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Calculation of FRP Reinforced Concrete Beam

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Abstract: *In this article, the calculation of reinforced concrete beams with fiber reinforced polymer (FRP) bar according to ACI 440.1R-06 (USA) and fib Bulletin 40 (European Union) codes is presented, and their differences are considered. In addition, in both calculation methods, reinforcement ratio and bending moment values are determined.*

Keywords: *FRP bar, reinforcement, concrete, concrete beam, tension zone, compression zone, deflection, corrosion, strength*

I. Introduction. The increase of steel demand as construction industry develops in recent years indicates that its reserves are decreasing as the time pass. Given the limited availability of iron ore, finding an alternative material and using it in load-bearing structures has become an urgent issue today. Fiber-reinforced polymer (FRP) bar has brought revolution for the construction that is in full alignment efforts to encourage nonmetallic solutions. The corrosion resistant rebar is an ideal replacement of steel reinforcement for the aggressive environment in the region. And the only one-quarter the weight of steel, FRP rebar is easy to transport and construct. FRP bars are not only corrosion-resistant, dielectric, non-magnetic material. Also, it is becoming the main material of the future construction elements of various sectors of civil and industrial buildings [1-6].

FRP rebar manufacture was stopped for a certain period at the end of the 20th century due to the lack of cheap and high-quality raw materials. However, in recent years, the volume of production of FRP reinforcement has increased. In some areas of construction, the demand for FRP rebar has appeared, for example:

- reduction of concrete structures strength due to the corrosion of steel reinforcement;
- reinforcement of building structures in a highly aggressive environment;
- ensuring the dielectric features of buildings and structures intended for various purposes;
- limited source of steel and its alloying substances;

All of the above are required the research on FRP rebar, further improvement of production technologies, development of scientific and technical and normative documents.

II. Main part. As we know, the limit state method is currently used in the calculation of building structures. The normative documents used in all countries are based on the limit state method. Many countries all over the world have own codes to design FRP reinforced concrete structures. In this article is intended to the calculation of concrete beam reinforced with FRP rebar. ACI 440.1R-06 (USA) and fib Bulletin 40 (European Union) calculation methods were compared.

Calculation on ACI 440.1R-06

Determination of the moment capacity of concrete beams reinforced with FRP rebar is illustrated in the flowchart of Fig. 1.

