International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 7 Issue 1, January - 2023, Pages: 78-82

Solving Constraint and Unconstraint Optimization Problems Using Approximate Methods

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Abstract— *In One-dimensional problem of numerical optimization, the Golden section search method and Lagrange multiplier method They an iterative optimization procedure of the first order. process for locating a function's local lowest and maximum values. the function can be minimized using this strategy in applied mathematics. this methods is typically taught at we implementation the new approach for Solving constraints and unconstraint numerical optimization. A solution's degree of goodness is determined by minimizing or optimizing an objective function (e.g., cost). when searching for a solution, constraints and without constraints and system model are utilized as a guide. instead, optimization aims to improve (or reduce) the value of the objective function while taking into consideration a variety of restrictions.*

Keywords— Golden section search method, Lagrange multiplier method, Numerical Optimization.

1. Introduction

Optimizing processes is one of the most powerful approaches in process integration. "Best" is a term used in optimization to describe the most advantageous option among a set of feasible alternatives mathematical modeling and numerical simulation. A mathematical model is a representation of physical reality that can be analyzed and calculated. We can compute the using numerical simulation, [1,2,3,13]. Calculate a model's solution on a computer in order to make a virtual duplicate of physical reality. PDEs (partial differential equations) or multivariable differential equations will be our major modeling tool in this inquiry (time and space, for example). Applied mathematics has a third fundamental feature: the mathematical study of models. Mathematical analysis is a necessary step It is possible to get some severe shocks from numerical solutions to physical models. A detailed understanding of the underlying mathematical ideas is required to fully appreciate them. and Nonlinear problems and applications are the driving force behind applied mathematics. Difficulties that do not have any random or stochastic aspects. Finally, something must be done in order for this to work. In our efforts to be simple and understandable, we may occasionally use ambiguous language.In our use of mathematics. We may ensure the more discerning reader that Numerical algorithms must be utilized. This goal is to show and analyze several algorithms that help us to better understand the world around us. To tackle real-world problems, all of the algorithms covered here may be put to work computer-aided to specific optimization issues. All of these algorithms are iterative in nature, beginning with a predetermined initial u_0condition. Each approach creates a sequence *(U _n) n ∈N* that converges under certain conditions, Optimization consists of an objective function that is a set of variables that reduce (maximize) and constraint set of variables that define the solution area through which the optimal solution is found. The optimization problems are one-dimensional each problem has special solutions, always numerical methods.an example of modeling that leads to the equation for heat flow.

2. Methodology

In this section, we talk about how to solve both one-dimensional problems using the golden section search and Lagrange multiplier algorithm of numerical optimization

2.1 One-dimensional problem of numerical optimization

There are problems that are easiest to solve are first-order problems, while the hardest are third-order problems. In practice, constrained problems are multidimensional , it usually boils down to unconstrained multidimensional problems which in turn, For unconstrained one-dimensional problems. In fact , nonlinear programming algorithms are available when minified function of one variable without restrictions. Therefore, if multidimensional optimization algorithms are effective one-dimensional optimization algorithms are also effective ,unconstrained and constrained algorithms must be built. The one-dimensional optimization problem is

$$\begin{cases} \text{minimize} & F(x) \\ \text{subject to} & X_L \leq X \leq X_U \end{cases}$$

where the function *f(x)* is one variable .There is a solution to this problem if f(x) is continuous in some range of x , i.e., *f(x)* has only one minimum in some range $X_L \leq X \leq X_U$ where X_L and X_U are the lower and upper limits of the minimizer x^* [4,5,6].

2.2 SOLVING ONE-DIMENSIONAL PROBLEM WITH UNCONSTRAINED OPTIMIZATION

One of the methods that solve one-dimensional problem with optimization unconstrained is golden section search method

2.2.1 GOLDEN SECTION SEARCH METHOD

Golden section search is a one-dimensional method used to find the minimum of a continuous function of one variable over a range without the use of derivatives. This method solves unconstrained problems. The following algorithm can be used to solving golden search method [7,8,9]

Determine x_L and x_U which is interval

Step1 : Determine two intermediate points x_1 and x_2 such that

$$x_1 = x_L + d \qquad x_2 = x_U - d \qquad \text{where} \qquad d = \frac{\sqrt{5}-1}{2}(x_U - x_L)$$

step 2 : Evaluate $f(x_1)$ and $f(x_2)$

if $f(x_1) > f(x_2)$, then determine new x_L, x_1, x_2 note that the only new calculation is done to determine the new x_1

$$x_L = x_2 \qquad x_2 = x_1 \qquad x_U = x_U \qquad x_1 = x_L + \frac{\sqrt{5}-1}{2}(x_U - x_L)$$

If $f(x_1) < f(x_2)$, then determine new x_L, x_1, x_2 note that the only new

calculation is done to determine the new x_2

$$x_U = x_1 \qquad x_1 = x_2 \qquad x_L = x_L \qquad x_2 = x_U + \frac{\sqrt{5}-1}{2}(x_U - x_L)$$

Step 3: if $x_U - x_L < \varepsilon$ (a sufficiently small number) then the maximum occurs at $\frac{x_U - x_L}{2}$ and stop iterating, else go to Step 2

