)$ (called edge set). An edge of a graph that joins a vertex to itself is called a loop. If two edges of a graph are joined by an vertex then these edges are called the edges ϱ incident with the edges $ϱ_1$. the set of ϱ is $\{ϱ_1 \in \mathcal{E}(\Omega): ϱ_1 \text{incident with } ϱ\}$ and the edges ϱ non incident with the edges $ϱ_1$. the set of ϱ is $\{ϱ_1 \in \mathcal{E}(\Omega): ϱ_1 \text{nonincident with } ϱ\}$. A sub graph of a graph Ω is a graph each of whose vertices belong to $Ʊ(\Omega)$ and each of whose edges belong to $\mathcal{E}(\Omega)$. An empty graph if the vertices set and edge set is empty. A degree of a vertex Ʋ in a graph Ω is the number of edges of Ω incident with Ʋ. Let $\Omega = (Ʊ(\Omega), \mathcal{E}(\Omega))$ be und. g. and a edge $ϱ \in \mathcal{E}(\Omega)$.The incident edges set of ϱ is denoted by $I\mathcal{E}(ϱ)$ and defined by $I\mathcal{E}(ϱ) = \{ϱ_1 \in \mathcal{E}(\Omega): ϱ_1 \text{incident with } ϱ\}$ and The non-incident edges set of ϱ is denoted by $NI\mathcal{E}(ϱ)$ and defined by $NI\mathcal{E}(ϱ) = \{ϱ_1 \in \mathcal{E}(\Omega): ϱ_1 \text{nonincident with } ϱ\}$. an und. g., $\Omega = (Ʊ(\Omega), \mathcal{E}(\Omega))$ the incident edges system (resp. non incident edges system) of a edge $ϱ \in \mathcal{E}(\Omega)$ is denoted by $I\mathcal{E}S(ϱ)$ (resp. $NI\mathcal{E}S(ϱ)$) and defined by : $I\mathcal{E}S(ϱ) = \{I\mathcal{E}(ϱ)\}$ (resp. $NI\mathcal{E}S(ϱ) = \{NI\mathcal{E}(ϱ)\}$). The Combined edges System of a edge $ϱ \in \mathcal{E}(\Omega)$ is denoted by $C\mathcal{E}S(ϱ)$ and defined by $C\mathcal{E}S(ϱ) = \{I\mathcal{E}S(ϱ), NI\mathcal{E}S(ϱ)\}$. A edge $ϱ \in \mathcal{E}(\Omega)$ is called isolated edge if $\{ϱ \in \mathcal{E}(\Omega);\ \exists C\mathcal{E}S(ϱ) \cap (\mathcal{E}(ƕ) - \{ϱ\}) = \phi\}$ A star graph of order n (denoted by S_n) is a graph that all edges are incident to each other. Let $\Omega = (Ʊ(\Omega), \mathcal{E}(\Omega))$ be an und. g. and suppose that $Þ_c: \mathcal{E}(\Omega) \to P(P(\mathcal{E}(\Omega)))$ is a mapping which assigns for each ϱ in $\mathcal{E}(\Omega)$ its Combined edges System in $P(P(\mathcal{E}(\Omega)))$. The pair $(\Omega, Þ_c)$is called the C-space.

2. Second New Approximation Operators Using Combined Edges Systems:

This section aims to present a set-theoretic foundation for granular computing with Combined edges systems. Considering the generalized approximation space $Ʒ = (Ʊ(\Omega), \mathcal{E}(\Omega))$, Using Combined edges systems, we developed a novel description of the lower and upper approximation operators. (incident, non incident, and Combined) edges systems are used to create the approximations. There is a comparison of these three approaches.

Definition 2.1: Let $Ʒ = (Ʊ(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space and $ƕ \subseteq \Omega$. Then

a) The second lower and upper approximations of ƕ using incident edges systems are denoted by $L_i^2(\mathcal{E}(ƕ))$ and $U_i^2(\mathcal{E}(ƕ))$ and defined by:

$$L_i^2(\mathcal{E}(ƕ)) = \{ϱ \in \mathcal{E}(\Omega)\,;\, I\mathcal{E}(ϱ) \subseteq \mathcal{E}(ƕ)\},$$
$$U_i^2(\mathcal{E}(ƕ)) = \{ϱ \in \mathcal{E}(\Omega)\,;\, I\mathcal{E}(ϱ) \cap \mathcal{E}(ƕ) \neq \emptyset\},$$

b) The second lower and upper approximations of ƕ using non incident edges systems are denoted by $L_n^2(\mathcal{E}(ƕ))$ and $U_n^2(\mathcal{E}(ƕ))$ and defined by:

$$L_n^2(\mathcal{E}(ƕ)) = \{ϱ \in \mathcal{E}(\Omega)\,;\, NI\mathcal{E}(ϱ) \subseteq \mathcal{E}(ƕ)\},$$
$$U_n^2(V(H)) = \{ϱ \in \mathcal{E}(\Omega)\,;\, NI\mathcal{E}(ϱ) \cap \mathcal{E}(ƕ) \neq \emptyset\},$$

c) The second lower and upper approximations of ƕ using Combined edges systems are denoted by $L_c^2(\mathcal{E}(ƕ))$ and $U_c^2(\mathcal{E}(ƕ))$ and defined by:

$$L_c^2(\mathcal{E}(\text{ƕ})) = \{\varrho \in \mathcal{E}(\Omega) \,;\, for\ some\ C\mathcal{E}(\varrho) \subseteq \mathcal{E}(\text{ƕ})\},$$
$$U_c^2(\mathcal{E}(\text{ƕ})) = \{\varrho \in \mathcal{E}(\Omega) \,;\, for\ all\ C\mathcal{E}(\varrho) \cap \mathcal{E}(\text{ƕ}) \neq \emptyset\}.$$

Definition 2.2: Let $\mathfrak{Z} = (\mho(\Omega), \mathcal{E}(\Omega))$ be a generalization approximation space and $\text{ƕ} \subseteq \Omega$. Then

a) The second boundary, positive and negative regions of ƕ using incident edges systems are denoted by $Bd_i^2(\mathcal{E}(\text{ƕ})), POS_i^2(\mathcal{E}(\text{ƕ}))$ and $NEG_i^2(\mathcal{E}(\text{ƕ}))$ and defined by:
$$Bd_i^2(\mathcal{E}(\text{ƕ})) = U_i^2(\mathcal{E}(\text{ƕ})) - L_i^2(\mathcal{E}(\text{ƕ})),$$
$$POS_i^2(\mathcal{E}(\text{ƕ})) = L_i^2(\mathcal{E}(\text{ƕ})),$$
$$NEG_i^2(\mathcal{E}(\text{ƕ})) = \mathcal{E}(\Omega) - U_i^2(\mathcal{E}(\text{ƕ})),$$

b) The second boundary, positive and negative regions of ƕ using non incident edges systems are denoted by $Bd_n^2(\mathcal{E}(\text{ƕ})), POS_n^2(\mathcal{E}(\text{ƕ}))$ and $NEG_n^2(\mathcal{E}(\text{ƕ}))$ and defined by:
$$Bd_n^2(\mathcal{E}(\text{ƕ})) = U_n^2(\mathcal{E}(\text{ƕ})) - L_n^2(\mathcal{E}(\text{ƕ})),$$
$$POS_n^2(\mathcal{E}(\text{ƕ})) = L_n^2(\mathcal{E}(\text{ƕ})),$$
$$NEG_n^2(\mathcal{E}(\text{ƕ})) = \mathcal{E}(\Omega) - U_n^2(\mathcal{E}(\text{ƕ})),$$

c) The second boundary, positive and negative regions of ƕ using Combined edges systems are denoted by $Bd_c^2(\mathcal{E}(\text{ƕ})), POS_c^2(\mathcal{E}(\text{ƕ}))$ and $NEG_c^2(\mathcal{E}(\text{ƕ}))$ and defined by:
$$Bd_c^2(\mathcal{E}(\text{ƕ})) = U_c^2(\mathcal{E}(\text{ƕ})) - L_c^2(\mathcal{E}(\text{ƕ})),$$
$$POS_c^2(\mathcal{E}(\text{ƕ})) = L_c^2(\mathcal{E}(\text{ƕ})),$$
$$NEG_c^2(\mathcal{E}(\text{ƕ})) = \mathcal{E}(\Omega) - U_c^2(\mathcal{E}(\text{ƕ})).$$

Definition 2.3 : Let $\mathfrak{Z} = (\mho(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space. The second accuracy of the approximation of a sub und. g. $\text{ƕ} \subseteq \Omega$ using (incident, non incident, and Combined) edges systems are denoted by($\zeta_i^2(\mathcal{E}(\text{ƕ})), \zeta_n^2(\mathcal{E}(\text{ƕ}))$ and $\zeta_c^2(\mathcal{E}(\text{ƕ}))$ and defined respectively by:
$$\zeta_i^2(\mathcal{E}(\text{ƕ})) = 1 - \frac{|Bd_i^2(\mathcal{E}(\text{ƕ}))|}{|\mathcal{E}(\Omega)|},$$
$$\zeta_n^2(\mathcal{E}(\text{ƕ})) = 1 - \frac{|Bd_n^2(\mathcal{E}(\text{ƕ}))|}{|\mathcal{E}(\Omega)|},$$
$$\zeta_c^2(\mathcal{E}(\text{ƕ})) = 1 - \frac{|Bd_c^2(\mathcal{E}(\text{ƕ}))|}{|\mathcal{E}(\Omega)|}.$$

It is obvious that $0 \le \zeta_i^2(\mathcal{E}(\text{ƕ})) \le 1$, $0 \le \zeta_n^2(\mathcal{E}(\text{ƕ})) \le 1$ and $0 \le \zeta_c^2(\mathcal{E}(\text{ƕ})) \le 1$. Moreover, if $\zeta_i^2(\mathcal{E}(\text{ƕ})) = 1$ or $\zeta_n^2(\mathcal{E}(\text{ƕ})) = 1$ or $\zeta_c^2(\mathcal{E}(\text{ƕ})) = 1$ then ƕ is called ƕ-definable (ƕ-exact) und. g. otherwise, it is called ƕ-rough.

