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Model operator $⊡_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}$ on Intuitionistic Fuzzy RG-algebras

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Abstract— *The aim of this paper is introducing the notion of operator $⊡_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}$ in intuitionistic fuzzy RG-subalgebras and intuitionistic fuzzy RG-ideals of RG-algebra, several theorems and properties are stated and proved.*

Keywords— RG-algebra, fuzzy RG-subalgebra, fuzzy RG-ideal, Intuitionistic Fuzzy Set, Model Operator$⊡_{\propto,\beta,\gamma,\delta,\tau,\varepsilon}$,, Extended Model Operator, Homomorphism.

1. Introduction

Zadeh proposed the concept of a subset of a set in 1965 as a way to illustrate the principle of relativity in the real world [23]. Atanassov was introduced the intuitionistic fuzzy subset concept and the intuitionistic fuzzy model operators † and ⊞ was introduced in 1986, [1]. Also, Atanassov was proposed new procedures defined over intuitionistic fuzzy subsets [2-8]. The extension on both the operators † and ⊞is the new operator Dα which represents both of them. Further, the extension of all the operators is the operator Fα, β called (α, β)-model operator.The operator $⊡_{\propto,\beta,\gamma,\delta,\tau,\varepsilon}$ was introduced by Atanassov [8]. The concept of fuzzy RG-subalgebras of RG-algebras was introduced by A.T. Hameed and S.M. Abrahem [14] and doutl of them [15]. The concept of intuitionistic fuzzy RG-subalgebras and intuitionistic fuzzy RG-ideals were introduced by A.T. Hameed and S.M. Abrahem [16]. In this paper, we study the effect of these generalized model operators as discussed above on intuitionistic fuzzy RG-algebras.

2. Preliminaries

Now, we give some definitions and preliminary results needed in the later sections.

Definition 2.1. [1]: An algebra $(X;*,ב)$ is called RG-algebra if the following axioms are satisfied: $\forall x, y, z \in X$,
(i) $x * ב = x$,
(ii) $x * y = (x * z) * (y * z)$,
(iii)$x * y = y * x = ב$ imply $x = y$.

Remark 2.2. [1]: In $(X;*,ב)$ an RG-algebra, we define a binary relation $(\le)$ by putting $x \le y$ if and only if $x * y = ב$.

Definition 2.3. [19]: Let $(X;*,ב)$ be an RG-algebra, a nonempty subset I of X is called **an RG-ideal of X** if $\forall x, y \in X$
i) $ב \in I$,
ii) $x * y \in I$ and $ב * x \in I$ imply $ב * y \in I$.

Proposition 2.4. [19]: In an RG-algebra $(X;*,ב)$, every RG-ideal is a subalgebra of X.

Proposition 2.5. [19,20]: In any RG-algebra $(X;*,ב)$, the following hold: $\forall x, y, z \in X$,
i) $x * x = ב$,
ii) $ב * (ב * x) = x$,
iii) $x * (x * y) = y$,
iv) $x * y = ב$ if and only if $y * x = ב$,
v) $x * ב = ב$ implies $x = ב$,
vi) $ב * (y * x) = x * y$.

Proposition 2.6. [19]: In any RG-algebra $(X;*,ב)$, the following hold: $\forall x, y, z \in X$,
i) $(x * y) * (ב * y) = (x * (ב * y)) * y = x$,
ii) $x * (x * (x * y)) = x * y$,
iii) $(x * y) * z = (x * z) * y$,
iv) $x * y = (z * y) * (z * x)$,
v) $((x * y) * (x * z)) * (z * y) = ב$.

Theorem 2.7. [20]:

If $f: (X; *,ב) \rightarrow (Y; *`,ב`)$ is a homomorphism of an RG-algebras respectively X ,Y, then
1) $f(ב) = ב'$.
2) f is injective if and only if $\ker f = \{ב\}$.

Definition 2.8. [23]:

Let$(X;*,ב)$ be a nonempty set, a fuzzy subset μ of X is a function $\mu: X \rightarrow [ב, 1]$.

Definition 2.9. [23]:
For any t∈ [$\beth$,1] and a fuzzy subset μ of a nonempty set **X**, the set
$U(\mu, t) = \{x \in X \mid \mu(x) \geq t\}$ is called **an upper level cut of μ**, and the set
$L(\mu, t) = \{x \in X \mid \mu(x) \leq t\}$ is called **a lower level cut of μ.**

Definition 2.10.[14]:
Let $(X;*,\beth)$ be an RG-algebra and S be a nonempty subset of X .Then S is called an RG-**subalgebra of X** if $x * y \in S$, for any $x, y \in S$.

Proposition 2.11. [14]:
In an RG-algebra $(X; *,\beth)$ every RG-ideal is a RG-subalgebra of X.

Definition 2.12.[14]:
Let $(X;*,\beth)$ be an RG-algebra, a fuzzy subset μ of X is called a **fuzzy RG-subalgebra of X**, if $\forall x, y \in X$, $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$.

Definition 2.13.[14]:
Let $(\mathbf{X};*,\mathbf{0})$ be an RG-algebra, a fuzzy subset **μ** of **X** is called a **fuzzy RG-ideal of X** if it satisfies the following conditions: $\forall \mathbf{x}, \mathbf{y} \in \mathbf{X}$,
(i) $\mu(\beth) \geq \mu(x)$,
(v) $\mu(\beth * y) \geq \min\{\mu(x * y), \mu(\beth * x)\}$.

Proposition 2.14. [14] :
Every fuzzy RG-ideal of RG-algebra $(X; *,\beth)$ is a fuzzy RG-subalgebra of X.

Proposition 2.15.[14]:
1- The intersection of any set of fuzzy RG-subalgebras of RG-algebra $(X; *,\beth)$ is also fuzzy RG-subalgebra of X.
2- The union of any set of fuzzy RG-subalgebras of RG-algebra is also fuzzy RG-subalgebra, where is chain (Noetherian).
3- The intersection of any set of fuzzy RG-ideals of RG-algebra $(X; *,\beth)$ is also fuzzy RG-ideal of X.
4- The union of any set of fuzzy RG-ideals of RG-algebra is also fuzzy RG-ideal, where is chain (Noetherian).

Definition 2.16.[10]:
Let $f:(X;*,\beth)\to(Y;*',\beth')$ be a homeomorphism from the set X into the set Y. If μ is a fuzzy subset of X, then the fuzzy subset $f(\mu)$ in Y defined by:

$$f(\mu)(y)=\begin{cases}\sup\{\mu(x): x \in f^{-1}(y)\} & \text{if } f^{-1}(y) = \{x \in X, f(x) = y\} \neq \emptyset \\ \beth & \text{otherwies}\end{cases}$$

is said to be **the image of μ under *f***.
Similarly, if β is a fuzzy subset of Y, then the fuzzy subset $\mu = (\beta \circ f)$ in X, (i.e. the fuzzy subset defined by $\mu(x) = \beta(f(x))$, for all x∈X) is called **the pre-image of β under** f.

Definition 2.17.[10]:
1) A fuzzy subset μ of algebra $(X; *,\beth)$ has inf property if for any subset T of X, there exist $t_0 \in T$ such that $\mu(t_0) = \inf_{t\in T} \mu(t)$.
2) A fuzzy subset μ of algebra $(X; *,\beth)$ has inf property if for any subset T of X, there exist $t_0 \in T$ such that $\mu(t_\beth) = \sup\{\mu(t) \mid t\in T\}$.

Remark 2.18.[19]:

A fuzzy subset A in X is defined as $A = \{(x, \mu_A(x)) \mid x \in X\}$ where $\mu_A(x)$ denotesto the degree of the membership value of x in A and $\beth \leq \mu_A(x) \leq 1$.

Definition 2.19. [1]:
An intuitionistic fuzzy subset A in a nonempty set X is an object having the form
$A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$or $A =(\mu_A, \nu_A)$, where the functions $\mu_A: X \to [\beth, 1]$ and $\nu_A: X \to [\beth, 1]$ denotes to the degree of the membership and the degree of non membership respectively, and $\beth \leq \mu_A(x), \nu_A(x) \leq 1$, for all x∈X.