Example 1 : Use golden section search to minimize

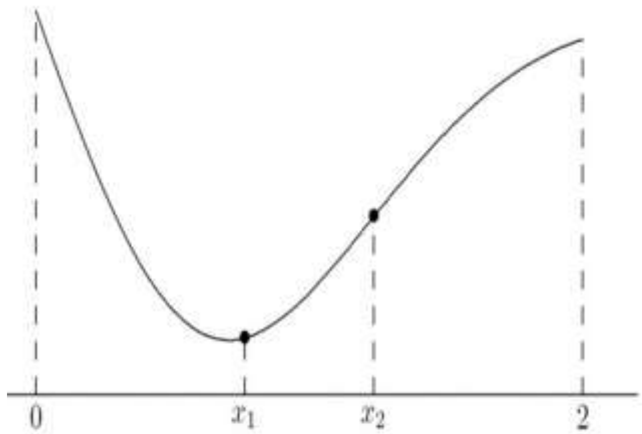
$$f(x) = 0.5 - x\, e^{(-x)^2}$$

The results of applying the golden section search method

]x_1	f_1	x_2	f_2
0.0764	0.074	1.236	0.074
0.472	0.122	0.764	0.074
0.764	0.074	0.944	0.113
0.652	0.074	0.764	0.074
0.584	0.085	0.652	0.074
0.652	0.074	0.695	0.071
0.695	0.071	0.721	0.071
0.679	0.072	0.695	0.071
0.695	0.071	0.705	0.071
0.705	0.071	0.711	0.071

In the example above, to find value of x that minimizes the objective function in the range [0,2] for the given number of iterations (N= 10), we employed the golden search approach, which incorporates the golden ratio ($\rho = \frac{\sqrt{5}-1}{2}$ = 0.382) as part of its calculation.

After finding ($x_1 = 0.764$, $x_2 = 1.236$, $f_1 = 0.074$, $f_2 = 0.074$) in iteration one, we find ($x_1 = 0.705$, $x_2 = 0.711$, $f_1 = 0.071$, $f_2 = 0.071$) in last iteration values have become decreasing.



2.3 SOLVING ONE-DIMENSIONAL PROBLEM WITH CONSTRAINT OPTIMIZATION

One of the methods that solve one-dimensional problem with optimization constrained is Lagrange multiplier method

2.3.1 LAGRANGE MULTIPLIER METHOD:

The Lagrangian multiples method is a simple and elegant way to find the local minima or local extrema of a function subject to the equality or inequality constraints. Lagrangian multiples are also called indefinite multiples. The general Lagrangian multiples method [10,11,12]:

$$\text{Minimize } f(x)$$

$$\text{Subject to:} \quad g_i(x) = 0$$

The method of Lagrange multipliers first constructs a function called the Lagrange function as given by the following expression.

$$L(x,\lambda) = f(x) + \lambda_1 g_1(x) + \lambda_2 g_2(x) + \dots + \lambda_n g_n(x)$$

Example 2 : solving the following minimization problem

$$\boldsymbol{minimize\ f(x) = x^2 + y^2}$$

$$\boldsymbol{subject\ to \quad x + 2y - 1 = 0}$$

The steps is to construct the Lagrange function:

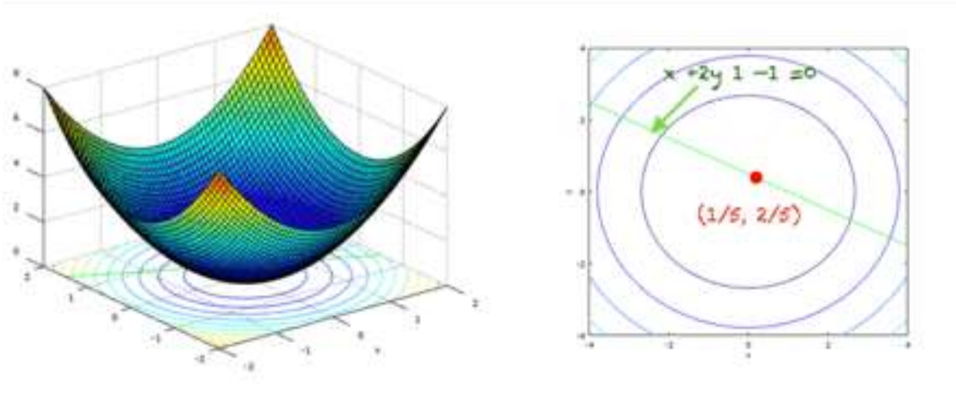
$$L(x,y,\lambda) = x^2 + y^2 + \lambda(x + 2y - 1)$$

$$\frac{\partial L}{\partial x} = 2x + \lambda = 0$$

$$\frac{\partial L}{\partial x} = 2y + 2\lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = x + 2y - 1 = 0$$

$x = \frac{1}{5}$, $y = \frac{2}{5}$, $\lambda = -\frac{2}{5}$



Graph of f(x, y) and the constraint   Contours of f(x, y) and the constraint

2.4 RESULTS AND DISCUSSION

In this article, we check how One-dimensional problem of numerical optimization, and it is used in solving problems containing constrained and unconstrained optimization by using golden section search method and Lagrange multiplier method when solving problems that contain a objective function and unconstraint. The results were (x_1 = 0.705, x_2 = 0.711, f_1 = 0.071, f_2 = 0.071) in last iteration values have become decreasing. As for when solving problems that contain a objective function and constraints The results were

$$x = \frac{1}{5} \quad , \quad y = \frac{2}{5} \quad , \quad \lambda = -\frac{2}{5}$$

3. CONCLUSION

In this study, we used two of the most important numerical optimization methods in applied mathematics, which is called the golden section search method, using to solve One-dimensional problem of numerical optimization that contain a objective function and unconstraint .And Lagrange multiplier method using to solve One-dimensional problem of numerical optimization that contain a objective function and constraint. In this study, we used a new approach to solve constrained and unconstrained numerical optimization problems in a more accurate way and in less time to find the optimal solution.

4. ACKNOWLEDGMENT

Many thanks to the journal's editorial staff, reviewers, and everyone who helped us out..

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