

Review of Extended for Statistical Distributions

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Abstract- This article is a review of extended statistical distributions. We study some of the transformations to obtain this extended.

Keywords- extended, transformations.

1. INTRODUCTION AND PRIMILINARIES

D. Filippo, *et al.*, studies extend Azzalini's Method when this extension is done in two stage see[1]. Z. Mohamed, S. B. Nadeem, study a new model called the Burr X Exponentiated Frechet Distribution see [2]. S-N. Manoel *et al.*, introduced a new class of models called the Marshall-Olkin extended Weibull family of distributions based on the work by Marshall and Olkin[3], K. K. Sankaran and K. Jayakumar, introduced a new family of distributions using truncated the Discrete Mittag- Leffler distribution[4]. N. Salman, study develops a new class of distributions called the extended odd Fréchet family of distributions for modifying existing standard distributions[5].

1.1 Definition: Extended statistical distributions refer to a class of probability distributions that go beyond standard distributions and are developed to model specific types of data or phenomena. These distributions often have unique characteristics or properties that make them suitable for specialized applications.

The term "extended" implies that these distributions expand the range of modeling capabilities beyond the commonly used distributions, such as the normal, uniform, or exponential distributions. They provide more flexible and tailored approaches to handle diverse data patterns and capture complex statistical behaviors.

Extended statistical distributions can have various forms, such as continuous or discrete, univariate or multivariate, parametric or non-parametric.

1.2 Some for Extended Statistical Distributions and properties

Extended statistical distributions typically refer to probability distributions that go beyond the commonly used distributions like the normal or exponential distributions. These extended distributions are often designed to model specific types of data or phenomena and may have unique properties tailored to their applications. Here are a few examples:

1. A New Extended Uniform Distribution DMLU(α, c, θ) [6]: The probability density function is given by:

$$g(x; \alpha, c, \theta) = \frac{\alpha \theta^\alpha (1+c)x^{\alpha-1}}{[\theta^\alpha + cx^\alpha]^2}$$

when $0 < x < \theta$, $\alpha > 0$, $c > 0$ and $\theta > 0$.

2. Inverted Rayleigh Distribution (EWIRD)[7]: The probability density function for this distribution:

$$f(x, \alpha, \theta) = \left(\frac{1 + \alpha^2}{\alpha^2} \right) \left(\frac{2\theta}{x^3} \right) e^{-\frac{\theta}{x^2} \left(\frac{1 + \alpha^2}{\alpha^2} \right)}$$

When ; $x > 0$, $\alpha, \theta > 0$

The extended distribution was derived using the approach discussed by Azzellini (1985). According to the following equation:

$$f_x(x, \alpha) = \frac{f_Y(x)F_Y(\alpha x)}{P[\alpha X_1 > X_2]} ; \alpha > 0 \text{ \& } x > 0$$

3. The Extended Frechet Distribution (BXEF) [8]: The probability density function is given by:

$$f(x; \theta, \lambda, \beta, \alpha) = \frac{2\alpha\beta\lambda\theta_1 - (1 - e^{-\alpha x^{-\beta}})^{\lambda}}{x^{\beta+1} e^{\alpha x^{-\beta} (1 - e^{-\alpha x^{-\beta}})^{2\lambda+1}}} \exp \left[- \left(\frac{1 - (1 - e^{-\alpha x^{-\beta}})^{\lambda}}{(1 - e^{-\alpha x^{-\beta}})^{\lambda}} \right)^2 \right] \times \{1 - \exp\}^{\theta-1}$$

4. Extended Weighted Exponential Distribution (EWED)[9]: The probability density function is:

$$f(x; \alpha, \beta, \lambda) = \frac{(1 + \alpha)^2 \lambda}{\alpha(\lambda(1 + \alpha) + \alpha\beta)} e^{-\lambda x} (\lambda + \beta - (\lambda + \beta + \alpha\beta\lambda x)e^{-\alpha\lambda x})$$

For $x > 0$ and 0 otherwise.

5. Extended Weighted Frechet Distribution [10]: The probability density function is:

$$f(x; \lambda, \theta, \alpha) = \frac{\lambda \alpha^{\lambda+1}}{\theta \alpha^{\lambda}} \left(\frac{\theta}{x}\right)^{\lambda+1} e^{-\left(\frac{\theta}{x}\right)^{\lambda} \left(\frac{\alpha^{\lambda+1}}{\alpha^{\lambda}}\right)} \quad x > 0$$

When using formula for Azzalini's methods.