Definition 2.20.[15]:
Let $(X; *,\beth)$ be an RG-algebra. μ be a fuzzy subset of X**,** μ is called **doubt fuzzy RG-subalgebra of X** if for all $x, y \in X$ $\mu(x * y) \leq \max\{\mu(x), \mu(y)\}$,

Definition 2.21.[15]:
Let $(\mathbf{X};*,\mathbf{0})$ be an RG-algebra, a fuzzy subset **μ** of **X** is called **a doubt fuzzy RG-ideal of X** if it satisfies the following conditions: $\forall \mathbf{x}, \mathbf{y} \in \mathbf{X}$,
1. $\mu(\beth) \leq \mu(x)$.
2. $\mu(\beth * y) \leq \max\{\mu(x * y), \mu(\beth * x)\}$.

Proposition 2.22.[15]:
Every doubt fuzzy RG-ideal of RG-algebra $(X; *,\beth)$ is a doubt fuzzy RG-subalgebra of X.

Proposition 2.23.[15]:

1- The intersection of any set of doubt fuzzy RG-subalgebras of RG-algebra $(X; *, ב)$ is also doubt fuzzy RG-subalgebra of X , where is chain (Arterian).
2- The union of any set of doubt fuzzy RG-subalgebras of RG-algebra is also doubt fuzzy RG-subalgebra.
3- The intersection of any set of doubt fuzzy RG-ideals of RG-algebra $(X; *, ב)$ is also doubt fuzzy RG-ideal of X, where is chain (Arterian).
4- The union of any set of doubt fuzzy RG-ideals of RG-algebra is also doubt fuzzy RG-ideal.

Definition 2.24.[1]:
If $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ and $B = \{(x, \mu_B(x), \nu_B(x)) \mid x \in X\}$ are two intuitionistic fuzzy subsets of X, then
1) $A \subseteq B$ if and only if $x \in X$, $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$.
2) $A = B$ if and only if $x \in X, \mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$.
3) $A \cap B = \{(x, (\mu_A \cap \mu_B)(x), (\nu_A \cup \nu_B)(x) \mid x \in X\}$.
4) $A \cup B = \{(x, (\mu_A \cup \mu_B)(x), (\nu_A \cap \nu_B)(x) \mid x \in X\}$.

Definition 2.25. [2,3]:
A mapping $f: (X; *, ב) \to (Y; *, ב)$ be a homomorphism of BCK-algebra and for any intuitionistic fuzzy set $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ of X , then
IFS $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ in Y, we define
IFS $A^f = \{(x, \mu_A^f(x), \nu_A^f(x)) \mid x \in X\}$ in X by $\mu_A^f(x) = \mu_A(f(x)), \nu_A^f(x) = \nu_A(f(x)), \forall x \in X$.

Remark. 2.26.[15]: Let $(X; *, ב)$ be a RG-algebra.
1) If μ_A is a RG-subalgebra of X, then $\bar{\mu}_A = 1 - \mu_A$ is a doubt fuzzy RG-subalgebra of X.
2) If ν_A is a doubt fuzzy RG-subalgebra of X, then $\bar{\nu}_A = 1 - \nu_A$ is a fuzzy RG-subalgebra of X.
3) If μ_A is a RG-ideal of RG-algebra of X, then $\bar{\mu}_A = 1 - \mu_A$ is a doubt fuzzy RG- ideal of X.
4) If ν_A is a doubt fuzzy RG- ideal of X, then $\bar{\nu}_A = 1 - \nu_A$ is a fuzzy RG-ideal of X.
5) Note that $\mu_A = \overline{\overline{\mu_A}}$, and $\nu_A = \overline{\overline{\nu_A}}$.

Definition 2.27.
Let $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ be an intuitionistic fuzzy subset of RG-algebra $(X; *, ב)$. A is said to be an **intuitionistic fuzzy RG-subalgebra of X** if
(IFS$_1$) $\mu_A(x * y) \ge \min\{\mu_A(x), \mu_A(y)\}$,
(IFS$_2$) $\nu_A(x * y) \le \max\{\nu_A(x), \nu_A(y)\}$.
That mean μ_A is a fuzzy RG-subalgebra and ν_A is a doubt fuzzy RG-subalgebra.

Proposition 2.28.
1- Every intuitionistic fuzzy RG-subalgebra $\{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ of RG-algebra $(X; *, ב)$, satisfies the inequalities $\mu_A(ב) \ge \mu_A(x)$ and $\nu_A(ב) \le \nu_A(x)$, for all $x \in X$.
2- Let $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ be an intuitionistic fuzzy RG-subalgebra of X if and only if the fuzzy set $\mu_A(x)$ and $\bar{\nu}_A(x)$ are fuzzy RG-subalgebra of X. (or, the fuzzy set $\nu_A(x)$ and $\bar{\mu}_A(x)$ are doubt fuzzy RG-subalgebra of X).

Theorem 2.29.
Let $f: (X; *, ב) \to (Y; *`, ב`)$ be a homomorphism of an RG-algebras $(X; *, ב)$, $(Y; *, ב)$ respectively.
1- If $A = (x, \mu_A(x), \nu_A(x))$ is an intuitionistic fuzzy RG-subalgebra of X with sup and inf properties, then $f(A) = (y, f(\mu_A)(y), f(\nu_A)(y))$ of A is an intuitionistic fuzzy RG-subalgebra of Y, were f is onto.
2- If $B = (\mu_B(x), \nu_B(x))$ is an intuitionistic fuzzy RG-subalgebra of Y with sup and inf properties, then the pre-image $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\nu_B))$ of B under f in X is an intuitionistic fuzzy RG-subalgebra of X.

Definition 2.30.
Let $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ be an intuitionistic fuzzy subset of RG-algebra $(X; *, ב)$. A is said to be **an intuitionistic fuzzy RG-ideal of X** if, for all $x, y \in X$, then
1) $\mu_A(ב) \ge \mu_A(x)$ and $\nu_A(ב) \le \nu_A(x)$,
2) $\mu_A(ב * y) \ge \min\{\mu_A(x * y), \mu_A(ב * x)\}$ and $\nu_A(ב * y) \le \max\{\nu_A(x * y), \nu_A(ב * x)\}$.
That means μ_A is a fuzzy RG-ideal and ν_A is a doubt fuzzy RG-ideal.

Proposition 2.31.
Let $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ be an intuitionistic fuzzy RG-ideal of RG-algebra $(X; *, ב)$, then A is an intuitionistic fuzzy RG-subalgebra of X.

Proposition 2.32.

Let $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ be an intuitionistic fuzzy RG-ideal of X if and only if the fuzzy set $\mu_A(x)$ and $\bar{\nu}_A(x)$ are fuzzy RG-ideal of X.

Theorem 2.33.

Let $f: (X; *, ב) \to (Y; *`, ב`)$ be a homomorphism of an RG-algebras (X; * ,ב) , (Y; * ,ב) respectively.

1- If $A = (x, \mu_A(x), \nu_A(x))$ is an intuitionistic fuzzy RG-ideal of X with sup and inf properties, then $f(A) = (y, f(\mu_A)(y), f(\nu_A)(y))$ of A is an intuitionistic fuzzy RG-ideal of Y, were f is onto.

2- If $B = (\mu_B(x), \nu_B(x))$ is an intuitionistic fuzzy RG-ideal of Y with sup and inf properties, then the pre-image $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\nu_B))$ of B under f in X is an intuitionistic fuzzy RG-ideal of X.

Definition 2.34.[6,7]:

For any intuitionistic fuzzy set $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ of a set X denoted by (IFS) and $\propto \in [ב, 1]$ the operator † : IFS(X) ⟶ IFS(X), : IFS(X) ⟶ IFS(X)are defined as $† A = \{(x, \mu_A(x), 1 - \mu_A(x)) \mid x \in X\}$, $⊞ A = \{(x, 1 - \nu_A(x), \nu_A(x)) \mid x \in X\}$,

Definition 2.35.[6,7]:

For any intuitionistic fuzzy set $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ of X and $\propto, \beta, \gamma, \delta, \varepsilon, \tau \in [ב, 1]$ and $\max(\propto - \varepsilon, \beta - \tau) + \gamma + \delta \le 1$, $\min(\propto - \varepsilon, \beta - \tau) + \gamma + \delta \ge ב$, then the operator $⊡_{\propto,\beta,\gamma,\delta,\tau,\varepsilon}$: IFS(X) ⟶ IFS(X) is defined as

$⊡_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A) = \{(x, \propto \mu_A(x) - \varepsilon \nu_A(x) + \gamma, \beta \nu_A(x) - \tau \mu_A(x) + \delta) \mid x \in X\}$.