Example 2.4: Let $\Omega = (\mho(\Omega), \mathcal{E}(\Omega))$ such that $\mho(\Omega) = \{\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4\}$ and $\mathcal{E}(\Omega) = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5\}$.

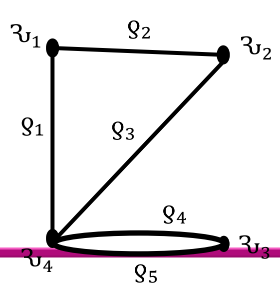

Figure 2.1: und. g. Ω given in Example (2.4).

We get:
$I\mathcal{E}(\varrho_1) = \{\varrho_2, \varrho_3, \varrho_4, \varrho_5\}, I\mathcal{E}(\varrho_2) = \{\varrho_1, \varrho_3\}, I\mathcal{E}(\varrho_3) = \{\varrho_1, \varrho_2, \varrho_4, \varrho_5\}, I\mathcal{E}(\varrho_4) = \{\varrho_1, \varrho_3, \varrho_5\}, I\mathcal{E}(\varrho_5) = \{\varrho_1, \varrho_3, \varrho_4\}$.
Also we have
$NI\mathcal{E}(\varrho_1) = \phi, NI\mathcal{E}(\varrho_2) = \{\varrho_4, \varrho_5\}, NI\mathcal{E}(\varrho_3) = \phi, NI\mathcal{E}(\varrho_4) = \{\varrho_2\}, NI\mathcal{E}(\varrho_5) = \{\varrho_2\}$. Then we obtain $C\mathcal{E}(\varrho_1) = \{\{\varrho_2, \varrho_3, \varrho_4, \varrho_5\}, \phi\}, C\mathcal{E}(\varrho_2) = \{\{\varrho_1, \varrho_3\}, \{\varrho_4, \varrho_5\}\}, C\mathcal{E}(\varrho_3) = \{\{\varrho_1, \varrho_2, \varrho_4, \varrho_5\}, \phi\}, C\mathcal{E}(\varrho_4) = \{\{\varrho_1, \varrho_3, \varrho_5\}, \{\varrho_2\}\}, C\mathcal{E}(\varrho_5) = \{\{\varrho_1, \varrho_3, \varrho_4\}, \{\varrho_2\}\}$.
Accordingly ,can be obtain the following table

Table 2.1: $L_i^2(\mathcal{E}(\text{ƕ}))$, $L_n^2(\mathcal{E}(\text{ƕ}))$ and $L_c^2(\mathcal{E}(\text{ƕ}))$ for all $\text{ƕ} \subseteq \Omega$

$\mathcal{E}(\text{ƕ})$	$L_i^2(\mathcal{E}(\text{ƕ}))$	$L_n^2(\mathcal{E}(\text{ƕ}))$	$L_c^2(\mathcal{E}(\text{ƕ}))$
$\{\varrho_1\}$	ϕ	$\{\varrho_1, \varrho_3\}$	$\{\varrho_1, \varrho_3\}$
$\{\varrho_2\}$	ϕ	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$
$\{\varrho_3\}$	ϕ	$\{\varrho_1, \varrho_3\}$	$\{\varrho_1, \varrho_3\}$
$\{\varrho_4\}$	ϕ	$\{\varrho_1, \varrho_3\}$	$\{\varrho_1, \varrho_3\}$
$\{\varrho_5\}$	ϕ	$\{\varrho_1, \varrho_3\}$	$\{\varrho_1, \varrho_3\}$
$\{\varrho_1, \varrho_2\}$	ϕ	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$
$\{\varrho_1, \varrho_3\}$	$\{\varrho_2\}$	$\{\varrho_1, \varrho_3\}$	$\{\varrho_1, \varrho_2, \varrho_3\}$
$\{\varrho_1, \varrho_4\}$	ϕ	$\{\varrho_1, \varrho_3\}$	$\{\varrho_1, \varrho_3\}$
$\{\varrho_1, \varrho_5\}$	ϕ	$\{\varrho_1, \varrho_3\}$	$\{\varrho_1, \varrho_3\}$
$\{\varrho_2, \varrho_3\}$	ϕ	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$
$\{\varrho_2, \varrho_4\}$	ϕ	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$
$\{\varrho_2, \varrho_5\}$	ϕ	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$
$\{\varrho_3, \varrho_4\}$	ϕ	$\{\varrho_1, \varrho_3\}$	$\{\varrho_1, \varrho_3\}$
$\{\varrho_3, \varrho_5\}$	ϕ	$\{\varrho_1, \varrho_3\}$	$\{\varrho_1, \varrho_3\}$
$\{\varrho_4, \varrho_5\}$	ϕ	$\{\varrho_1, \varrho_2, \varrho_3\}$	$\{\varrho_1, \varrho_2, \varrho_3\}$
$\{\varrho_1, \varrho_2, \varrho_3\}$	$\{\varrho_2\}$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\mathcal{E}(\Omega)$
$\{\varrho_1, \varrho_2, \varrho_4\}$	ϕ	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$
$\{\varrho_1, \varrho_2, \varrho_5\}$	ϕ	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$
$\{\varrho_2, \varrho_3, \varrho_4\}$	ϕ	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$
$\{\varrho_2, \varrho_3, \varrho_5\}$	ϕ	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$
$\{\varrho_3, \varrho_4, \varrho_1\}$	$\{\varrho_2, \varrho_5\}$	$\{\varrho_1, \varrho_3\}$	$\{\varrho_1, \varrho_2, \varrho_3, \varrho_5\}$
$\{\varrho_3, \varrho_4, \varrho_5\}$	ϕ	$\{\varrho_1, \varrho_2, \varrho_3\}$	$\{\varrho_1, \varrho_2, \varrho_3\}$
$\{\varrho_4, \varrho_5, \varrho_1\}$	ϕ	$\{\varrho_1, \varrho_2, \varrho_3\}$	$\{\varrho_1, \varrho_2, \varrho_3\}$
$\{\varrho_4, \varrho_5, \varrho_2\}$	ϕ	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\{\varrho_1, \varrho_3, \varrho_5\}$	$\{\varrho_2, \varrho_4\}$	$\{\varrho_1, \varrho_3\}$	$\{\varrho_1, \varrho_2, \varrho_3, \varrho_4\}$
$\{\varrho_1, \varrho_2, \varrho_3, \varrho_4\}$	$\{\varrho_2, \varrho_5\}$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\mathcal{E}(\Omega)$
$\{\varrho_1, \varrho_2, \varrho_3, \varrho_5\}$	$\{\varrho_2, \varrho_4\}$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\mathcal{E}(\Omega)$
$\{\varrho_2, \varrho_3, \varrho_4, \varrho_5\}$	$\{\varrho_1\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\{\varrho_2, \varrho_4, \varrho_5\}$	$\{\varrho_1, \varrho_2, \varrho_3\}$	$\mathcal{E}(\Omega)$
$\{\varrho_1, \varrho_2, \varrho_4, \varrho_5\}$	$\{\varrho_3\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
ϕ	ϕ	ϕ	$\{\varrho_1, \varrho_3\}$

Table 2.2: $U_i^2(\mathcal{E}(\text{ƕ}))$, $U_n^2(\mathcal{E}(\text{ƕ}))$ and $U_c^2(\mathcal{E}(\text{ƕ}))$ for all $\text{ƕ} \subseteq \Omega$.

$\mathcal{E}(\text{ƕ})$	$U_i^2(\mathcal{E}(\text{ƕ}))$	$U_n^2(\mathcal{E}(\text{ƕ}))$	$U_c^2(\mathcal{E}(\text{ƕ}))$
$\{\varrho_1\}$	$\{\varrho_2, \varrho_3, \varrho_4, \varrho_5\}$	ϕ	ϕ
$\{\varrho_2\}$	$\{\varrho_1, \varrho_3\}$	$\{\varrho_4, \varrho_5\}$	ϕ
$\{\varrho_3\}$	$\{\varrho_1, \varrho_2, \varrho_4, \varrho_5\}$	ϕ	ϕ
$\{\varrho_4\}$	$\{\varrho_1, \varrho_3, \varrho_5\}$	$\{\varrho_2\}$	ϕ

International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 7 Issue 1, January - 2023, Pages: 60-66