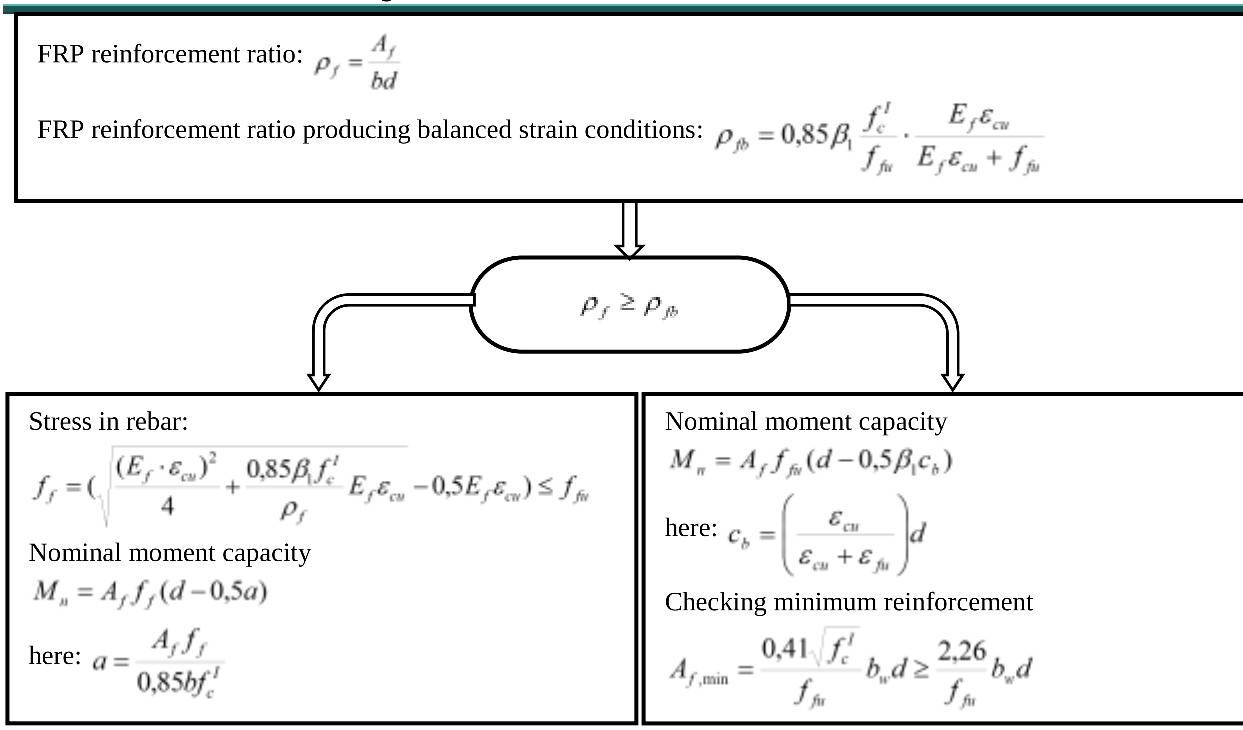

Fig. 1. The flowchart of determining nominal moment capacity of concrete beam reinforced with FRP bar on ACI 440.1R-06

Flexural strength of concrete beam as follows:

$$\phi M_n \geq M_u \tag{1}$$

Nominal moment capacity M_n taking into account strength reduction factor ϕ is required to be higher than factored moment at section M_u. In Eq. (1) ϕ coefficient ensuring strength can be determined from Eq. (2) and fig. 2.

$$\phi = \begin{cases} 0,55 \quad for\ \rho_f \leq \rho_{fb} \\ 0,3 + 0,25 \frac{\rho_f}{\rho_{fb}} \quad for\ \rho_{fb} < \rho_f < 1,4\rho_{fb} \\ 0,65 \quad for\ \rho_f \geq 1,4\rho_{fb} \end{cases} \tag{2}$$

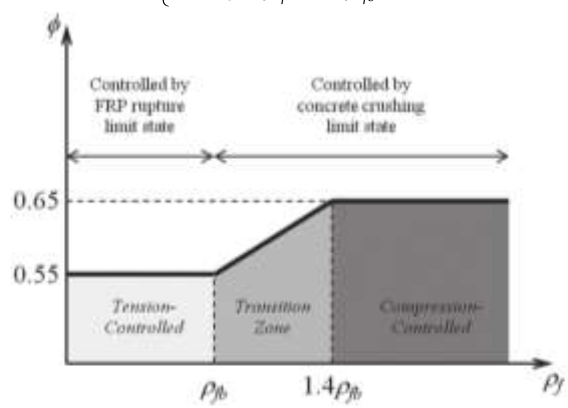

Fig. 2. Relationship between reinforcement ration and strength reduction factor

Three cases of failure of FRP reinforced concrete beams are considered in the calculations: concrete crushing in the compression zone, concrete crushing in the compression zone and FRP rebar rupture simultaneously, and FRP rebar rupture (Fig. 3). The failure character is determined by the balanced ratio for reinforcement and actual reinforcement ratio values. There are different expressions for determining the normal bending moment for both cases of failure [1].

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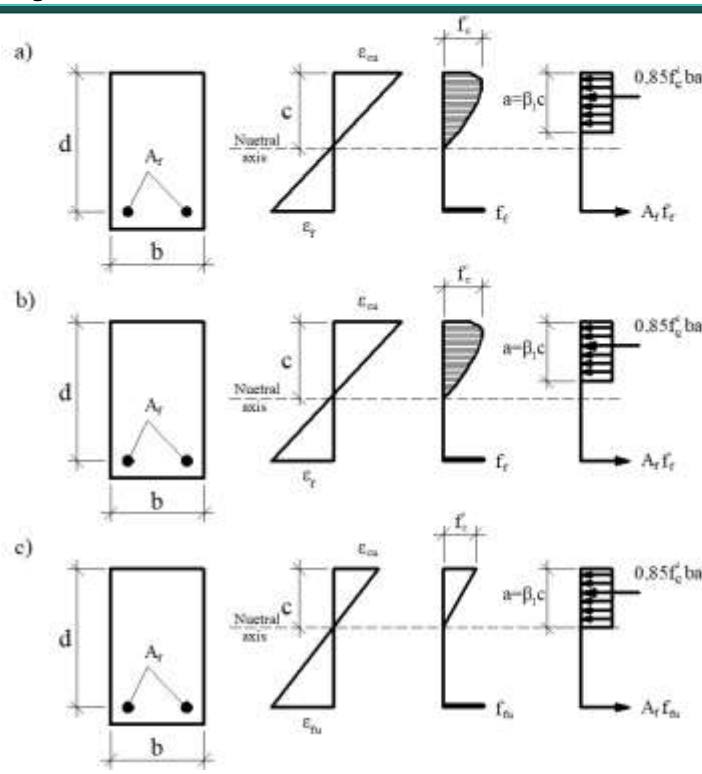