If $(\varepsilon = ב = \tau)$, then $\max(\propto, \beta) + \gamma + \delta \le 1$, $\min(\propto, \beta) + \gamma + \delta \ge ב$, and the operator $⊡_{\propto,\beta,\gamma,\delta}$: IFS(X) ⟶ IFS(X) is defined as

$⊡_{\propto,\beta,\gamma,\delta}(A) = \{(x, \propto \mu_A(x) + \gamma, \beta \nu_A(x) + \delta) \mid x \in X\}$.

Definition 2.36. [18]:

A mapping $f: (X; *, ב) \to (Y; *, ב)$be a homeomorphism of BCK-algebra and for any intuitionistic fuzzy set $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ of X , then IFS $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ in Y, we define IFS $A^f = \{(x, \mu_A^f(x), \nu_A^f(x)) \mid x \in X\}$ in X by $\mu_A^f(x) = \mu_A(f(x))$, $\nu_A^f(x) = \nu_A(f(x))$, $\forall x \in X$.

Definition 2.37.[10]:

Let μ and β be fuzzy subsets of a set . The Cartesian product of μ and β is defined by $(\mu \times \beta)(x, y) = \min\{\mu(x), \beta(y)\}$, , for all $, y \in S$.

Lemma 2.38.[10]:

Let S be a set and μ and β be fuzzy subsets of S . Then

(1) $\mu \times \beta$is a fuzzy relation on S,

(2) $(\mu \times \beta)_t = \mu_t \times \beta_t$, for all $t \in [ב, 1]$.

Definition 2.39. [10]:

If R is a fuzzy relation on sets S and β is a fuzzy subset of , then R is a fuzzy relation on β if $R(x, y) = \min\{\beta(x), \beta(y)\}$,, for all $x, y \in S$.

Remark 2.40.[10]:

Let $(X; *, ב)$ and $(Y; *', ב')$ be RG-algebras, we define $(\cdot)$ on $X \times Y$ by: for all $(x, y), (u, v) \in X \times Y$, $(x, y) \cdot (u, v) = (x * u, y *' v)$. Then clearly $(X \times Y; \cdot, (ב, ב'))$

3. Model operator $⊡_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}$ on Intuitionistic Fuzzy RG-subalgebras of RG-algebra

In this section, we study the effect of model operator $⊡_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}$ in intuitionistic fuzzy RG-subalgebras of RG-algebra.

Theorem 3.1.

Let $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ be an intuitionistic fuzzy set of a RG-algebra $(X; *, ב)$. A be an intuitionistic fuzzy RG-subalgebra of X if and only if † A and ⊞ A are intuitionistic fuzzy RG-subalgebras of X.

Proof

If $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ is an intuitionistic fuzzy RG-subalgebra of X, then μ_A and $\bar{\nu}_A$are fuzzy RG-subalgebras and ν_A and $\bar{\mu}_A$ are a doubt fuzzy subalgebras by Proposition (2.28(2)). Hence † A and ⊞ A are intuitionistic fuzzy RG-subalgebras.

Conversely, let $† A = \{(x, \mu_A(x), 1 - \mu_A(x)) \mid x \in X\}$ and $⊞ A = \{(x, 1 - \nu_A(x), \nu_A(x)) \mid x \in X\}$ are intuitionistic fuzzy RG-subalgebras, then the fuzzy subsets $\mu_A$and $\bar{\nu}_A$ are fuzzy RG-subalgebras of X and ν_A and $\bar{\mu}_A$ are doubt fuzzy RG-subalgebras of X, by Proposition (2.28(2)).

Hence $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ intuitionistic fuzzy RG-subalgebra. ■

Lemma 3.2.

If IFS A $=\{(x, \mu_A(x), \nu_A(x)) | x \in X\}$ is an intuitionistic RG-subalgebra of X, then $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A) =$
$\{(x, \propto \mu_A(x) + \varepsilon \bar{\nu}_A(x) + \gamma - \varepsilon, \beta \nu_A(x) + \tau\bar{\mu}_A(x) + \delta - \tau) | x \in X\}$.

Proof

Since $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A) = \{(x, \propto \mu_A(x) - \varepsilon \nu_A(x) + \gamma, \beta \nu_A(x) - \tau\mu_A(x) + \delta) | x \in X\}$,
and $\bar{\nu}_A = 1 - \bar{\nu}_A$, $\bar{\mu}_A = 1 - \bar{\mu}_A$, thus $-\varepsilon \nu_A(x) = -\varepsilon(1 - \bar{\nu}_A)(x) = \bar{\nu}_A(x) - \varepsilon$ and $-\varepsilon \mu_A(x) = -\tau(1 - \bar{\mu}_A)(x) = \bar{\mu}_A(x) - \tau$. Hence
$\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A) = \{(x, \propto \mu_A(x) + \varepsilon \bar{\nu}_A(x) + \gamma - \varepsilon, \beta \nu_A(x) + \tau\bar{\mu}_A(x) + \delta - \tau) | x \in X\}$.

Proposition 3.3.

If A and B be two intuitionistic fuzzy subsets of RG-algebra X and RG-algebra Y respectively and $f: (X; *, ב) \to (Y; *`, ב`)$ be a mapping, then

1) $f(\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A)) \subseteq \boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(f(A))$
2) $f^{-1}(\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}B) = \boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(f^{-1}(B))$

Proof:

1) Let x$\in X$, then
$f(\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A))(x) = (x, \mu(f(\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A))(x), \nu(f(\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A))(x))$
Also,
$f(\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau} A)(x) = (\mu(f(\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau} A)(x)), (\nu(f(\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}A)(x))$
$= (\mu \boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau} A)(f(x)), \nu(\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau} A)(f(x))).$
Now,

$$\begin{aligned}\mu(f(\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A))(x) &= sup_{t\in f(x)}\ \mu\ (\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A))(t)\\ &= sup_{t\in f(x)}\ (\propto \mu_A(t) - \varepsilon\, \nu_A(t) + \gamma)\\ &= \propto sup_{t\in f(x)}\ \mu_A(t) - \varepsilon\ sup_{t\in f(x)}\ \nu_A(t) + \gamma\\ &\le \propto sup_{t\in y}\ \mu_{f(A)}(t) - \varepsilon\, sup_{t\in y}\, \nu_{f(A)}(t) + \gamma\\ &= \propto \mu_{f(A)}(f(x)) - \varepsilon\, \nu_{f(A)}(f(x)) + \gamma\end{aligned}$$

By Definition (2.25), $f(\mu(A)) \subseteq \mu(f(A))$
Similarly, we can prove

$$\begin{aligned}\nu(f(\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A))(x) &= sup_{t\in f(x)}\ \nu\ (\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A))(t)\\ &= sup_{t\in f(x)}\ (\beta\nu_A(t) - \tau\, \mu_A(t) + \delta)\\ &= \beta sup_{t\in f(x)}\ \nu_A(t) - \tau\ sup_{t\in f(x)}\ \mu_A(t) + \delta\\ &\le \beta sup_{t\in y}\ \nu_{f(A)}(t) - \tau\, sup_{t\in y}\, \mu_{f(A)}(t) + \delta\\ &= \beta\, \nu_{f(A)}(f(x)) - \tau\, \mu_{f(A)}(f(x)) + \delta\end{aligned}$$

By Definition (2.25), $f(\nu(A)) \subseteq \nu(f(A))$

Therefore $f(\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A)) \subseteq \boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(f(A))$.

2)
$f^{-1}(\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau} B)(y) = (y, \mu(f^{-1}(\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau} B)(y)), (\nu(f^{-1}(\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}B)(y)))$
Also,
$f^{-1}(\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau} A)(y) = (\mu(f^{-1}(\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau} B)(y)), (\nu(f^{-1}(\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}B)(y))$
$= (\mu \boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau} B) * (f^{-1}(y)), \nu(\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau} B)(f^{-1}(y)))$
Now,

$$\begin{aligned}\mu \boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(B))(f^{-1}(y)) &= \propto \mu_B(f^{-1}(y)) - \varepsilon\, \nu_B(f^{-1}(y)) + \gamma\\ &= \propto \mu_B(f^{-1}(y)) - \varepsilon\, \nu_B(f^{-1}(y)) + \gamma\\ &= \propto \mu_{f^{-1}(B)}(y) - \varepsilon\, \nu_{f^{-1}(B)}(y) + \gamma\end{aligned}$$

$= \mu \boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(f^{-1}(B))(y)$, and

$$\begin{aligned}\nu \boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(B))(f^{-1}(y)) &= \beta\nu_B(f^{-1}(y)) - \tau\, \mu_B(f^{-1}(y)) + \delta\\ &= \beta\nu_B(f^{-1}(y)) - \tau\, \mu_B(f(y)) + \delta\\ &= \beta\nu_{f^{-1}(B)}(y) - \tau\, \mu_{f^{-1}(B)}(y) + \delta\end{aligned}$$

$= \mu \boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(f^{-1}(B))(y)$.