2. Some Application for Extended Statistical Distribution

Extending statistical distribution methods can have a wide range of applications in various fields. Here are some practical applications of extended statistical distribution methods:

1- Environmental Science:

Extreme Value Analysis: For analyzing extreme events like floods, hurricanes, or heat waves, extended distributions can capture the tail behavior of extreme values more accurately.

2- Biostatistics and Health Sciences:

Survival Analysis: In medical research, extended distributions can be used for modeling survival data with non-Gaussian features, such as the Weibull-skew-normal distribution.

Disease Modeling: When modeling disease outbreaks, incorporating skewness in the distribution of infectious periods or transmission rates can provide more realistic insights.

3- Engineering:

- **Reliability Engineering:** In engineering, especially when dealing with complex systems and equipment, extended distributions can improve the estimation of system reliability and failure probabilities.

- **Quality Control:** Skewness-aware distributions can be applied in quality control to account for skewed process variations.

4- Economics:

- **Income Distribution Modeling:** Extended distributions can be used to model income or wealth distributions, where skewness and heavy tails are often observed.

5- Environmental Economics:

-Natural Resource Valuation: In environmental economics, distributions that account for skewness can be applied to assess the economic value of natural resources

6- Energy and Climate Modeling:

- **Energy Demand Forecasting:** Distributions that capture non-Gaussian features can be applied to forecast energy demand, considering factors like weather variations.

7- Marketing and Customer Analytics:

- **Customer Behavior Modeling:** Skewness-aware distributions can be used to model customer behavior, such as purchase amounts or response rates in marketing campaigns.

8- Geosciences:

-**Hydrology and Precipitation Modeling:** Extended distribution methods are used to model precipitation patterns and estimate flood risk in hydrology.

9- Machine Learning and Data Science:

-**Anomaly Detection:** Extended statistical distributions can improve the accuracy of anomaly detection algorithms in identifying rare or unexpected events in large datasets.

In each of these fields, the choice of an extended statistical distribution method should be based on the data's specific characteristics and the problem's nature. Proper model selection and validation are essential to ensure that the chosen distribution accurately represents the underlying data and provides meaningful insights for decision-making or research.

5. References

1. D. Filippo, *et al.*, "An extension of Azzalini's method", (2018).
2. Z. Mohamed, S. B. Nadeem, "The extended frechet distribution: Properties and application" *Pak.j.stat.oper.res.* Vol. XIII, No.3, pp529-543, 2017.
3. S.N. Manoel, *et al.*, "The Marshall-Olkin extended Weibull family of distributions", *Journal of Statistical Distributions and Applications*, 1:9, pp2-24, 2014.
4. K. K. Sankaran and K. Jayakumar, "A New Extended Uniform Distribution", *International Journal of Statistical Distributions and Applications*, 2(3), pp 35-41, 2016.
5. N. Suleman, "Extended Odd Fréchet-G Family of Distributions", *Journal of Probability and Statistics*, Article ID 2931326, Ps. 12, 2018.
6. K.K. Sankaran and K. Jayakumar, "A New Extended Uniform Distribution", *International Journal of Statistical Distributions and Applications*, Vol.2, No.3, PP.35-41, 2016.
7. S. Y. Abhimanyu , S. K. Singh and S. Umesh, " Statistical Properties and Different Methods of Estimation for Extended Weighted Inverted Rayleigh Distribution", *Statistics in Transition new series*, Vol.21, No.2, PP.119-141, 2020.
8. Z. Mohamed & S. B. Nadeem., " The Extended Frechet Distribution: Properties and Applications", *Pakistan Journal of Statistics and Operation Research*, 2017.
9. M. Abbas & J. Leila., "An Extended Weighted Exponential Distribution" , *Journal of Modern Applied Statistical Methods*, vol.16, No.1, PP.296-307, 2017.
10. W. N. Mahdi & H. A-S. Nahla, "Statistical Properties & Different Methods of Estimation of a New Extended Weighted Frechet Distribution", *Turkish Journal of Computer and Mathematics Education*, Vol.12, No.6, PP. 1011-1029, 2021.