Fig. 3. Stress and strain distribution in the cross section:
a) failure due to concrete crushing; b) balanced failure condition; c) failure due to FRP rupture

FRP reinforcement ratio can be determined following equation

$$\rho_f = \frac{A_f}{bd} \tag{3}$$

here: A_f - cross section of longitudinal FRP rebar;

b – width of beam cross section;

d – distance from outermost compression fiber to the center of tension rebar;

Balanced reinforcement ratio

$$\rho_{fb} = 0{,}85\beta_1 \frac{f_c^I}{f_{fu}} \cdot \frac{E_f \varepsilon_{cu}}{E_f \varepsilon_{cu} + f_{fu}} \tag{1.4}$$

here: β_1 factor taken as 0,85 for concrete strength $f_c^I < 4$ ksi or 28 MPa (1 ksi = 6,895 MPa). For strength $f_c^I \geq 4$, the factor reduced continuously at a rate of 0,05 per each 1 ksi of strength in excess of 4 ksi, but is not taken less than 0,65.

f_c^I - compressive strength of concrete;

f_{fu} - tensile strength of FRP rebar;

E_f - elastic modulus of FRP reinforcement;

ε_{cu} - ultimate strain of compressive concrete;

If $\rho_f \geq \rho_{fb}$, failure characterizes due to concrete crushing. Nominal reinforcement ratio M_n can be determined from Eq. 5:

$$M_n = A_f f_f (d - 0{,}5a) \tag{5}$$

here: $a = \frac{A_f f_f}{0{,}85 b f_c^I}$.

Stress in rebar f_f:

$$f_f = (\sqrt{\frac{(E_f \cdot \varepsilon_{cu})^2}{4} + \frac{0{,}85\beta_1 f_c^I}{\rho_f} E_f \varepsilon_{cu}} - 0{,}5 E_f \varepsilon_{cu}) \leq f_{fu} \tag{6.}$$

If $\rho_f < \rho_{fb}$, failure starts from tensile FRP rebar rupture. Nominal moment capacity M_n as follows:

$$M_n = A_f f_{fu}(d - 0{,}5\beta_1 c_b) \qquad (7)$$

here: $c_b = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{fu}} d$ - distance from outermost compressive fiber to neutral axis.

ε_{fu} - ultimate tensile strain of rebar;

If a section is tension controlled $\rho_f \le \rho_{fb}$, a minimum reinforcement should be provided to prevent failure on concrete cracking.

$$A_{f,\min} = \frac{0{,}41\sqrt{f_c^I}}{f_{fu}} b_w d \ge \frac{2{,}26}{f_{fu}} b_w d \qquad (8)$$

here: b_w - thickness of web for T shape beams.

Calculation on fib Bulletin 40

Determination of the moment capacity of FRP reinforced concrete beams is shown in the flowchart of Fig. 4.

FRP reinforcement ratio: $\rho_f = \frac{A_f}{bd}$

FRP balanced reinforcement ratio: $\rho_{fb} = \frac{0{,}81(f_{ck}+8)\varepsilon_{cu}}{f_{fk}(\frac{f_{fk}}{E_{fk}} + \varepsilon_{cu})}$, or $\rho_{fb} = 2{,}1\left(\frac{f_{cu}}{40}\right)\left(\frac{E_f}{110\cdot 10^3}\right)^{0,7}\left(\frac{500}{f_f}\right)^{1,6}$

$\rho_f \ge \rho_{fb}$

Ultimate moment:

$M_u = \eta f_{cd} b d^2 (\lambda\xi)(1 - 0{,}5\lambda\xi)$

here $f_{cd} = \frac{\alpha_{cc} f_{ck}}{\gamma_c}$

$f_{ck} \le 50\ MPa \quad \begin{cases} \lambda = 0{,}8 \\ \eta = 1 \end{cases}$

$50 \le f_{ck} \le 90\ MPa \quad \begin{cases} \lambda = 0{,}8 - \left(\frac{f_{ck} - 50}{400}\right) \\ \eta = 1 - \left(\frac{f_{ck} - 50}{200}\right) \end{cases}$