Hence $f^{-1}(\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(B)) = \boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(f^{-1}(B))$. ■

Proposition 3.4.

If IFS $A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$ is an intuitionistic RG-subalgebra of X, then $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A)$ is also is an intuitionistic RG-subalgebra of X.

Proof

Since $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A) = \{(x, \propto \mu_A(x) + \varepsilon\, \bar{\nu}_A(x) + \gamma - \varepsilon, \beta\, \nu_A(x) + \tau\bar{\mu}_A(x) + \delta - \tau) | x \in X\}$, by Lemma (3.2), then for any $x, y \in X$,

$$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x * y) = \propto \mu_A(x * y) + \varepsilon\, \bar{\nu}_A(x * y) + \gamma - \varepsilon$$
$$\geq \propto (\min\{\mu_A(x), \mu_A(y)\}) + \varepsilon \min\{\bar{\nu}_A(x), \bar{\nu}_A(y)\} + \gamma - \varepsilon$$
$$= (\min\{\propto \mu_A(x), \propto \mu_A(y)\}) + \min\{\varepsilon\bar{\nu}_A(x), \varepsilon\bar{\nu}_A(y)\} + \gamma - \varepsilon$$
$$= \min\{\propto \mu_A(x) + \varepsilon\bar{\nu}_A(x) + \gamma - \varepsilon, \propto \mu_A(y) + \varepsilon\bar{\nu}_A(y) + \gamma - \varepsilon\}$$
$$= \min\{\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x), \mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(y)\}.$$

$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)$ is RG-subalgebra of X.

$$\nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x * y) = \beta\, \nu_A(x * y) + \tau\, \bar{\mu}_A(x * y) + \delta - \tau$$
$$\leq \beta(\max\{\nu_A(x), \nu_A(y)\}) + \tau \max\{\bar{\mu}_A(x), \bar{\mu}_A(y)\} + \delta - \tau$$
$$= (\max\{\beta\nu_A(x), \beta\nu_A(y)\}) + \max\{\tau\bar{\mu}_A(x), \tau\bar{\mu}_A(y)\} + \delta - \tau$$
$$= \max\{\beta\nu_A(x) + \tau\bar{\mu}_A(x) + \delta - \tau, \beta\mu_A(y) + \tau\bar{\mu}_A(y) + \delta - \tau\}$$
$$= \max\{\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x), \mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(y)\}.$$

$\nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)$ is a doubt RG-subalgebra of X.

Hence $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A)$ be an intuitionistic fuzzy RG-subalgebra of X. ■

Remark 3.5.

The converse of Proposition (3.4) is not true as the following example:

Example 3.6.

Let X = {ב, 1, 2, 3} in which $*$ is defined by the following table:

*	ב	1	2	3
ב	ב	1	2	3
1	1	ב	3	2
2	2	3	ב	1
3	3	2	1	ב

Then (X; $*$, ב) is an RG-algebra. Define μ_A and ν_A as the following

X	ב	1	2	3
μ_A	0.6	0.6	0.2	0.4
ν_A	0.7	0.2	0.3	0.2

$A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$ is not an intuitionistic, since μ_A are not fuzzy RG-subalgebra of X, since $(\mu_A)(1 * 3) = (\mu_A)(2) = 0.2 \ngeq \min\{\mu_A(1), \mu_A(3)\} = 0.4$.

X	ב	1	2	3
$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}$	0.48	0.42	0.34	0.36
$\nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}$	0.12	0.22	0.22	0.16

If we take $\propto = 0.2, \beta = 0.2, \gamma = 0.3, \delta = 0.2, \varepsilon = 0, \tau = 0.2$, $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A)$ be an intuitionistic fuzzy RG-subalgebra of X.

Proposition 3.7.

If A is an intuitionistic RG-subalgebra of X, then $\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(ב) \geq \mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x)$ and

$v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(ב) \leq v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x), \forall x \in X.$

Proof

Since A is an intuitionistic RG-subalgebra of X, $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A)$ is also an intuitionistic RG-subalgebra of X by Proposition (3.4). Now, for any x ∈X,

$$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(ב) = \mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x*x) \geq \min\left\{\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x), \mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x)\right\}$$

$= \mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x)$ and

$$v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(ב) = v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x*x) \leq \max\left\{v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x), v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x)\right\}$$

$= v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x)$.■

Proposition 3.8.

If $A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$ and $B = \{(x, \mu_B(x), \nu_B(x)) | x \in X\}$ are intuitionistic RG-subalgebras of X, then the intersection on intuitionistic of A and B, $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A \cap B)$ is intuitionistic fuzzy RG-subalgebra of X.

Proof

Let $x, y \in X$ by using Definition (2.24(3))

$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)(x*y) = \propto \mu_{A\cap B}(x*y) - \varepsilon\, \nu_{A\cup B}(x*y) + \gamma$. By Lemma (3.2),

$$\begin{aligned}
\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)(x*y) &= \propto \mu_{A\cap B}(x*y) + \varepsilon\, \bar{\nu}_{A\cap B}(x*y) + \gamma - \varepsilon \\
&\geq \propto (\min\{\mu_{A\cap B}(x), \mu_{A\cap B}(y)\}) + \varepsilon \min\{\bar{\nu}_{A\cap B}(x), \bar{\nu}_{A\cap B}(y)\} + \gamma - \varepsilon \\
&\geq \min\{\propto \mu_{A\cap B}(x), \propto \mu_{A\cap B}(y)\} + \min\{\varepsilon\, \bar{\nu}_{A\cap B}(x), \varepsilon\, \bar{\nu}_{A\cap B}(y)\} + \gamma - \varepsilon \\
&= \min\{\propto \mu_{A\cap B}(x) + \varepsilon\, \bar{\nu}_{A\cap B}(x) + \gamma - \varepsilon, \propto \mu_{A\cap B}(y) + \varepsilon\, \bar{\nu}_{A\cap B}(y) + \gamma - \varepsilon\} \\
&\geq \min\{\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)(x), \mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)(y)\}
\end{aligned}$$

$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)$ is fuzzy RG-subalgebra of X. Now,

$v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)(x) = \beta\, \nu_{A\cup B}(x) - \tau\mu_{A\cap B}(x) + \delta$. By Lemma (3.2),

$$\begin{aligned}
v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cup B)(x*y) &= \beta\nu_{A\cup B}(x*y) + \tau\, \bar{\mu}_{A\cup B}(x*y) + \delta - \tau \\
&\leq \beta(\max\{\nu_{A\cup B}(x), \nu_{A\cup B}(y)\}) + \tau \max\{\bar{\mu}_{A\cup B}(x), \bar{\mu}_{A\cup B}(y)\} + \delta - \tau \\
&\leq \max\{\beta\nu_{A\cup B}(x), \beta\nu_{A\cup B}(y)\} + \max\{\tau\bar{\mu}_{A\cup B}(x), \tau\bar{\mu}_{A\cup B}(y)\} + \delta - \tau \\
&\leq \max\{\beta\nu_{A\cup B}(x) + \tau\, \bar{\mu}_{A\cup B}(x) + \delta - \tau, \beta\nu_{A\cup B}(y) + \tau\bar{\mu}_{A\cup B}(y) + \delta - \tau\} \\
&\leq \max\{v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cup B)(x), v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cup B)(y)\}
\end{aligned}$$

$v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cup B)$ is a doubt fuzzy RG-subalgebra of X.

Hence the intersection on intuitionistic of A and B $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A \cap B)$ be an intuitionistic fuzzy RG-subalgebra of X. ■

Proposition 3.9.

If $A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$ and $B = \{(x, \mu_B(x), \nu_B(x)) | x \in X\}$ are intuitionistic RG-subalgebras of X, then the union on intuitionistic of A and B $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A \cup B)$ is intuitionistic fuzzy RG-subalgebra of X, where v_{A_i} is chain (Arterian) and μ_{A_i}, chain (Notherian).