$\{\varrho_5\}$	$\{\varrho_1,\varrho_3,\varrho_4\}$	$\{\varrho_2\}$	ϕ
$\{\varrho_1,\varrho_2\}$	$\mathcal{E}(\Omega)$	$\{\varrho_4,\varrho_5\}$	$\{\varrho_4,\varrho_5\}$
$\{\varrho_1,\varrho_3\}$	$\mathcal{E}(\Omega)$	ϕ	ϕ
$\{\varrho_1,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2\}$	$\{\varrho_2\}$
$\{\varrho_1,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2\}$	$\{\varrho_2\}$
$\{\varrho_2,\varrho_3\}$	$\mathcal{E}(\Omega)$	$\{\varrho_4,\varrho_5\}$	$\{\varrho_4,\varrho_5\}$
$\{\varrho_2,\varrho_4\}$	$\{\varrho_1,\varrho_3,\varrho_5\}$	$\{\varrho_2,\varrho_4,\varrho_5\}$	$\{\varrho_5\}$
$\{\varrho_2,\varrho_5\}$	$\{\varrho_1,\varrho_3,\varrho_4\}$	$\{\varrho_2,\varrho_4,\varrho_5\}$	$\{\varrho_4\}$
$\{\varrho_3,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2\}$	$\{\varrho_2\}$
$\{\varrho_3,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2\}$	$\{\varrho_2\}$
$\{\varrho_4,\varrho_5\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_2\}$	ϕ
$\{\varrho_1,\varrho_2,\varrho_3\}$	$\mathcal{E}(\Omega)$	$\{\varrho_4,\varrho_5\}$	$\{\varrho_4,\varrho_5\}$
$\{\varrho_1,\varrho_2,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2,\varrho_4,\varrho_5\}$	$\{\varrho_2,\varrho_4,\varrho_5\}$
$\{\varrho_1,\varrho_2,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2,\varrho_4,\varrho_5\}$	$\{\varrho_2,\varrho_4,\varrho_5\}$
$\{\varrho_2,\varrho_3,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2,\varrho_4,\varrho_5\}$	$\{\varrho_2,\varrho_4,\varrho_5\}$
$\{\varrho_2,\varrho_3,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2,\varrho_4,\varrho_5\}$	$\{\varrho_2,\varrho_4,\varrho_5\}$
$\{\varrho_3,\varrho_4,\varrho_1\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2\}$	$\{\varrho_2\}$
$\{\varrho_3,\varrho_4,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2\}$	$\{\varrho_2\}$
$\{\varrho_4,\varrho_5,\varrho_1\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2\}$	$\{\varrho_2\}$
$\{\varrho_4,\varrho_5,\varrho_2\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_2,\varrho_4,\varrho_5\}$	$\{\varrho_4,\varrho_5\}$
$\{\varrho_1,\varrho_3,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2\}$	$\{\varrho_2\}$
$\{\varrho_1,\varrho_2,\varrho_3,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2,\varrho_4,\varrho_5\}$	$\{\varrho_2,\varrho_4,\varrho_5\}$
$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2,\varrho_4,\varrho_5\}$	$\{\varrho_2,\varrho_4,\varrho_5\}$
$\{\varrho_2,\varrho_3,\varrho_4,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2,\varrho_4,\varrho_5\}$	$\{\varrho_2,\varrho_4,\varrho_5\}$
$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2\}$	$\{\varrho_2\}$
$\{\varrho_1,\varrho_2,\varrho_4,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2,\varrho_4,\varrho_5\}$	$\{\varrho_2,\varrho_4,\varrho_5\}$
$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\{\varrho_2,\varrho_4,\varrho_5\}$	$\{\varrho_2,\varrho_4,\varrho_5\}$
ϕ	ϕ	ϕ	ϕ

Table 2.3: $Bd_{\mathrm{i}}^2(\mathcal{E}(ƕ))$, $Bd_{\mathrm{n}}^2(\mathcal{E}(ƕ))$ and $Bd_{\mathrm{c}}^2(\mathcal{E}(ƕ))$ for all $ƕ \subseteq \Omega$.

$\mathcal{E}(ƕ)$	$Bd_{\mathrm{i}}^2(\mathcal{E}(ƕ))$	$Bd_{\mathrm{n}}^2(\mathcal{E}(ƕ))$	$Bd_{\mathrm{c}}^2(\mathcal{E}(ƕ))$
$\{\varrho_1\}$	$\{\varrho_2,\varrho_3,\varrho_4,\varrho_5\}$	ϕ	ϕ
$\{\varrho_2\}$	$\{\varrho_1,\varrho_3\}$	ϕ	ϕ
$\{\varrho_3\}$	$\{\varrho_1,\varrho_2,\varrho_4,\varrho_5\}$	ϕ	ϕ
$\{\varrho_4\}$	$\{\varrho_1,\varrho_3,\varrho_5\}$	$\{\varrho_2\}$	ϕ
$\{\varrho_5\}$	$\{\varrho_1,\varrho_3,\varrho_4\}$	$\{\varrho_2\}$	ϕ
$\{\varrho_1,\varrho_2\}$	$\mathcal{E}(\Omega)$	ϕ	ϕ
$\{\varrho_1,\varrho_3\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	ϕ	ϕ
$\{\varrho_1,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2\}$	$\{\varrho_2\}$
$\{\varrho_1,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2\}$	$\{\varrho_2\}$
$\{\varrho_2,\varrho_3\}$	$\mathcal{E}(\Omega)$	ϕ	ϕ
$\{\varrho_2,\varrho_4\}$	$\{\varrho_1,\varrho_3,\varrho_5\}$	$\{\varrho_2\}$	ϕ
$\{\varrho_2,\varrho_5\}$	$\{\varrho_1,\varrho_3,\varrho_4\}$	$\{\varrho_2\}$	ϕ
$\{\varrho_3,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2\}$	$\{\varrho_2\}$
$\{\varrho_3,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2\}$	$\{\varrho_2\}$
$\{\varrho_4,\varrho_5\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	ϕ	ϕ
$\{\varrho_1,\varrho_2,\varrho_3\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	ϕ	ϕ
$\{\varrho_1,\varrho_2,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2\}$	$\{\varrho_2\}$
$\{\varrho_1,\varrho_2,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2\}$	$\{\varrho_2\}$
$\{\varrho_2,\varrho_3,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2\}$	$\{\varrho_2\}$
$\{\varrho_2,\varrho_3,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2\}$	$\{\varrho_2\}$
$\{\varrho_3,\varrho_4,\varrho_1\}$	$\{\varrho_1,\varrho_3,\varrho_4\}$	$\{\varrho_2\}$	ϕ
$\{\varrho_3,\varrho_4,\varrho_5\}$	$\mathcal{E}(\Omega)$	ϕ	ϕ
$\{\varrho_4,\varrho_5,\varrho_1\}$	$\mathcal{E}(\Omega)$	ϕ	ϕ
$\{\varrho_4,\varrho_5,\varrho_2\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	ϕ	ϕ
$\{\varrho_1,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_3,\varrho_5\}$	$\{\varrho_2\}$	ϕ
$\{\varrho_1,\varrho_2,\varrho_3,\varrho_4\}$	$\{\varrho_1,\varrho_3,\varrho_4\}$	$\{\varrho_2\}$	ϕ
$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_3,\varrho_4\}$	$\{\varrho_2\}$	ϕ
$\{\varrho_2,\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_2,\varrho_3,\varrho_4,\varrho_5\}$	ϕ	ϕ
$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_1,\varrho_3\}$	ϕ	ϕ
$\{\varrho_1,\varrho_2,\varrho_4,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_4,\varrho_5\}$	ϕ	ϕ
$\mathcal{E}(\Omega)$	ϕ	ϕ	ϕ
ϕ	ϕ	ϕ	ϕ

Table 2.2: $NEG_{\mathrm{i}}^2(\mathcal{E}(ƕ))$, $NEG_{\mathrm{n}}^2(\mathcal{E}(ƕ))$ and $NEG_{\mathrm{c}}^2(\mathcal{E}(ƕ))$ for all $ƕ \subseteq \Omega$.

$\mathcal{E}(ƕ)$	$NEG_{\mathrm{i}}^2(\mathcal{E}(ƕ))$	$NEG_{\mathrm{n}}^2(\mathcal{E}(ƕ))$	$\mathcal{E}(\Omega)$
$\{\varrho_1\}$	$\{\varrho_1\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3\}$
$\{\varrho_2\}$	$\{\varrho_2,\varrho_4,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3\}$	$\mathcal{E}(\Omega)$
$\{\varrho_3\}$	$\{\varrho_3\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$
$\{\varrho_4\}$	$\{\varrho_2,\varrho_4\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$
$\{\varrho_5\}$	$\{\varrho_2,\varrho_5\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3\}$
$\{\varrho_1,\varrho_2\}$	ϕ	$\{\varrho_1,\varrho_2,\varrho_3\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_4\}$
$\{\varrho_1,\varrho_3\}$	ϕ	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_1,\varrho_4\}$	ϕ	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$
$\{\varrho_1,\varrho_5\}$	ϕ	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$
$\{\varrho_2,\varrho_3\}$	ϕ	$\{\varrho_1,\varrho_2,\varrho_3\}$	$\mathcal{E}(\Omega)$
$\{\varrho_2,\varrho_4\}$	$\{\varrho_2,\varrho_4\}$	$\{\varrho_1,\varrho_3\}$	$\{\varrho_1,\varrho_2,\varrho_3\}$
$\{\varrho_2,\varrho_5\}$	$\{\varrho_2,\varrho_5\}$	$\{\varrho_1,\varrho_3\}$	$\{\varrho_1,\varrho_3\}$
$\{\varrho_3,\varrho_4\}$	ϕ	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_1,\varrho_3\}$
$\{\varrho_3,\varrho_5\}$	ϕ	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_1,\varrho_3\}$
$\{\varrho_4,\varrho_5\}$	$\{\varrho_2\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_1,\varrho_3\}$
$\{\varrho_1,\varrho_2,\varrho_3\}$	ϕ	$\{\varrho_1,\varrho_2,\varrho_3\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$
$\{\varrho_1,\varrho_2,\varrho_4\}$	ϕ	$\{\varrho_1,\varrho_3\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$
$\{\varrho_1,\varrho_2,\varrho_5\}$	ϕ	$\{\varrho_1,\varrho_3\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$
$\{\varrho_2,\varrho_3,\varrho_4\}$	ϕ	$\{\varrho_1,\varrho_3\}$	$\{\varrho_1,\varrho_2,\varrho_3\}$
$\{\varrho_2,\varrho_3,\varrho_5\}$	ϕ	$\{\varrho_1,\varrho_3\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$
$\{\varrho_3,\varrho_4,\varrho_1\}$	ϕ	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_1,\varrho_3\}$
$\{\varrho_3,\varrho_4,\varrho_5\}$	ϕ	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_1,\varrho_3\}$
$\{\varrho_4,\varrho_5,\varrho_1\}$	ϕ	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_1,\varrho_3\}$
$\{\varrho_4,\varrho_5,\varrho_2\}$	$\{\varrho_2\}$	$\{\varrho_1,\varrho_3\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$
$\{\varrho_1,\varrho_3,\varrho_5\}$	ϕ	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_1,\varrho_3\}$
$\{\varrho_1,\varrho_2,\varrho_3,\varrho_4\}$	ϕ	$\{\varrho_1,\varrho_3\}$	$\mathcal{E}(\Omega)$
$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	ϕ	$\{\varrho_1,\varrho_3\}$	$\{\varrho_1,\varrho_2,\varrho_3\}$
$\{\varrho_2,\varrho_3,\varrho_4,\varrho_5\}$	ϕ	$\{\varrho_1,\varrho_3\}$	$\mathcal{E}(\Omega)$
$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	ϕ	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$
$\{\varrho_1,\varrho_2,\varrho_4,\varrho_5\}$	ϕ	$\{\varrho_1,\varrho_3\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$
$\mathcal{E}(\Omega)$	ϕ	$\{\varrho_1,\varrho_3\}$	$\{\varrho_1,\varrho_3\}$
ϕ	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$