$\xi = \frac{x}{d} = \frac{\varepsilon_{cu}}{\varepsilon_f + \varepsilon_{cu}}$

$\varepsilon_f = \frac{-\varepsilon_{cu} + \sqrt{\varepsilon_{cu}^2 + \frac{4\eta\alpha_{cc} f_{ck} \lambda \varepsilon_{cu}}{\gamma_c \rho_f E_f}}}{2}$

Below equation can be used to calculate the stress developed in the FRP bar and, thus, verify that failure due to FRP rupture is avoided

$\sigma_f = \varepsilon_f E_f < \frac{f_{fk}}{\gamma_f}$

Ultimate moment:

$M_u = \frac{A_f f_{fk}}{\gamma_f}(1 - \frac{\xi}{2})$

here $\xi = \frac{x}{d} = \frac{\varepsilon_{cu}}{\varepsilon_f + \varepsilon_{cu}}$

$F_c = F_T \rightarrow bd\xi \frac{\int_0^{\varepsilon_c} f_c d\varepsilon_c}{\varepsilon_c} = \frac{A_f f_{fk}}{\gamma_f}$

$f_c = f_{cd}\left[1 - \left(1 - \frac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right]$ for $0 \le \varepsilon_c \le \varepsilon_{c2}$

or

$f_c = f_{cd}$ for $\varepsilon_{c2} \le \varepsilon_c \le \varepsilon_{cu}$

Checking minimum reinforcement

$A_{f,\min} = 0{,}26 \frac{f_{ctm}}{f_{fk}} bd \ge 0{,}0013 bd$

Fig. 4. The flowchart of determining nominal moment capacity of concrete beam reinforced with FRP bar on fib Bulletin 40

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Just like ACI 440.1R-06, fib Bulletin 40 also observes three cases of failure. Ultimate moment value is determined depending on reinforcement ratio and balanced reinforcement ratio. The balanced reinforcement ratio determined by expression (9) is taken from Eurocode 2 proposed by Pilakoutas K. (2002) for reinforced concrete members, or El-Ghandour A. (1999) can be determined from the empirical expression (10).

$$\rho_{fb} = \frac{0{,}81(f_{ck}+8)\varepsilon_{cu}}{f_{fk}\left(\frac{f_{fk}}{E_{fk}}+\varepsilon_{cu}\right)} \tag{9}$$

$$\rho_{fb} = 2{,}1\left(\frac{f_{cu}}{40}\right)\left(\frac{E_f}{110\cdot 10^3}\right)^{0{,}7}\left(\frac{500}{f_f}\right)^{1{,}6} \tag{10}$$

here: E_f - elastic modulus of FRP;

E_{fk} - characteristic value of elastic modulus of FRP;

f_c, f_c^I - cylinder compressive strength of concrete;

f_{cu} - cube compressive strength of concrete;

f_{ck} - characteristic value of concrete compressive strength;

f_f, f_{fu} - ultimate tensile strength of FRP rebar;

f_{fk} - characteristic value of tensile strength of FRP rebar;

ε_{cu} - concrete ultimate strain;

Ultimate moment in bending elements is determined for two cases:

1. Reinforcement ratio higher than balanced reinforcement ratio $\rho_f \geq \rho_{fb}$, it characterizes concrete crushing , thus ultimate moment can be determined from Eq. 11

$$M_u = \eta f_{cd} b d^2(\lambda\xi)(1-0{,}5\lambda\xi) \tag{11}$$

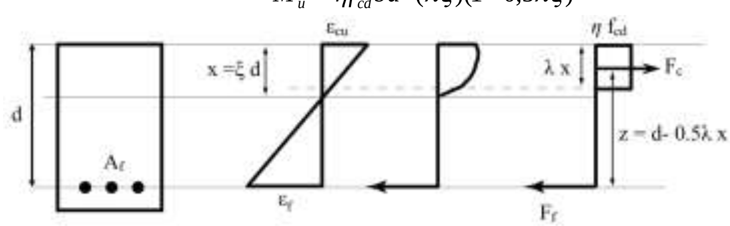

Fig. 5. Simplified stress block proposed for concrete elements reinforced with FRP

here: f_{cd} - design value of ultimate compressive strength of concrete

$$f_{cd} = \frac{\alpha_{cc} f_{ck}}{\gamma_c} \tag{12}$$

$$f_{ck} \leq 50\ MPa \quad \begin{cases}\lambda = 0{,}8,\\ \eta = 1\end{cases} \qquad 50 \leq f_{ck} \leq 90\ MPa \quad \begin{cases}\lambda = 0{,}8-\left(\frac{f_{ck}-50}{400}\right)\\ \eta = 1-\left(\frac{f_{ck}-50}{200}\right)\end{cases}$$