Proof

Let $x, y \in X$ by using Definition (2.24(4))

$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A\cup B)(x*y) = \propto \mu_{A\cup B}(x*y) - \varepsilon\, \nu_{A\cap B}(x*y) + \gamma$. By Lemma (3.2),

$$\begin{aligned}
\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cup B)(x*y) &= \propto \mu_{A\cup B}(x*y) + \varepsilon\, \bar{\nu}_{A\cup B}(x*y) + \gamma - \varepsilon \\
&\geq \propto (\max\{\mu_{A\cup B}(x), \mu_{A\cup B}(y)\}) + \varepsilon \max\{\bar{\nu}_{A\cup B}(x), \bar{\nu}_{A\cup B}(y)\} + \gamma - \varepsilon \\
&\geq \max\{\propto \mu_{A\cup B}(x), \propto \mu_{A\cup B}(y)\} + \max\{\varepsilon\, \bar{\nu}_{A\cup B}(x), \varepsilon\, \bar{\nu}_{A\cup B}(y)\} + \gamma - \varepsilon \\
&= \max\{\propto \mu_{A\cup B}(x) + \varepsilon\, \bar{\nu}_{A\cap B}(x) + \gamma - \varepsilon, \propto \mu_{A\cap B}(y) + \varepsilon\bar{\nu}_{A\cap B}(y) + \gamma - \varepsilon\} \\
&\geq \max\{\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cup B)(x), \mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cup B)(y)\}
\end{aligned}$$

$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cup B)$ is fuzzy RG-subalgebra of X. Now,

$v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)(x) = \beta\, \nu_{A\cup B}(x) - \tau\mu_{A\cap B}(x) + \delta$. By Lemma (3.2),

$$\begin{aligned}
v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)(x*y) &= \beta\nu_{A\cap B}(x*y) + \tau\, \bar{\mu}_{A\cap B}(x*y) + \delta - \tau \\
&\leq \beta(\min\{\nu_{A\cap B}(x), \nu_{A\cap B}(y)\}) + \tau \min\{\bar{\mu}_{A\cap B}(x), \bar{\mu}_{A\cap B}(y)\} + \delta - \tau \\
&\leq \min\{\beta\nu_{A\cap B}(x), \beta\nu_{A\cap B}(y)\} + \min\{\tau\bar{\mu}_{A\cap B}(x), \tau\bar{\mu}_{A\cap B}(y)\} + \delta - \tau \\
&\leq \min\{\beta\nu_{A\cap B}(x) + \tau\, \bar{\mu}_{A\cap B}(x) + \delta - \tau, \beta\nu_{A\cap B}(y) + \tau\bar{\mu}_{A\cap B}(y) + \delta - \tau\}
\end{aligned}$$

$$\leq \min\{\nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)(x), \nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)(y)\}$$

$\nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)$ is a doubt fuzzy RG-subalgebra of X.

Hence the union on intuitionistic of A and B $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A \cap B)$ be an intuitionistic fuzzy RG-subalgebra of X. ■

Proposition 3.10.

If A and B are two an intuitionistic fuzzy RG-subalgebra of RG-algebra X, then $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A \times B)$ be an intuitionistic fuzzy RG-subalgebra of $X \times X$.

Proof:

Let $(x_1, x_2), (y_1, y_2) \in X \times X$, by using Definition (2.37)

$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \times B)(x_1 * y_1, x_2 * y_2) = \propto \mu_{A\times B}(x_1 * y_1, x_2 * y_2) - \varepsilon\, \nu_{A\times B}(x_1 * y_1, x_2 * y_2) + \gamma$. By Lemma (3.2),

$$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \times B)(x_1 * y_1, x_2 * y_2)$$
$$= \propto \mu_{A\times B}(x_1 * y_1, x_2 * y_2) + \varepsilon\, \bar{\nu}_{A\times B}(x_1 * y_1, x_2 * y_2) + \gamma - \varepsilon$$
$$\geq \propto (\min\{\mu_A(x_1 * y_1), \mu_B(x_2 * y_2)\}) + \varepsilon \min\{\bar{\nu}_A(x_1 * y_1), \bar{\nu}_B(x_2 * y_2)\} + \gamma - \varepsilon$$
$$\geq \propto (\min\{\min\{\mu_A(x_1), \mu_A(y_1)\}, \min\{\mu_B(x_2), \mu_B(y_2)\}\}) + \varepsilon (\min\{\min\{\bar{\nu}_A(x_1), \bar{\nu}_A(y_1)\}, \min\{\bar{\nu}_B(x_2), \bar{\nu}_B(y_2)\}\}) + \gamma - \varepsilon$$
$$\geq \propto (\min\{\min\{\mu_A(x_1), \mu_B(x_2)\}, \min\{\mu_A(y_1), \mu_B(y_2)\}\}) + \varepsilon (\min\{\min\{\bar{\nu}_A(x_1), \bar{\nu}_B(x_2)\}, \min\{\bar{\nu}_A(y_1), \bar{\nu}_B(y_2)\}\}) + \gamma - \varepsilon$$
$$\geq \propto (\min\{\mu_{A\times B}(x_1, x_2), \mu_{A\times B}(y_1, y_2)\}) + \varepsilon (\min\{\nu_{A\times B}(x_1, x_2), \nu_{A\times B}(y_1, y_2)\}) + \gamma - \varepsilon$$
$$\geq \min\{\propto \mu_{A\times B}(x_1, x_2) + \varepsilon\bar{\nu}_{A\times B}(x_1, x_2) + \gamma - \varepsilon, \propto \mu_{A\times B}(y_1, y_2) + \varepsilon\, \bar{\nu}_{A\times B}(y_1, y_2) + \gamma - \varepsilon\}$$

$= \min\{\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \times B)(x_1, x_2), \mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \times B)(y_1, y_2)\}$. Thus

$$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \times B)((x_1, x_2) * (y_1, y_2)) \geq \min\{\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \times B)(x_1, x_2), \mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \times B)(y_1, y_2)\}$$

$$\nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \times B)((x_1 * y_1, x_2 * y_2))$$
$$= \beta\, \nu_{A\times B}((x_1 * y_1, x_2 * y_2)) + \tau\, \bar{\mu}_{A\times B}((x_1 * y_1, x_2 * y_2)) + \delta - \tau$$
$$\leq \beta(\max\{\nu_A((x_1 * y_1)), \nu_B((x_2 * y_2))\}) + \tau \max\{\bar{\mu}_A(x_1 * y_1), \bar{\mu}_B((x_2 * y_2))\} + \delta - \tau$$
$$\leq \beta(\max\{\max\{\nu_A(x_1), \nu_A(y_1)\}, \max\{\nu_B(x_2), \nu_B(y_2)\}\}) + \tau (\max\{\max\{\bar{\mu}_A(x_1), \bar{\mu}_A(y_1)\}, \max\{\bar{\mu}_B(x_2), \bar{\mu}_B(y_2)\}\}) + \delta - \tau$$
$$\leq \beta(\max\{\max\{\nu_A(x_1), \nu_B(x_2)\}, \max\{\nu_A(y_1), \nu_B(y_2)\}\}) + \tau (\max\{\max\{\bar{\mu}_A(x_1), \bar{\mu}_B(x_2)\}, \max\{\bar{\mu}_A(y_1), \bar{\mu}_B(y_2)\}\}) + \delta - \tau$$
$$\leq \max\{\beta\nu_{A\times B}(x_1, x_2) + \tau\bar{\mu}_{A\times B}(x_1, x_2) + \delta - \tau, \beta\nu_{A\times B}(y_1, y_2) + \tau\, \bar{\mu}_{A\times B}((y_1, y_2)) + \delta - \tau\}$$
$$= \max\{\nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \times B)(x_1, x_2), \nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \times B)((y_1, y_2))\}$$

Thus

$\nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \times B)(x_1, x_2) * (y_1, y_2) \leq \max\{\nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \times B)(x_1, x_2), \nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \times B)(y_1, y_2)\}$.■

4. Model operator $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}$ on Intuitionistic Fuzzy RG-ideals of RG-algebra

In this section, we study the effect of model operator $\boxdot_{\propto,\boldsymbol{\beta},\boldsymbol{\gamma},\boldsymbol{\delta},\boldsymbol{\varepsilon},\boldsymbol{\tau}}$ in intuitionistic fuzzy RG-ideal of RG-algebra.