Table 2.4: $\zeta_{\mathrm{i}}^2(\mathcal{E}(ƕ))$, $\zeta_{\mathrm{n}}^2(\mathcal{E}(ƕ))$ and $\zeta_{\mathrm{c}}^2(\mathcal{E}(ƕ))$ for all $ƕ \subseteq \Omega$.

$\mathcal{E}(ƕ)$	$\zeta_{\mathrm{i}}^2(\mathcal{E}(ƕ))$	$\zeta_{\mathrm{n}}^2(\mathcal{E}(ƕ))$	$\zeta_{\mathrm{c}}^2(\mathcal{E}(ƕ))$
$\{\varrho_1\}$	1/5	1	1
$\{\varrho_2\}$	3/5	1	1
$\{\varrho_3\}$	1/5	1	1

$\{\varrho_4\}$	2/5	4/5	1
$\{\varrho_5\}$	2/5	4/5	1
$\{\varrho_1,\varrho_2\}$	0	1	1
$\{\varrho_1,\varrho_3\}$	1/5	1	1
$\{\varrho_1,\varrho_4\}$	0	4/5	4/5
$\{\varrho_1,\varrho_5\}$	0	4/5	4/5
$\{\varrho_2,\varrho_3\}$	0	1	1
$\{\varrho_2,\varrho_4\}$	2/5	4/5	1
$\{\varrho_2,\varrho_5\}$	2/5	4/5	1
$\{\varrho_3,\varrho_4\}$	0	4/5	4/5
$\{\varrho_3,\varrho_5\}$	0	4/5	4/5
$\{\varrho_4,\varrho_5\}$	1/5	1	1
$\{\varrho_1,\varrho_2,\varrho_3\}$	1/5	1	1
$\{\varrho_1,\varrho_2,\varrho_4\}$	0	4/5	4/5
$\{\varrho_1,\varrho_2,\varrho_5\}$	0	4/5	4/5
$\{\varrho_2,\varrho_3,\varrho_4\}$	0	4/5	4/5
$\{\varrho_2,\varrho_3,\varrho_5\}$	0	4/5	4/5
$\{\varrho_3,\varrho_4,\varrho_1\}$	2/5	4/5	1
$\{\varrho_3,\varrho_4,\varrho_5\}$	0	1	1
$\{\varrho_4,\varrho_5,\varrho_1\}$	0	1	1
$\{\varrho_4,\varrho_5,\varrho_2\}$	1/5	1	1
$\{\varrho_1,\varrho_3,\varrho_5\}$	2/5	4/5	1
$\{\varrho_1,\varrho_2,\varrho_3,\varrho_4\}$	2/5	4/5	1
$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	2/5	4/5	1
$\{\varrho_2,\varrho_3,\varrho_4,\varrho_5\}$	1/5	1	1
$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	3/5	1	1
$\{\varrho_1,\varrho_2,\varrho_4,\varrho_5\}$	1/5	1	1
$\mathcal{E}(\Omega)$	1	1	1
ϕ	1	1	1

Theorem 2.5. Let $\mathfrak{Z}=(\mho(\Omega),\mathcal{E}(\Omega))$ be a generalized approximation space and $ƕ\subseteq\Omega$. Then

(1) $L_c^2(\mathcal{E}(ƕ))=L_i^2(\mathcal{E}(ƕ))\cup L_n^2(\mathcal{E}(ƕ))$.
(2) $U_c^2(\mathcal{E}(ƕ))=U_i^2(\mathcal{E}(ƕ))\cap U_n^2(\mathcal{E}(ƕ))$.
(3) $Bd_c^2(\mathcal{E}(ƕ))=Bd_i^2(\mathcal{E}(ƕ))\cap Bd_n^2(\mathcal{E}(ƕ))$.
(4) $NEG_c^2(\mathcal{E}(ƕ))=NEG_i^2(\mathcal{E}(ƕ))\cup NEG_n^2(\mathcal{E}(ƕ))$.
(5) $\zeta_c^2(\mathcal{E}(ƕ))\geq max\{\zeta_i^2(\mathcal{E}(ƕ)),\zeta_n^2(\mathcal{E}(ƕ))\}$.

Proof:

(1) Let $\varrho\in(L_i^2(\mathcal{E}(ƕ))\cup L_n^2(\mathcal{E}(ƕ)))\Leftrightarrow\varrho\in L_i^2(\mathcal{E}(ƕ))\vee\varrho\in L_n^2(\mathcal{E}(ƕ))\Leftrightarrow I\mathcal{E}(\varrho)\subseteq\mathcal{E}(ƕ)\vee NI\mathcal{E}(\varrho)\subseteq\mathcal{E}(ƕ)\Leftrightarrow\exists\ C\mathcal{E}(\varrho)$ such that $C\mathcal{E}(\varrho)\subseteq\mathcal{E}(ƕ)\Leftrightarrow\varrho\in L_c^2(\mathcal{E}(ƕ))$,hence $L_c^2(\mathcal{E}(ƕ))=L_i^2(\mathcal{E}(ƕ))\cup L_n^2(\mathcal{E}(ƕ))$.

(2) Let $\varrho\in\left(U_i^2(\mathcal{E}(ƕ))\cap U_n^2(\mathcal{E}(ƕ))\right)\Leftrightarrow\varrho\in U_i^2\left(\mathcal{E}(ƕ)\right)\wedge\varrho\in U_n^2(\mathcal{E}(ƕ))\Leftrightarrow(I\mathcal{E}(\varrho)\cap\mathcal{E}(ƕ)\neq\emptyset)\wedge(NI\mathcal{E}(\varrho)\cap\mathcal{E}(ƕ)\neq\emptyset)\Leftrightarrow$ for all $C\mathcal{E}(\varrho)$, $C\mathcal{E}(\varrho)\cap\mathcal{E}(ƕ)\neq\emptyset\Leftrightarrow\varrho\in U_c^2(\mathcal{E}(ƕ))$ so, $U_c^2(\mathcal{E}(ƕ))=U_i^2(\mathcal{E}(ƕ))\cap U_n^2(\mathcal{E}(ƕ))$.

(3) Let $\varrho\in Bd_c^2\left(\mathcal{E}(ƕ)\right)\Rightarrow\varrho\in U_c^2(\mathcal{E}(ƕ))\wedge\varrho\notin L_c^2(\mathcal{E}(ƕ))$ since $\varrho\in U_c^2(\mathcal{E}(ƕ))$ by Theorem $(2.5(2))$ we get $\varrho\in\left(U_i^2(\mathcal{E}(ƕ))\cap U_n^2(\mathcal{E}(ƕ))\right)\Rightarrow\varrho\in U_i^2(\mathcal{E}(ƕ))$ and $\varrho\in U_n^2(\mathcal{E}(ƕ))$. Since $\varrho\notin L_c^2(\mathcal{E}(ƕ))$ by Theorem $(2.5(1))$ we get $\varrho\notin\left(L_i^2(\mathcal{E}(ƕ))\cup L_n^2(\mathcal{E}(ƕ))\right)\Rightarrow\varrho\notin L_i^2(\mathcal{E}(ƕ))$ and $\varrho\notin L_n^2(\mathcal{E}(ƕ))$ and hence $\varrho\in Bd_i^2\left(\mathcal{E}(ƕ)\right)$ and $\varrho\in Bd_n^2\left(\mathcal{E}(ƕ)\right)\Rightarrow\varrho\in\left(Bd_i^2(\mathcal{E}(ƕ))\cap Bd_n^2(\mathcal{E}(ƕ))\right)$. Conversely, $\varrho\in\left(Bd_i^2(\mathcal{E}(ƕ))\cap Bd_n^2(\mathcal{E}(ƕ))\right)\Rightarrow\varrho\in Bd_i^2(\mathcal{E}(ƕ))$ and $\varrho\in Bd_n^2(\mathcal{E}(ƕ))$ since $\varrho\in Bd_i^2(\mathcal{E}(ƕ))\Rightarrow\varrho\in U_i^2(\mathcal{E}(ƕ))$ and $\varrho\notin L_i^2(\mathcal{E}(ƕ))$ and since $\varrho\in Bd_n^2(\mathcal{E}(ƕ))\Rightarrow\varrho\in U_n^2(\mathcal{E}(ƕ))$ and $\varrho\notin L_n^2(\mathcal{E}(ƕ))$ and hence $\varrho\in\left(U_i^2(\mathcal{E}(ƕ))\cap U_n^2(\mathcal{E}(ƕ))\right)$ by Theorem $(2.5(2))$ we get $\varrho\in U_c^2(\mathcal{E}(ƕ))$ and $\varrho\notin\left(L_i^2(\mathcal{E}(ƕ))\cup L_n^2(\mathcal{E}(ƕ))\right)$ by Theorem $(2.5(1))$ we get $\varrho\notin L_c^2(\mathcal{E}(ƕ))$ then $\varrho\in Bd_c^2\left(\mathcal{E}(ƕ)\right)$