α_{cc} - coefficient taking into account the long term effects on the compressive strength and unfavourable effects resulting from the way the load is applied;

λ - coefficient taking into account effective height of compression zone;

η - concrete strength coefficient;

γ_c - concrete safety coefficient;

d - effective height of section;

b - width of section;

ξ - relationship between compression zone and effective height;

$$\xi = \frac{x}{d} = \frac{\varepsilon_{cu}}{\varepsilon_f + \varepsilon_{cu}} \tag{13}$$

бу ерда: x - the height of compression zone;

ε_f - strain in longitudinal reinforcement;

$$\varepsilon_f = \frac{-\varepsilon_{cu} + \sqrt{\varepsilon_{cu}^2 + \frac{4\eta\alpha_{cc} f_{ck} \lambda \varepsilon_{cu}}{\gamma_c \rho_f E_f}}}{2} \tag{14}$$

Stress in FRP bar is determined by Eq. (15), but with the exception of rebar rupture

$$\sigma_f = \varepsilon_f E_f < \frac{f_{fk}}{\gamma_f} \tag{15}$$

σ_f - tensile stress in FRP reinforcement;

γ_f - safety coefficient of FRP bar;

2. If reinforcement ratio RC section is below balanced reinforcement ratio $\rho_f < \rho_{fb}$, the expected type of flexural failure is FRP rupture and, to calculate ultimate moment Eq. 16, it is necessary to determine the concrete compressive strain ε_c at which FRP rupture occurs. It can be obtained through an iterative procedure by solving Eq. 17 and 18.

$$M_u = \frac{A_f f_{fk}}{\gamma_f}(1 - \frac{\xi}{2}) \tag{16}$$

$$\xi = \frac{x}{d} = \frac{\varepsilon_{cu}}{\varepsilon_f + \varepsilon_{cu}} \tag{17}$$

$$F_c = F_T \rightarrow bd\xi \frac{\int_0^{\varepsilon_c} f_c d\varepsilon_c}{\varepsilon_c} = \frac{A_f f_{fk}}{\gamma_f} \tag{18}$$

here: A_f - cross section area of longitudinal FRP bar;

ε_c - compressive strain in concrete;

here f_c is calculated from Eq. 19. The values proposed by Eurocode 2 are used for concrete strain ε_{c2} and ε_{cu}.

$$f_c = f_{cd}\left[1 - \left(1 - \frac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right] \text{ for } 0 \le \varepsilon_c \le \varepsilon_{c2}$$

or (19)

$$f_c = f_{cd} \qquad \text{for } \varepsilon_{c2} \le \varepsilon_c \le \varepsilon_{cu}$$

here: n –factor depending on the characteristic strength of concrete;

ε_{c2} - strain in maximum strength of concrete;

ε_{cu2} - ultimate concrete strain;

To ensure ultimate moment is higher than cracking moment of the section, a minimum limit may be applied on the amount of longitudinal rebar

$$A_{f,\min} = 0{,}26 \frac{f_{ctm}}{f_{fk}} bd \ge 0{,}0013bd \tag{20}$$

here: f_{ctm} - average value of concrete tensile strength.

III. Conclusion. If reinforcement ratio is higher balanced reinforcement ratio $\rho_f \ge \rho_{fb}$, according to ACI 440.1R-06, stress in tension rebar is initially determined then nominal moment capacity. However in fib Bulletin 40, taking into account reinforcement strain, ultimate moment capacity is determined by using strength of concrete in compression zone then FRP rupture condition will be checked.

In both cases if condition $\rho_f < \rho_{fb}$, ultimate moment is determined by considering reinforcement stress. In fib Bulletin 40, to determine the height of compression zone it's required to calculate concrete strain when FRP ruptured. In both codes to ensure minimum reinforcement ratio is checked. As a result of that ultimate moment M_u should not exceed cracking moment M_{crc}.

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