Theorem 4.1.

Let $A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$ be an intuitionistic fuzzy set of RG-algebra $(X; *, ב)$. A is an intuitionistic fuzzy RG-ideal of X if and only if $\dagger$ A and $\boxplus$ A are intuitionistic fuzzy RG-ideals of X.

Proof

If $A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$ is an intuitionistic fuzzy RG-ideal of X, then μ_A and $\bar{\nu}_A$ are fuzzy RG-ideals and ν_A and $\bar{\mu}_A$ are a doubt fuzzy ideals by Proposition (2.32). Hence $\dagger$ A and $\boxplus$ A are intuitionistic fuzzy RG-ideals.

Conversely, let $\dagger A = \{(x, \mu_A(x), 1 - \mu_A(x)) | x \in X\}$ and $\boxplus A = \{(x, 1 - \nu_A(x), \nu_A(x)) | x \in X\}$ are intuitionistic fuzzy RG-ideals, then the fuzzy subsets μ_A and $\bar{\nu}_A$ are fuzzy RG-ideals of X and ν_A and $\bar{\mu}_A$ are doubt fuzzy RG-ideals of X, by Proposition (2.32).

Hence $A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$ intuitionistic fuzzy RG-ideal. ■

Proposition 4.2.

If IFS $A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$ is an intuitionistic RG-ideal of X, then $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A)$ is also is an intuitionistic RG-ideal of X.

Proof

Since $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A) = \{(x, \propto \mu_A(x) + \varepsilon\, \bar{\nu}_A(x) + \gamma - \varepsilon, \beta\, \nu_A(x) + \tau\bar{\mu}_A(x) + \delta - \tau) | x \in X\}$, by Lemma (3.2), then for any $x, y \in X$,

$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(ב) = \propto \mu_A(ב) + \varepsilon\, \bar{\nu}_A(ב) + \gamma - \varepsilon \geq \mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x) = \propto \mu_A(x) + \varepsilon\, \bar{\nu}_A(x) + \gamma - \varepsilon$ and

$$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(ב * y) = \propto \mu_A(ב * y) + \varepsilon\, \bar{\nu}_A(ב * y) + \gamma - \varepsilon$$
$$\geq \propto (\min\{\mu_A(x * y), \mu_A(ב * x)\}) + \varepsilon \min\{\bar{\nu}_A(x * y), \bar{\nu}_A(ב * x)\} + \gamma - \varepsilon$$

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$= (\min\{\propto \mu_A(x*y), \propto \mu_A(ב*x)\}) + \min\{\varepsilon \bar{v}_A(x*y), \varepsilon \bar{v}_A(ב*x)\} + \gamma - \varepsilon$

$= \min\{\propto \mu_A(x*y) + \varepsilon \bar{v}_A(x*y) + \gamma - \varepsilon, \propto \mu_A(ב*x) + \varepsilon \bar{v}_A(ב*x) + \gamma - \varepsilon\}$

$= \min\{\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x*y), \mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(ב*x)\}$.

$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)$ is RG-ideal of X.

$v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(ב) = \beta v_A(ב) + \tau \bar{\mu}_A(ב) + \delta - \tau \le v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x) = \beta v_A(x) + \tau \bar{\mu}_A(x) + \delta - \tau$ and

$v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(ב*y) = \beta v_A(ב*y) + \tau \bar{\mu}_A(ב*y) + \delta - \tau$

$\le \beta(\max\{v_A(x*y), v_A(ב*x)\}) + \tau \max\{\bar{\mu}_A(x*y), \bar{\mu}_A(ב*x)\} + \delta - \tau$

$= (\max\{\beta v_A(x*y), \beta v_A(ב*x)\}) + \max\{\tau \bar{\mu}_A(x*y), \tau \bar{\mu}_A(ב*x)\} + \delta - \tau$

$= \max\{\beta v_A(x*y) + \tau \bar{\mu}_A(x*y) + \delta - \tau, \beta \mu_A(ב*x) + \tau \bar{\mu}_A(ב*x) + \delta - \tau\}$

$= \max\{\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x*y), \mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(ב*x)\}$.

$v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)$ is a doubt RG-ideal of X.

Hence $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A)$ be an intuitionistic fuzzy RG-subalgebra of X.■

Remark 4.3.

The converse of Proposition (4.2) is not true as the following example:

Example 4.4.

Let X = {ב, 1, 2, 3} in which $*$ is defined by the following table:

*	ב	1	2	3
ב	ב	1	2	3
1	1	ב	3	2
2	2	3	ב	1
3	3	2	1	ב

Then (X; $*$, ב) is an RG-algebra. Define μ_A and v_A as the following

X	ב	1	2	3
μ_A	0.6	0.6	0.4	0.2
v_A	0.7	0.2	0.2	0.3

$A = \{(x, \mu_A(x), v_A(x)) | x \in X\}$ is not an intuitionistic, since μ_A are not fuzzy RG-ideal of X, since $(\mu_A)(ב*3) = 0.2 \ngeq \min\{\mu_A(1*3), \mu_A(ב*3)\} = 0.4$.

X	ב	1	2	3
$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}$	0.48	0.42	0.36	0.34
$v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}$	0.12	0.22	0.16	0.22

If we take $\propto = 0.2, \beta = 0.2, \gamma = 0.3, \delta = 0.2, \varepsilon = 0, \tau = 0.2$, $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A)$ be an intuitionistic fuzzy RG-ideal of X.

Proposition 4.5.

If $A = \{(x, \mu_A(x), v_A(x)) | x \in X\}$ and $B = \{(x, \mu_B(x), v_B(x)) | x \in X\}$ are intuitionistic RG-ideals of X, then the intersection on intuitionistic of A and B, $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A \cap B)$ is intuitionistic fuzzy RG-ideal of X.

Proof

Let $x, y \in X$ by using Definition (2.24(3))

$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)(x*y) = \propto \mu_{A\cap B}(x*y) - \varepsilon v_{A\cup B}(x*y) + \gamma$. By Lemma (3.2),

$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(ב) = \propto \mu_A(ב) + \varepsilon \bar{v}_A(ב) + \gamma - \varepsilon \ge \mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x) = \propto \mu_A(x) + \varepsilon \bar{v}_A(x) + \gamma - \varepsilon$ and

$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)(ב*y) = \propto \mu_{A\cap B}(ב*y) + \varepsilon \bar{v}_{A\cap B}(ב*y) + \gamma - \varepsilon$

$\ge \propto (\min\{\mu_{A\cap B}(x*y), \mu_{A\cap B}(ב*y)\}) + \varepsilon \min\{\bar{v}_{A\cap B}(x*y), \bar{v}_{A\cap B}(ב*y)\} + \gamma - \varepsilon$

$\ge \min\{\propto \mu_{A\cap B}(x*y), \propto \mu_{A\cap B}(ב*y)\} + \min\{\varepsilon \bar{v}_{A\cap B}(x*y), \varepsilon \bar{v}_{A\cap B}(ב*y)\} + \gamma - \varepsilon$

$= \min\{\propto \mu_{A\cap B}(x*y) + \varepsilon \bar{v}_{A\cap B}(x*y) + \gamma - \varepsilon, \propto \mu_{A\cap B}(ב*y) + \varepsilon \bar{v}_{A\cap B}(ב*y) + \gamma - \varepsilon\}$

$\ge \min\{\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)(x*y), \mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)(ב*y)\}$

$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)$ is fuzzy RG-ideal of X. Now,

$\nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)(x) = \beta\, \nu_{A\cup B}(x) - \tau\mu_{A\cap B}(x) + \delta$. By Lemma (3.2),

$\nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(\beth) = \beta\, \nu_A(\beth) + \tau\, \bar{\mu}_A(\beth) + \delta - \tau \le \nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x) = \beta\, \nu_A(x) + \tau\, \bar{\mu}_A(x) + \delta - \tau$ and

$$\begin{aligned}\nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cup B)(\beth * y) &= \beta\nu_{A\cup B}(\beth * y) + \tau\, \bar{\mu}_{A\cup B}(\beth * y) + \delta - \tau \\ &\le \beta(\max\{\nu_{A\cup B}(x * y), \nu_{A\cup B}(\beth * x)\}) + \tau \max\{\bar{\mu}_{A\cup B}(x * y), \bar{\mu}_{A\cup B}(\beth * x)\} + \delta - \tau \\ &\le \max\{\beta\nu_{A\cup B}(x * y), \beta\nu_{A\cup B}(\beth * x)\} + \max\{\tau\bar{\mu}_{A\cup B}(x * y), \tau\bar{\mu}_{A\cup B}(\beth * x)\} + \delta - \tau \\ &\le \max\{\beta\nu_{A\cup B}(x * y) + \tau\, \bar{\mu}_{A\cup B}(x * y) + \delta - \tau, \beta\nu_{A\cup B}(\beth * x) + \tau\bar{\mu}_{A\cup B}(\beth * x) + \delta - \tau\} \\ &\le \max\{\nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cup B)(x * y), \nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cup B)(\beth * x)\}\end{aligned}$$

$\nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cup B)$ is a doubt fuzzy RG-ideal of X.