(4) Let $\varrho\in NEG_c^2\left(\mathcal{E}(ƕ)\right)\Rightarrow\varrho\notin U_c^2(\mathcal{E}(ƕ))$ by Theorem $(2.5(2))$ we get $U_c^2(\mathcal{E}(ƕ))=U_i^2(\mathcal{E}(ƕ))\cap U_n^2(\mathcal{E}(ƕ))\Rightarrow\varrho\notin[U_i^2(\mathcal{E}(ƕ))\cap U_n^2(\mathcal{E}(ƕ))]\Rightarrow\varrho\notin U_i^2\left(\mathcal{E}(ƕ)\right)\vee\varrho\notin U_n^2(\mathcal{E}(ƕ))\Rightarrow\varrho\in NEG_i^2\left(\mathcal{E}(ƕ)\right)\vee\varrho\in NEG_n^2\left(\mathcal{E}(ƕ)\right)$ thus $NEG_c^2(\mathcal{E}(ƕ))\subseteq NEG_i^2(\mathcal{E}(ƕ))\cup NEG_n^2(\mathcal{E}(ƕ))---(1)$.
Let $\varrho\in[NEG_i^2(\mathcal{E}(ƕ))\cup NEG_n^2(\mathcal{E}(ƕ))]\Rightarrow\varrho\in NEG_i^2(\mathcal{E}(ƕ))\vee\varrho\in NEG_n^2(\mathcal{E}(ƕ))\Rightarrow\varrho\notin U_i^2\left(\mathcal{E}(ƕ)\right)\vee\varrho\notin U_n^2(\mathcal{E}(ƕ))\Rightarrow\varrho\notin[U_i^2(\mathcal{E}(ƕ))\cap U_n^2(\mathcal{E}(ƕ))]$ by Theorem $(2.5(2))$ we get $U_c^2(\mathcal{E}(ƕ))=U_i^2(\mathcal{E}(ƕ))\cap U_n^2(\mathcal{E}(ƕ))\Rightarrow\varrho\notin U_c^2(\mathcal{E}(ƕ))\Rightarrow\varrho\in NEG_c^2(\mathcal{E}(ƕ))$ thus $NEG_i^2(\mathcal{E}(ƕ))\cup NEG_n^2(\mathcal{E}(ƕ))\subseteq NEG_c^2(\mathcal{E}(ƕ))---(2)$.
From (1) and (2) we get $NEG_c^2(\mathcal{E}(ƕ))=NEG_i^2(\mathcal{E}(ƕ))\cup NEG_n^2(\mathcal{E}(ƕ))$.

(5) By Theorem $(2.5(3))$ we get $Bd_c^2\left(\mathcal{E}(ƕ)\right)=Bd_i^2\left(\mathcal{E}(ƕ)\right)\cap Bd_n^2\left(\mathcal{E}(ƕ)\right)\Rightarrow Bd_c^2\left(\mathcal{E}(ƕ)\right)\subseteq Bd_i^2\left(\mathcal{E}(ƕ)\right)$ and hence $\left|Bd_c^2\left(\mathcal{E}(ƕ)\right)\right|\leq\left|Bd_i^2\left(\mathcal{E}(ƕ)\right)\right|\Rightarrow$

$$\frac{\left|Bd_c^2\left(\mathcal{E}(ƕ)\right)\right|}{|\mathcal{E}(\Omega)|}\leq\frac{\left|Bd_i^2\left(\mathcal{E}(ƕ)\right)\right|}{|\mathcal{E}(\Omega)|}\Rightarrow 1-\frac{\left|Bd_c^2\left(\mathcal{E}(ƕ)\right)\right|}{|\mathcal{E}(\Omega)|}\geq 1-\frac{\left|Bd_i^2\left(\mathcal{E}(ƕ)\right)\right|}{|\mathcal{E}(\Omega)|}$$

$\Rightarrow\ \zeta_c^2(\mathcal{E}(ƕ))\geq\zeta_i^2(\mathcal{E}(ƕ))$In the same way we get $\zeta_c^2(\mathcal{E}(ƕ))\geq\zeta_n^2(\mathcal{E}(ƕ))$ thus $\zeta_c^2(\mathcal{E}(ƕ))\geq max\{\zeta_i^2\left(\mathcal{E}(ƕ)\right),\zeta_n^2\left(\mathcal{E}(ƕ)\right)\}$.

Proposition 2.6: Let $\mathfrak{Z}=(\mho(\Omega),\mathcal{E}(\Omega))$ be a generalized approximation space and $ƕ,ƙ\subseteq\Omega$. Then.

(1) $L_c^2(\mathcal{E}(\Omega))=\mathcal{E}(\Omega)$,
(2) if $\mathcal{E}(ƕ)\subseteq\mathcal{E}(ƙ)$,then $L_c^2(\mathcal{E}(ƕ))\subseteq L_c^2(\mathcal{E}(ƙ))$,
(3) $L_c^2(\mathcal{E}(ƕ)\cap\mathcal{E}(ƙ))\subseteq L_c^2(\mathcal{E}(ƕ))\ \cap\ L_c^2(\mathcal{E}(ƙ))$,
(4) $L_c^2(\mathcal{E}(ƕ))\cup L_c^2(\mathcal{E}(ƙ))\subseteq L_c^2(\mathcal{E}(ƕ)\cup\mathcal{E}(ƙ))$,
(5) $L_c^2(\mathcal{E}(ƕ))=\mathcal{E}(\Omega)-[U_c^2(\mathcal{E}(\Omega))-\mathcal{E}(ƕ))]$,

Proof:

(1) by definition $(2.1(c))$. We get $L_c^2(\mathcal{E}(\Omega))=\mathcal{E}(\Omega)$.

(2) let $\mathcal{E}(ƕ)\subseteq\mathcal{E}(ƙ)$ and $\varrho\in L_c^2(\mathcal{E}(ƕ))$,then $\exists\ C\mathcal{E}(\varrho)$ such that $C\mathcal{E}(\varrho)\subseteq\mathcal{E}(ƕ)$ so $\varrho\in L_c^2(\mathcal{E}(ƕ))\subseteq\mathcal{E}(ƕ)\subseteq\mathcal{E}(ƙ)$. thus we have $\varrho\in\mathcal{E}(ƙ)$ and there exist $C\mathcal{E}(\varrho)$ such that $C\mathcal{E}(\varrho)\subseteq\mathcal{E}(ƕ)\subseteq\mathcal{E}(ƙ)$. hence, $\varrho\in L_c^2(\mathcal{E}(ƙ))$ and so $L_c^2(\mathcal{E}(ƕ))\subseteq L_c^2(\mathcal{E}(ƙ))$.

(3) Since $(\mathcal{E}(ƕ)\cap\mathcal{E}(ƙ))\subseteq\mathcal{E}(ƕ)$ by proposition $(2.6.(2))$ we get $L_c^2(\mathcal{E}(ƕ)\cap\mathcal{E}(ƙ))\subseteq L_c^2\left(\mathcal{E}(ƕ)\right)---(1)$. And since $(\mathcal{E}(ƕ)\cap\mathcal{E}(ƙ))\subseteq\mathcal{E}(ƙ)$ by proposition $(2.6.(4))$ we get $L_c^2(\mathcal{E}(ƕ)\cap\mathcal{E}(ƙ))\subseteq L_c^2(\mathcal{E}(ƙ))---(2)$. From (1) and (2) we get $L_c^2(\mathcal{E}(ƕ)\cap\mathcal{E}(ƙ))\subseteq L_c^2\left(\mathcal{E}(ƕ)\right)\ \cap L_c^2\left(\mathcal{E}(ƙ)\right)$.

International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 7 Issue 1, January - 2023, Pages: 60-66

(4) Since $\mathcal{E}(ƕ) \subseteq (\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ))$ by proposition $(2.6.(2))$ we get $L_c^2(\mathcal{E}(ƕ)) \subseteq L_c^2(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ)) - - - (1)$. And since $\mathcal{E}(ƙ) \subseteq (\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ))$ by proposition $(2.6.(4))$ we get $L_c^2(\mathcal{E}(ƙ)) \subseteq L_c^2(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ)) - - - (2)$. From (1) and (2) we get $L_c^2(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ)) \subseteq L_c^2(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ))$.