Hence the intersection on intuitionistic of A and B $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A \cap B)$ be an intuitionistic fuzzy RG-ideal of X. ■

Proposition 4.6.

If $A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$ and $B = \{(x, \mu_B(x), \nu_B(x)) | x \in X\}$ are intuitionistic RG-ideals of X, then the union on intuitionistic of A and B $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A \cup B)$ is intuitionistic fuzzy RG-ideal of X, where ν_{A_i} is chain (Arterian) and μ_{A_i} chain (Notherian).

Proof

Let $x, y \in X$ by using Definition (2.24(4))

$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A\cup B)(x * y) = \propto \mu_{A\cup B}(x * y) - \varepsilon\, \nu_{A\cap B}(x * y) + \gamma$. By Lemma (3.2),

$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(\beth) = \propto \mu_A(\beth) + \varepsilon\, \bar{\nu}_A(\beth) + \gamma - \varepsilon \ge \mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x) = \propto \mu_A(x) + \varepsilon\, \bar{\nu}_A(x) + \gamma - \varepsilon$ and

$$\begin{aligned}\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cup B)(\beth * y) &= \propto \mu_{A\cup B}(\beth * y) + \varepsilon\, \bar{\nu}_{A\cup B}(\beth * y) + \gamma - \varepsilon \\ &\ge \propto (\max\{\mu_{A\cup B}(x * y), \mu_{A\cup B}(\beth * x)\}) + \varepsilon \max\{\bar{\nu}_{A\cup B}(x * y), \bar{\nu}_{A\cup B}(\beth * x)\} + \gamma - \varepsilon \\ &\ge \max\{\propto \mu_{A\cup B}(x * y), \propto \mu_{A\cup B}(\beth * x)\} + \max\{\varepsilon\, \bar{\nu}_{A\cup B}(x * y), \varepsilon\, \bar{\nu}_{A\cup B}(\beth * x)\} + \gamma - \varepsilon \\ &= \max\{\propto \mu_{A\cup B}(x * y) + \varepsilon\, \bar{\nu}_{A\cap B}(x * y) + \gamma - \varepsilon, \propto \mu_{A\cap B}(\beth * x) + \varepsilon\bar{\nu}_{A\cap B}(\beth * x) + \gamma - \varepsilon\} \\ &\ge \max\{\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cup B)(x * y), \mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cup B)(\beth * x)\}\end{aligned}$$

$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cup B)$ is fuzzy RG-ideal of X. Now,

$\nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)(x) = \beta\, \nu_{A\cup B}(x) - \tau\mu_{A\cap B}(x) + \delta$. By Lemma (3.2),

$\nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(\beth) = \beta\, \nu_A(\beth) + \tau\, \bar{\mu}_A(\beth) + \delta - \tau \le \nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A)(x) = \beta\, \nu_A(x) + \tau\, \bar{\mu}_A(x) + \delta - \tau$ and

$$\begin{aligned}\nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)(\beth * y) &= \beta\nu_{A\cap B}(\beth * y) + \tau\, \bar{\mu}_{A\cap B}(\beth * y) + \delta - \tau \\ &\le \beta(\min\{\nu_{A\cap B}(x * y), \nu_{A\cap B}(\beth * x)\}) + \tau \min\{\bar{\mu}_{A\cap B}(x * y), \bar{\mu}_{A\cap B}(\beth * x)\} + \delta - \tau \\ &\le \min\{\beta\nu_{A\cap B}(x * y), \beta\nu_{A\cap B}(\beth * x)\} + \min\{\tau\bar{\mu}_{A\cap B}(x * y), \tau\bar{\mu}_{A\cap B}(\beth * x)\} + \delta - \tau \\ &\le \min\{\beta\nu_{A\cap B}(x * y) + \tau\, \bar{\mu}_{A\cap B}(x * y) + \delta - \tau, \beta\nu_{A\cap B}(\beth * x) + \tau\bar{\mu}_{A\cap B}(\beth * x) + \delta - \tau\} \\ &\le \min\{\nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)(x * y), \nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)(\beth * x)\}\end{aligned}$$

$\nu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \cap B)$ is a doubt fuzzy RG-ideal of X.

Hence the union on intuitionistic of A and B $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A \cap B)$ be an intuitionistic fuzzy RG-ideal of X. ■

Proposition 4.7.

If A and B are two an intuitionistic fuzzy RG-ideal of RG-algebra X, then $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A \times B)$ be an intuitionistic fuzzy RG-ideal of $X \times X$.

Proof:

Let $(x_1, x_2), (y_1, y_2) \in X \times X$, by Definition (2.37)

$$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \times B)(\beth, \beth) = \propto \mu_{A\times B}(\beth, \beth) - \varepsilon\, \nu_{A\times B}(\beth, \beth) + \gamma \ge \mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \times B)(x_1, x_2) = \propto \mu_{A\times B}(x_1, x_2) - \varepsilon\, \nu_{A\times B}(x_1, x_2) + \gamma$$

And

$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \times B)(x_1 * y_1, x_2 * y_2) = \propto \mu_{A\times B}(x_1 * y_1, x_2 * y_2) - \varepsilon\, \nu_{A\times B}(x_1 * y_1, x_2 * y_2) + \gamma$. By Lemma (3.2),

$$\begin{aligned}\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A \times B)(\beth * y_1, \beth * y_2) \\ &= \propto \mu_{A\times B}(\beth * y_1, \beth * y_2) + \varepsilon\, \bar{\nu}_{A\times B}(\beth * y_1, \beth * y_2) + \gamma - \varepsilon \\ &= \propto (\min\{\mu_A(\beth * y_1), \mu_B(\beth * y_2)\}) + \varepsilon\, (\min\{\bar{\nu}_A(\beth * y_1), \bar{\nu}_B(\beth * y_2) + \gamma - \varepsilon\end{aligned}$$

$\ge \propto (\min\{\min\{\mu_A(x_1 * y_1), \mu_A(\beth * x_1)\}, \min\{\mu_B(x_2 * y_2), \mu_B(\beth * x_2)\}\}) + \varepsilon\, (\min\{\min\{\bar{\nu}_A(x_1 * y_1), \bar{\nu}_A(\beth * x_1)\}, \min\{\bar{\nu}_B(x_2 * y_2), \bar{\nu}_B(\beth * x_2)\}\}) + \gamma - \varepsilon$.

$\ge \propto (\min\{\min\{\mu_A(x_1 * y_1), \mu_B(x_2 * y_2)\}, \min\{\mu_A(\beth * x_1), \mu_B(\beth * x_2)\}\}) + \varepsilon\, (\min\{\min\{\bar{\nu}_A(x_1 * y_1), \bar{\nu}_B(x_2 * y_2)\}, \min\{\bar{\nu}_A(\beth * x_1), \bar{\nu}_B(\beth * x_2)\}\}) + \gamma - \varepsilon$.

$\geq \propto (\min\{\mu_{A\times B}(x_1 * y_1, x_2 * y_2), \mu_{A\times B}(ב * x_1, ב * x_2)\}) + \varepsilon (\min\{\bar{v}_{A\times B}(x_1 * y_1, x_2 * y_2), \bar{v}_{A\times B}(ב * x_1, ב * x_2)\}) + \gamma - \varepsilon$.

$\geq (\min\{\propto \mu_{A\times B}(x_1 * y_1, x_2 * y_2), \propto \mu_{A\times B}(ב * x_1, ב * x_2)\}) + (\min\{\varepsilon \bar{v}_{A\times B}(x_1 * y_1, x_2 * y_2), \varepsilon \bar{v}_{A\times B}(ב * x_1, ב * x_2)\}) + \gamma - \varepsilon$

$\geq \min\{\propto \mu_{A\times B}(x_1 * y_1, x_2 * y_2) + \varepsilon \bar{v}_{A\times B}(x_1 * y_1, x_2 * y_2) + \gamma - \varepsilon, \propto \mu_{A\times B}(ב * x_1, ב * x_2) + \varepsilon \bar{v}_{A\times B}(ב * x_1, ב * x_2) + \gamma - \varepsilon\}$.

$\geq \min\{\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A\times B)((x_1 * y_1, x_2 * y_2), \mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A\times B)(ב * x_1, ב * x_2)\}$. Thus

$\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A\times B)(ב * y_1, ב * y_2) \geq \min\{\mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A\times B)((x_1 * y_1, x_2 * y_2), \mu_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A\times B)(ב * x_1, ב * x_2)\}$.

And

$v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A\times B)(ב * y_1, ב * y_2)$

$= \beta v_{A\times B}(ב * y_1, ב * y_2) + \tau\ \bar{\mu}_{A\times B}(ב * y_1, ב * y_2) + \delta - \tau$

$= \beta(\max\{v_A(ב * y_1), v_B(ב * y_2)\}) + \tau (\max\{\bar{\mu}_A(ב * y_1), \bar{\mu}_B(ב * y_2) + \delta - \tau$

$\leq \beta(\max\{\max\{v_A(x_1 * y_1), v_A(ב * x_1)\}, \max\{v_B(x_2 * y_2), v_B(ב * x_2)\}\}) + \tau (\max\{\max\{\bar{\mu}_A(x_1 * y_1), \bar{\mu}_A(ב * x_1)\}, \max\{\bar{\mu}_B(x_2 * y_2), \bar{\mu}_B(ב * x_2)\}\}) + \delta - \tau$.

$\leq \beta(\max\{\max\{v_A(x_1 * y_1), v_B(x_2 * y_2)\}, \max\{v_A(ב * x_1), v_B(ב * x_2)\}\}) + \tau (\max\{\max\{\bar{\mu}_A(x_1 * y_1), \bar{\mu}_B(x_2 * y_2)\}, \max\{\bar{\mu}_A(ב * x_1), \bar{\mu}_B(ב * x_2)\}\}) + \delta - \tau$.

$\leq \beta(\max\{v_{A\times B}(x_1 * y_1, x_2 * y_2), v_{A\times B}(ב * x_1, ב * x_2)\}) + \tau (\max\{\bar{\mu}_{A\times B}(x_1 * y_1, x_2 * y_2), \bar{\mu}_{A\times B}(ב * x_1, ב * x_2)\}) + \delta - \tau$.

$\leq (\max\{\beta\, v_{A\times B}(x_1 * y_1, x_2 * y_2), \beta\, v_{A\times B}(ב * x_1, ב * x_2)\}) + (\max\{\tau\, \bar{\mu}_{A\times B}(x_1 * y_1, x_2 * y_2), \tau\, \bar{\mu}_{A\times B}(ב * x_1, ב * x_2)\}) + \delta - \tau$

$\leq \max\{\beta\, v_{A\times B}(x_1 * y_1, x_2 * y_2) + \tau\, \bar{\mu}_{A\times B}(x_1 * y_1, x_2 * y_2) + \delta - \tau, \beta\, v_{A\times B}(ב * x_1, ב * x_2) + \tau\, \bar{\mu}_{A\times B}(ב * x_1, ב * x_2) + \delta - \tau\}$.

$\leq \max\{v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A\times B)((x_1 * y_1, x_2 * y_2), v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A\times B)(ב * x_1, ב * x_2)\}$. Thus

$v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A\times B)(ב * y_1, ב * y_2) \leq \max\{v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A\times B)((x_1 * y_1, x_2 * y_2), v_{\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}}(A\times B)(ב * x_1, ב * x_2)\}$.

Hence $\boxdot_{\propto,\beta,\gamma,\delta,\varepsilon,\tau}(A\times B)$ be an intuitionistic fuzzy RG-ideal of $X \times X$.■

References

[1] K.T. Atanassov, **Intuitionistic Fuzzy Sets**, Fuzzy Sets and Systems, vol.20, no.1(1986), pp: 87-96.

[2] K.T. Atanassov, **New Operations Defined over the Intuitionistic Fuzzy Sets**, Fuzzy Sets and Systems, vol.61, no.2(1994), pp:137-142.

[3] K. T. Atanassov, **On Intuitionistic Fuzzy Sets Theory**. Published by Springer-Verlag Berlin: Heidelberg, 2012.

[4] K. T. Atanassov, **Intuitionistic Fuzzy Sets Theory and Applications**, Studies in Fuzziness and Soft Computing, vol. 35, Physica-Verlag, Heidelberg, New York, 1999.

[5] K. T. Atanassov, **Two operators on intuitionistic fuzzy sets** , Comptes Rendus de 1' Academie bulgare des Sciences, vol.41, no.5 (1988), pp: 35-38.

[6] K. T. Atanassov, **More on intuitionistic fuzzy sets** , Fuzzy Sets and Systems, vol.33 (1989), pp:37-45.

[7] K. T. Atanassov, **A new intuitionistic fuzzy model operator**, Notes on Intuitionistic Fuzzy Sets, vol.9, no.2 (2003), pp:56-60.

[8] K. T. Atanassov, **The most general form of one type of intuitionistic fuzzy model operators**., Part 2, Notes on Intuitionistic Fuzzy Sets, vol.14, no.1 (2008), pp:27-32.

[9] K. Dencheva , **Extension of intuitionistic fuzzy modal operators and ,** Proc. of the Second Int. IEEE Symp. on Intelligent Systems, Varna, June 22-24 June, vol.3 (2004), pp:21-22.

[10] A.T. Hameed, **Fuzzy Ideals of Some Algebras**, PH. Sc. Thesis, AinShams University, Faculty of Sciences, Egypt, 2015.

[11] AT. Hameed, and B.H. Hadi, **Intuitionistic Fuzzy AT-Ideals on AT-algebras,** Journal of Adv Research in Dynamical & Control Systems, vol.10, 10-Special Issue, 2018.

[12] A.T. Hameed, A.S. abed and A.H. Abed, **TL-ideals of BCC-algebras**, Jour of Adv Research in Dynamical & Control Systems, Vol. 10, 11- Special Issue, (2018).

[13] A.T. Hameed, **Intuitionistic Fuzzy AT-ideals of AT-algebras**, LAP LEMBERT Academic Publishing , Germany, 2019.

[14] A.T. Hameed, S.M. Abrahem and A.H. Abed, **Fuzzy RG-Ideals of RG-algebra**, (2022), **to appear**.

[15] A.T. Hameed, S.M. Abrahem and A.H. Abed, **Doubt Fuzzy RG-ideals of RG-algebra**, (2022), **to appear**.

[16] A.T. Hameed and S.M. Abrahem, **Intuitionistic Fuzzy RG-ideals of RG-algebra**, (2022), **to appear**.

[17] S.M. Mostafa, M.A. Abd-Elnaby and O.R. Elgendy, **Intuitionistic Fuzzy KU-ideals in KU-algebras**, Int. Journal of Mathematical Sciences and Applications, vol.1, no.3(2011), pp:1379-1384.

[18] P. Murugadas, S. Sriram and T. Muthuraji, **Model operators in intuitionistic fuzzy matrices**, International Journal of Computer Applications, vol.90, no.17 (2014), pp:1-4.

[19] R.A.K. Omar, **On RG-algebra,** Pure Mathematical Sciences, vol.3, no.2 (2014), pp.59-70.

[20] P. Patthanangkoor, **RG-Homomorphism and Its Properties**, Thai Journal of Science and Technology, vol.7, no.5 (2018), pp: 452-459.

[21] T. Senapati, M. Bhowmik, and M. Pol, **Atanassov's intuitionistic fuzzy translations of intuitionistic fuzzy subalgebras and ideals in BCK/BCI-algebras,** Journal Eurasian Mathmatical, vol. 6, no. 1(2015), pp:96-114.

[22] T. Senapati, M. Bhowmik, and M. Pol, **Atanassov's intuitionistic fuzzy translations of intuitionistic fuzzy H-ideals in BCK/BCI-algebras,** vol. 19, no. 1(2013), pp:32-47.

[23] L.A. Zadeh, **Fuzzy Sets**, Inform. And Control, vol.8(1965), pp: 338-353.