(5) let $ϱ \in L_c^2(\mathcal{E}(ƕ)) \Rightarrow ϱ \in \mathcal{E}(ƕ)$ and $\exists\ C\mathcal{E}(ϱ) \subseteq \mathcal{E}(ƕ) \Rightarrow ϱ \in \mathcal{E}(\Omega) - [\mathcal{E}(\Omega) - \mathcal{E}(ƕ)]$ and $\exists\ C\mathcal{E}(ϱ): C\mathcal{E}(ϱ) \cap [\mathcal{E}(\Omega) - \mathcal{E}(ƕ)] = \emptyset \Rightarrow ϱ \notin U_c^2(\mathcal{E}(\Omega) - \mathcal{E}(ƕ)) \Rightarrow ϱ \in \mathcal{E}(\Omega) - U_c^2(\mathcal{E}(\Omega) - \mathcal{E}(ƕ)) \Rightarrow L_c^2(\mathcal{E}(ƕ)) \subseteq \mathcal{E}(\Omega) - U_c^2(\mathcal{E}(\Omega) - \mathcal{E}(ƕ)) - - - (1)$. $ϱ \in \mathcal{E}(\Omega) - U_c^2(\mathcal{E}(\Omega) - \mathcal{E}(ƕ)) \Rightarrow ϱ \in \mathcal{E}(\Omega)\ and\ ϱ \notin U_c^2(\mathcal{E}(\Omega) - \mathcal{E}(ƕ)) \Rightarrow \exists\ C\mathcal{E}(ϱ): C\mathcal{E}(ϱ) \cap [\mathcal{E}(\Omega) - \mathcal{E}(ƕ)] = \emptyset\ and\ ϱ \in \mathcal{E}(\Omega) - [\mathcal{E}(\Omega) - \mathcal{E}(ƕ)]\ and\ \exists\ C\mathcal{E}(ϱ) \subseteq \mathcal{E}(ƕ) \Rightarrow ϱ \in L_c^2(\mathcal{E}(ƕ)) \Rightarrow \mathcal{E}(\Omega) - U_c^2(\mathcal{E}(\Omega) - \mathcal{E}(ƕ)) \subseteq L_c^2(\mathcal{E}(ƕ)) - - - (2)$. From (1) and (2) we get $L_c^2(\mathcal{E}(ƕ)) = \mathcal{E}(\Omega) - [U_c^2(\mathcal{E}(\Omega) - \mathcal{E}(ƕ))]$.

Proposition 2.7. Let $\mathfrak{Z} = (Ʊ(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space and $ƕ, ƙ \subseteq \Omega$. Then.

(1) $U_c^2(\emptyset) = \emptyset$,
(2) if $\mathcal{E}(ƕ) \subseteq \mathcal{E}(ƙ)$,then $U_c^2(\mathcal{E}(ƕ)) \subseteq U_c^2(\mathcal{E}(ƙ))$,
(3) $U_c^2(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)) \subseteq U_c^2(\mathcal{E}(ƕ)) \cap U_c^2(\mathcal{E}(ƙ))$,
(4) $U_c^2(\mathcal{E}(ƕ)) \cup U_c^2(\mathcal{E}(ƙ)) \subseteq U_c^2(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ))$,
(5) $U_c^2(\mathcal{E}(ƕ)) = \mathcal{E}(\Omega) - [L_c^2(\mathcal{E}(\Omega)) - \mathcal{E}(ƕ))]$

Proof:

(1) by definition $(2.1(c))$. We get $U_c^2(\emptyset) = \emptyset$.

(2) Let $\mathcal{E}(ƕ) \subseteq \mathcal{E}(ƙ)$ and $ϱ \in U_c^2(\mathcal{E}(ƕ)) \Rightarrow \forall\ C\mathcal{E}(ϱ): C\mathcal{E}(ϱ) \cap [\mathcal{E}(\Omega) - \mathcal{E}(ƕ)] \neq \emptyset$ and since $\mathcal{E}(ƕ) \subseteq \mathcal{E}(ƙ) \Rightarrow \forall\ C\mathcal{E}(ϱ): C\mathcal{E}(ϱ) \cap [\mathcal{E}(\Omega) - \mathcal{E}(ƙ)] \neq \emptyset \Rightarrow ϱ \in U_c^2(\mathcal{E}(ƙ))$ and hence $U_c^2(\mathcal{E}(ƕ)) \cap U_c^2(\mathcal{E}(ƙ))$.

(3) Since $(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)) \subseteq \mathcal{E}(ƕ)$ by proposition $(2.7.(2))$ we get $U_c^2(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)) \subseteq U_c^2(\mathcal{E}(ƕ)) - - - (1)$. And since $(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)) \subseteq \mathcal{E}(ƙ)$ by proposition $(2.7.(2))$ we get $U_c^2(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)) \subseteq U_c^2(\mathcal{E}(ƙ)) - - - (2)$. From (1) and (2) we get $U_c^2(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)) \subseteq U_c^2(\mathcal{E}(ƕ)) \cap U_c^2(\mathcal{E}(ƙ))$.

(4) Since $\mathcal{E}(ƕ) \subseteq (\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ))$ by proposition $(2.7.(2))$ we get $U_c^2(\mathcal{E}(ƕ)) \subseteq U_c^2(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ)) - - - (1)$. And since $\mathcal{E}(ƙ) \subseteq (\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ))$ by proposition $(2.7.(2))$ we get $U_c^2(\mathcal{E}(ƙ)) \subseteq U_c^2(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ)) - - - (2)$. From (1) and (2) we get $U_c^2(\mathcal{E}(ƕ)) \cup U_c^2(\mathcal{E}(ƙ)) \subseteq U_c^2(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ))$.

(5) by proposition $(2.6.(5))$ $L_c^2(\mathcal{E}(ƕ)) = \mathcal{E}(\Omega) - [U_c^2(\mathcal{E}(\Omega) - \mathcal{E}(ƕ))] \Rightarrow \mathcal{E}(\Omega) - L_c^2(\mathcal{E}(ƕ)) = \mathcal{E}(\Omega) - (\mathcal{E}(\Omega) - [U_c^2(\mathcal{E}(\Omega) - \mathcal{E}(ƕ))]) \Rightarrow U_c^2(\mathcal{E}(\Omega) - \mathcal{E}(ƕ)) = \mathcal{E}(\Omega) - L_c^2(\mathcal{E}(ƕ))$. Now we replace $\mathcal{E}(\Omega) - \mathcal{E}(ƕ)$ for $\mathcal{E}(ƕ)$ we get $U_c^2(\mathcal{E}(ƕ)) = \mathcal{E}(\Omega) - L_c^2(\mathcal{E}(\Omega) - \mathcal{E}(ƕ))$.

Remark 2.8. Let $\mathfrak{Z} = (Ʊ(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space and $ƕ, ƙ \subseteq \Omega$. Then the following statements are not necessarily true:

(1) $L_c^2(\mathcal{E}(ƕ)) \subseteq \mathcal{E}(ƕ)$,
(2) $L_c^2(\emptyset) = \emptyset$,
(3) $L_c^2(\mathcal{E}(ƕ)) = L_c^2(L_c^2(\mathcal{E}(ƕ)))$,
(4) $L_c^2(\mathcal{E}(ƕ)) = U_c^2(L_c^2(\mathcal{E}(ƕ)))$,
(5) $\mathcal{E}(ƕ) \subseteq L_c^2(U_c^2(\mathcal{E}(ƕ)))$,
(6) $L_c^2(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ)) = L_c^2(\mathcal{E}(ƕ)) \cup L_c^2(\mathcal{E}(ƙ))$,
(7) $\mathcal{E}(ƕ) \subseteq U_c^2(\mathcal{E}(ƕ))$,
(8) $U_c^2(\mathcal{E}(\Omega)) = \mathcal{E}(\Omega)$,
(9) $U_c^2(\mathcal{E}(ƕ)) = U_c^2(U_c^2(\mathcal{E}(ƕ)))$,
(10) $U_c^2(\mathcal{E}(ƕ)) = L_c^2(U_c^2(\mathcal{E}(ƕ)))$,
(11) $U_c^2(L_b^2(\mathcal{E}(ƕ))) \subseteq \mathcal{E}(ƕ)$,
(12) $U_c^2(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ)) = U_c^2(\mathcal{E}(ƕ)) \cup U_c^2(\mathcal{E}(ƙ))$,
(13) $L_c^2(\mathcal{E}(ƕ)) \subseteq U_c^2(\mathcal{E}(ƕ))$.

The following example is applied to show this remark.

Example 2.9: In Example (2.4) we get

(1) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{Ʋ_1, Ʋ_4\}$ and $\mathcal{E}(ƕ) = \{ϱ_1\}$. Then $L_c^2(\mathcal{E}(ƕ)) = \{ϱ_1, ϱ_3\}$,.Therefore, $L_c^2(\mathcal{E}(ƕ)) \nsubseteq \mathcal{E}(ƕ)$.

(2) $L_c^2(\mathcal{E}(ƕ)) = \{ϱ_1, ϱ_3\}$ Therefore, $L_c^2(\mathcal{E}(ƕ)) \neq \emptyset$.

(3) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{Ʋ_3, Ʋ_4\}$ and $\mathcal{E}(ƕ) = \{ϱ_5\}$. Then $L_c^2(\mathcal{E}(ƕ)) = \{ϱ_1, ϱ_3\}, L_c^2(L_c^2(\mathcal{E}(ƕ)) = \{ϱ_1, ϱ_2, ϱ_3\}$. Therefore, $L_c^2(\mathcal{E}(ƕ)) \neq L_c^2(L_c^2(\mathcal{E}(ƕ))$.

(4) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{Ʋ_1, Ʋ_4\}$ and $\mathcal{E}(ƕ) = \{ϱ_1\}$. Then $L_c^2(\mathcal{E}(ƕ)) = \{ϱ_1, ϱ_3\}, U_c^2(L_c^2(\mathcal{E}(ƕ)) = \emptyset$. Therefore, $L_c^2(\mathcal{E}(ƕ)) \neq U_c^2(L_c^2(\mathcal{E}(ƕ))$.

(5) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{Ʋ_3, Ʋ_4\}$ and $\mathcal{E}(ƕ) = \{ϱ_4\}$. Then $U_c^2(\mathcal{E}(ƕ)) = \emptyset, L_c^2(U_c^2(\mathcal{E}(ƕ)) = \{ϱ_1, ϱ_3\}$. Therefore, $\mathcal{E}(ƕ) \nsubseteq L_c^2(U_c^2(\mathcal{E}(ƕ))$.

(6) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{Ʋ_3, Ʋ_4\}$ and $\mathcal{E}(ƕ) = \{ϱ_5\}$.And $ƙ = (Ʊ(ƙ), \mathcal{E}(ƙ))$ such that $Ʊ(ƙ) = \{Ʋ_3, Ʋ_4\}$ and $\mathcal{E}(ƙ) = \{ϱ_4\}$ Then $L_c^2(\mathcal{E}(ƕ)) = \{ϱ_1, ϱ_3\}$, $L_c^2(\mathcal{E}(ƙ)) = \{ϱ_1, ϱ_3\}$, $L_c^2(\mathcal{E}(ƕ)) \cup L_c^2(\mathcal{E}(ƙ)) = \{ϱ_1, ϱ_3\}$, $L_c^2(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ)) = \{ϱ_1, ϱ_2, ϱ_3\}$Therefore, $L_c^2(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ)) \neq L_c^2(\mathcal{E}(ƕ)) \cup L_c^2(\mathcal{E}(ƙ))$.

(7) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{Ʋ_1, Ʋ_4\}$ and $\mathcal{E}(ƕ) = \{ϱ_1\}$. Then $U_c^2(\mathcal{E}(ƕ)) = \emptyset$,Therefore, $\mathcal{E}(ƕ) \nsubseteq U_c^2(\mathcal{E}(ƕ))$.

(8) $U_c^2(\mathcal{E}(\Omega)) = \{ϱ_2, ϱ_4, ϱ_5\}$, Therefore, $U_c^2(\mathcal{E}(\Omega)) \neq \mathcal{E}(\Omega)$.

(9) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{Ʋ_1, Ʋ_3, Ʋ_4\}$ and $\mathcal{E}(ƕ) = \{ϱ_1, ϱ_4\}$. Then $U_c^2(\mathcal{E}(ƕ)) = \{ϱ_2\}$, $U_c^2(U_c^2(\mathcal{E}(ƕ)) = \emptyset$. Therefore, $U_c^2(\mathcal{E}(ƕ)) \neq U_c^2(U_c^2(\mathcal{E}(ƕ))$.

(10) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{Ʋ_1, Ʋ_2, Ʋ_4\}$ and $\mathcal{E}(ƕ) = \{ϱ_1, ϱ_2\}$. Then $U_c^2(\mathcal{E}(ƕ)) = \{ϱ_4, ϱ_5\}$, $L_c^2(U_c^2(\mathcal{E}(ƕ)) = \{ϱ_1, ϱ_2, ϱ_3\}$. Therefore, $U_c^2(\mathcal{E}(ƕ)) \neq L_c^2(U_c^2(\mathcal{E}(ƕ))$.

(11) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{Ʋ_1, Ʋ_2, Ʋ_3, Ʋ_4\}$ and $\mathcal{E}(ƕ) = \{ϱ_1, ϱ_3, ϱ_4\}$. Then $L_c^2(\mathcal{E}(ƕ)) = \{ϱ_1, ϱ_2, ϱ_3, ϱ_5\}$, $U_c^2(L_c^2(\mathcal{E}(ƕ)) = \{ϱ_2, ϱ_4, ϱ_5\}$. Therefore, $U_c^2(L_c^2(\mathcal{E}(ƕ)) \nsubseteq \mathcal{E}(ƕ)$.

(12) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{Ʋ_1, Ʋ_4\}$ and $\mathcal{E}(ƕ) = \{ϱ_1\}$.And $ƙ = (Ʊ(ƙ), \mathcal{E}(ƙ))$ such that $Ʊ(ƙ) = \{Ʋ_1, Ʋ_2\}$ and $\mathcal{E}(ƙ) = \{ϱ_2\}$ Then $U_c^2(\mathcal{E}(ƕ)) = \emptyset$, $U_c^2(\mathcal{E}(ƙ)) = \emptyset, U_c^2(\mathcal{E}(ƕ)) \cup U_c^2(\mathcal{E}(ƙ)) = \emptyset, U_c^2(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ)) = \{ϱ_4, ϱ_5\}$ Therefore, $U_c^2(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ)) \neq U_c^2(\mathcal{E}(ƕ)) \cup U_c^2(\mathcal{E}(ƙ))$.

(13) Let $\mathfrak{h} = (\mho(\mathfrak{h}), \mathcal{E}(\mathfrak{h}))$ such that $\mho(\mathfrak{h}) = \{\Im_1, \Im_4\}$ and $\mathcal{E}(\mathfrak{h}) = \{\varrho_1\}$. Then $L_c^2(\mathcal{E}(\mathfrak{h})) = \{\varrho_1, \varrho_3\}$, $U_c^2(\mathcal{E}(\mathfrak{h})) = \emptyset$. Therefore, $L_c^2(\mathcal{E}(\mathfrak{h})) \nsubseteq U_c^2(\mathcal{E}(\mathfrak{h}))$.

Proposition 2.10: Let $\mathfrak{Z} = (\mho(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space and Ω is antisymmetric und. g. and $\mathfrak{h} \subseteq \Omega$. Then.

(1) $\mathcal{E}(\mathfrak{h}) \subseteq L_c^2(\mathcal{E}(\mathfrak{h}))$.

(2) $U_c^2(\mathcal{E}(\mathfrak{h})) \subseteq \mathcal{E}(\mathfrak{h})$.

Proof:

(1) Let Ω be an antisymmetric und. g. and $\mathfrak{h} \subseteq \Omega$. Let $\varrho \in \mathcal{E}(\mathfrak{h})$ and $\varrho \notin L_c^2(\mathcal{E}(\mathfrak{h})) \Rightarrow \forall\, C\mathcal{E}(\varrho) \nsubseteq \mathcal{E}(\mathfrak{h})$ and this contradiction with $\forall\, \varrho \in \mathcal{E}(\mathfrak{h}) \Rightarrow \mathrm{I}\mathcal{E}(\varrho) \subseteq \mathcal{E}(\mathfrak{h})$ because Ω is antisymmetric and hence $\varrho \in L_c^2(\mathcal{E}(\mathfrak{h})) \Rightarrow \mathcal{E}(\mathfrak{h}) \subseteq L_c^2(\mathcal{E}(\mathfrak{h}))$

(2) Let Ω be an antisymmetric und. g. and $\mathfrak{h} \subseteq \Omega$. Let $\varrho \in U_c^2(\mathcal{E}(\mathfrak{h}))$ and $\varrho \notin \mathcal{E}(\mathfrak{h})$ since Ω be an antisymmetric und. g $\Rightarrow \mathrm{I}\mathcal{E}(\varrho) \cap \mathcal{E}(\mathfrak{h}) = \emptyset$ and this contradiction, Then $\varrho \in U_c^2(\mathcal{E}(\mathfrak{h}))$ and hence $\varrho \in \mathcal{E}(\mathfrak{h}) \Rightarrow U_c^2(\mathcal{E}(\mathfrak{h})) \subseteq \mathcal{E}(\mathfrak{h})$.

Proposition 2.11: Let $\mathfrak{Z} = (\mho(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space and Ω is antisymmetric und. g. and $\mathfrak{h} \subseteq \Omega$. Then.

(1) $L_c^2(\mathcal{E}(\mathfrak{h})) = L_c^2\left(L_c^2(\mathcal{E}(\mathfrak{h}))\right)$.

(2) $U_c^2(\mathcal{E}(\mathfrak{h})) = U_c^2(U_c^2(\mathcal{E}(\mathfrak{h})))$.

(3) $U_c^2(\mathcal{E}(\mathfrak{h})) \subseteq L_c^2(\mathcal{E}(\mathfrak{h}))$

Proof:

(1) Let Ω be an antisymmetric und. g. and $\mathfrak{h} \subseteq \Omega$. By Proposition $(2.10(1))$ we get $\mathcal{E}(\mathfrak{h}) \subseteq L_c^2(\mathcal{E}(\mathfrak{h}))$ and by proposition $(2.6(2))$ we get $L_c^2(\mathcal{E}(\mathfrak{h})) \subseteq L_c^2\left(L_c^2(\mathcal{E}(\mathfrak{h}))\right) ---(1)$. To prove $L_c^2\left(L_c^2(\mathcal{E}(\mathfrak{h}))\right) \subseteq L_c^2(\mathcal{E}(\mathfrak{h}))$. Suppose that $L_c^2\left(L_c^2(\mathcal{E}(\mathfrak{h}))\right) \nsubseteq L_c^2(\mathcal{E}(\mathfrak{h})) \Longrightarrow \exists \varrho \in L_c^2\left(L_c^2(\mathcal{E}(\mathfrak{h}))\right) \wedge \varrho \notin L_c^2(\mathcal{E}(\mathfrak{h}))$ since $\varrho \notin L_c^2(\mathcal{E}(\mathfrak{h})) \Longrightarrow \forall\, C\mathcal{E}(\varrho) \nsubseteq \mathcal{E}(\mathfrak{h})$ and this contradiction since Ω is antisymmetric and hence $\varrho \in L_c^2(\mathcal{E}(\mathfrak{h}))$ thus $L_c^2\left(L_c^2(\mathcal{E}(\mathfrak{h}))\right) \subseteq L_c^2(\mathcal{E}(\mathfrak{h})) ---(2)$. From (1) and (2) we get $L_c^2(\mathcal{E}(\mathfrak{h})) = L_c^2\left(L_c^2(\mathcal{E}(\mathfrak{h}))\right)$.

(2) Let Ω be an antisymmetric und. g. and $\mathfrak{h} \subseteq \Omega$. By Proposition $(2.10(2))$ we get $U_c^2(\mathcal{E}(\mathfrak{h})) \subseteq \mathcal{E}(\mathfrak{h})$ and by proposition $(2.7(2))$ we get $U_c^2\left(U_c^2(\mathcal{E}(\mathfrak{h}))\right) \subseteq U_c^2(\mathcal{E}(\mathfrak{h})) ---(1)$. To prove $U_c^2(\mathcal{E}(\mathfrak{h})) \subseteq U_c^2\left(U_c^2(\mathcal{E}(\mathfrak{h}))\right)$. Let $\varrho \in U_c^2(\mathcal{E}(\mathfrak{h})) \Longrightarrow \forall\, C\mathcal{E}(\varrho) \cap \mathcal{E}(\mathfrak{h}) \neq \emptyset$ since $U_c^2(\mathcal{E}(\mathfrak{h})) \subseteq \mathcal{E}(\mathfrak{h}) \Longrightarrow \forall\, C\mathcal{E}(\varrho) \cap U_c^2(\mathcal{E}(\mathfrak{h})) \neq \emptyset \Longrightarrow \varrho \in U_c^2\left(U_c^2(\mathcal{E}(\mathfrak{h}))\right)$ thus $U_c^2(\mathcal{E}(\mathfrak{h})) \subseteq U_c^2\left(U_c^2(\mathcal{E}(\mathfrak{h}))\right) ---(2)$. From (1) and (2) we get $U_c^2(\mathcal{E}(\mathfrak{h})) = U_c^2(U_c^2(\mathcal{E}(\mathfrak{h})))$.

(3) Let Ω be an antisymmetric und. g. and $\mathfrak{h} \subseteq \Omega$. By Proposition $(2.10(2))$ we get $U_c^2(\mathcal{E}(\mathfrak{h})) \subseteq \mathcal{E}(\mathfrak{h})$ and By Proposition $(2.10(1))$ we get $\mathcal{E}(\mathfrak{h}) \subseteq L_c^2(\mathcal{E}(\mathfrak{h}))$ and hence $U_c^2(\mathcal{E}(\mathfrak{h})) \subseteq L_c^2(\mathcal{E}(\mathfrak{h}))$.

Lemme 2.12: Let $\mathfrak{Z} = (\mho(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space and Ω be an star und.g. and $\mathfrak{h} \subseteq \Omega$. Then.

(1) $L_c^2(\mathcal{E}(\mathfrak{h})) = \mathcal{E}(\Omega)$.

(2) $U_c^2(\mathcal{E}(\mathfrak{h})) = \emptyset$.

(3) $B_c^2(\mathcal{E}(\mathfrak{h})) = \emptyset$.

Proof:

(1) Let Ω be an star und.g. and $\mathfrak{h} \subseteq \Omega$ Since Ω is star then $\forall\, \varrho \in \mathcal{E}(\Omega)$ we have $N\mathrm{I}\mathcal{E}(\varrho) = \emptyset$ and $N\mathrm{I}\mathcal{E}(\varrho) \subseteq \mathcal{E}(\mathfrak{h}) \Longrightarrow \exists\, C\mathcal{E}(\varrho)$ such that $C\mathcal{E}(\varrho) \subseteq \mathcal{E}(\mathfrak{h}) \Longrightarrow \varrho \in L_c^2(\mathcal{E}(\mathfrak{h}))$, thus for all $\varrho \in \mathcal{E}(\Omega)$ we get $\varrho \in L_c^2(\mathcal{E}(\mathfrak{h}))$. Hence, $L_c^2(\mathcal{E}(\mathfrak{h})) = \mathcal{E}(\Omega)$.

(2) Let Ω be an star und.g. and $\mathfrak{h} \subseteq \Omega$ Since Ω is star then $\forall\, \varrho \in \mathcal{E}(\Omega)$ we have $N\mathrm{I}\mathcal{E}(\varrho) = \emptyset$ and $N\mathrm{I}\mathcal{E}(\varrho) \cap \mathcal{E}(\mathfrak{h}) = \emptyset \Longrightarrow \exists\, C\mathcal{E}(\varrho)$ such that $C\mathcal{E}(\varrho) \cap \mathcal{E}(\mathfrak{h}) = \emptyset \Longrightarrow \varrho \notin U_c^2(\mathcal{E}(\mathfrak{h}))$ thus for all $\varrho \in \mathcal{E}(\Omega)$ we get $\notin U_c^2(\mathcal{E}(\mathfrak{h}))$. Hence, $U_c^2(\mathcal{E}(\mathfrak{h})) = \emptyset$.

(3) Let Ω be an star und.g. and $\mathfrak{h} \subseteq \Omega$. Suppose that $B_c^2(\mathcal{E}(\mathfrak{h})) \neq \emptyset \Longrightarrow \exists\, \varrho \in B_c^2(\mathcal{E}(\mathfrak{h})) \Longrightarrow \varrho \in U_c^2(\mathcal{E}(\mathfrak{h})) \wedge \varrho \notin L_c^2(\mathcal{E}(\mathfrak{h})) \Longrightarrow \forall C\mathcal{E}(\varrho) \cap \mathcal{E}(\mathfrak{h}) \neq \emptyset \Longrightarrow N\mathrm{I}\mathcal{E}(\varrho) \cap \mathcal{E}(\mathfrak{h}) \neq \emptyset$ and this contradiction since Ω is star thus $\varrho \notin B_c^2(\mathcal{E}(\mathfrak{h}))$ and hence $B_c^2(\mathcal{E}(\mathfrak{h})) = \emptyset$.

Proposition2.13: Let $\mathfrak{Z} = (\mho(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space and Ω be an star und.g. and $\mathfrak{h} \subseteq \Omega$. Then.

(1) $L_c^2(\mathcal{E}(\mathfrak{h})) = L_c^2\left(L_c^2(\mathcal{E}(\mathfrak{h}))\right)$.

(2) $\mathcal{E}(\mathfrak{h}) \subseteq L_c^2\left(U_c^2(\mathcal{E}(\mathfrak{h}))\right)$.

(3) $L_c^2(\mathcal{E}(\mathfrak{h}) \cup \mathcal{E}(\mathfrak{k})) = L_c^2(\mathcal{E}(\mathfrak{h})) \cup L_c^2(\mathcal{E}(\mathfrak{k}))$.

(4) $U_c^2(\mathcal{E}(\mathfrak{h})) = U_c^2\left(U_c^2(\mathcal{E}(\mathfrak{h}))\right)$.

(5) $U_c^2\left(L_c^2(\mathcal{E}(\mathfrak{h}))\right) \subseteq \mathcal{E}(\mathfrak{h})$.

(6) $U_c^2(\mathcal{E}(\mathfrak{h}) \cup \mathcal{E}(\mathfrak{k})) = U_c^2(\mathcal{E}(\mathfrak{h})) \cup U_c^2(\mathcal{E}(\mathfrak{k}))$.

Proof:

Let Ω be an star und.g. and $\mathfrak{h} \subseteq \Omega$.

(1) Since Ω is star by Lemme (2.12) we get $L_c^2(\mathcal{E}(\mathfrak{h})) = \mathcal{E}(\Omega)$ and $L_c^2\left(L_c^2(\mathcal{E}(\mathfrak{h}))\right) = L_c^2(\mathcal{E}(\Omega)) = \mathcal{E}(\Omega)$ and hence $L_c^2(\mathcal{E}(\mathfrak{h})) = L_c^2\left(L_c^2(\mathcal{E}(\mathfrak{h}))\right)$.

And we can proof (2), (3), (4), (5), (6), by the same way in (1).

Proposition2.14: Let $\mathfrak{Z} = (\mho(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space and Ω be an star und.g. then for all $\mathfrak{h} \subseteq \Omega$ is ($\mathfrak{h}$-exact).

Proof:

Let Ω be an star und.g. and $\mathfrak{h} \subseteq \Omega$. By Lemme $(2.12(3))$ we get $B_c^2(\mathcal{E}(\mathfrak{h})) = \emptyset \Longrightarrow \left|Bd_c^2(\mathcal{E}(\mathfrak{h}))\right| = 0$ and by Definition (2.3) we get

$\zeta_c^2(\mathcal{E}(\mathfrak{h})) = 1 - \frac{|Bd_c^2(\mathcal{E}(\mathfrak{h}))|}{|\mathcal{E}(\Omega)|} = 1 - \frac{0}{|\mathcal{E}(\Omega)|} = 1$ thus $\mathfrak{h}$-exact.

3. REFERENCES:

4. K.S. Al'Dzhabri, A. Hamzha. and Y.S. Essa. On DG-topological spaces associated with directed graphs.

Journal of Discrete Mathematical Sciences & Cryptography,32(5), (2020) 1039-1046. Doi.org/10.1080/09720529.2020.1714886

5. K.S. Al'Dzhabri and M. F. Hani, On Certain Types of Topological Spaces Associated with Digraphs.Journal of Physics:Conference Series 1591(2020)012055.
6. K.S. Al'Dzhabri and et al, DG-domination topology in Digraph. Journal of Prime Research in Mathematics 2021, 17(2), 93–100. http://jprm.sms.edu.pk/
7. K.S. Al'Dzhabri, Enumeration of connected components of acyclic digraph. Journal of Discrete Mathematical Sciences and Cryptography, 2021, 24(7), 2047–2058. DOI: 10.1080/09720529.2021.1965299
8. L. W. Beineke and R. J. Wilson, Topics in topological graph theory, Cambridge University Press, 2009
9. F. Harary, Graph Theory (Addison-Wesley, Reading Mass, 1969).
10. M. Moller, General Topology, Authors, Notes, (2007).
11. M. Shokry, Approximations structures generated by trees vertices, International Journal of Engineering Science and Technology (IJEST), Vol.4 (2), February (2012), pp.406-418. 10.
12. M. Shokry, Closure concepts generated by directed trees; Journal of Mathematics & Computer Sciences, Vol.21 (1), (2010), pp.45-51.
13. S.S. Ray, Graph Theory with Algorithms and its Applications: In Applied Sciences and Technology, Springer, New Delhi, (2013).