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On The RG-subalgebras and Ideals with Homomorphism of RG-algebra

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***Abstract* -The aim of this paper is introducing the notion of RG − subalgebras, ideals and q-ideal of RG − algebra, several theorems, properties are stated and proved. The relations on RG − algebras are also studied.**

Keywords— RG − algebra, RG − subalgebra, ideal, fuzzy ideal, homomorphisms of RG − algebra, the cartesian product of fuzzy ideals.

1. Introduction

BCK − algebras form an important class of logical algebras introduced by K. Iseki [13] and was extensively investigated by several researchers. The class of all BCK − algebras is quasi variety. J. Meng and Y. B. Jun posed an interesting problem (solved in [16]) whether the class of all BCK − algebras is a variety. In connection with this problem, Komori introduced in [15] a notion of BCC algebras. W. A. Dudek (cf. [4]) redefined the notion of BCC − algebras by using a dual form of the ordinary definition in the sense of Y. Komori and studied ideals and congruences of BCC − algebras. In [20,21]), C. Prabpayak and U. Leerawat introduced a new algebraic structure, which is called KU − algebra. They gave the concept of homomorphisms of KU − algebras and investigated some related properties. L. A. Zadeh [23] introduced the notion of fuzzy subsets. At present this concept has been applied to many mathematical branches, such as group, functional analysis, probability theory, topology, and so on. In 1991, O. G. Xi [22] applied this concept to BCK − algebras, and he introduced the notion of fuzzy sub − algebras (ideals) of the BCK − algebras with respect to minimum, and since then Jun et al studied fuzzy ideals (cf. [14]), and moreover several fuzzy structures in BCC − algebras are considered (cf. [17]). In [6], the anti − fuzzy AB − ideals of AB − algebras and in [11], the anti − fuzzy AT − ideals of AT − algebras were introduced. Several theorems are stated and proved. In 2013, S. M. Mostafa, and A. T. Hameed introduced On KUS − algebras. In 2014, R. A. K. Omar introduced on RG − algebra. In 2018, P. Patthanangkoor have been given RG − homomorphism and Its Properties. In 2020, A. T. Hameed and N.J. Raheem introduced hyper SA − algebra. In this paper, the study of new algebraic structures was started with the introduction of the concept of RG-subalgebras, ideals and q −ideals of RG-algebra and we describe how to deal with the homomorphism of image and inverse image of them.

2. *Preliminaries*

Now, we give some definitions and preliminary results needed in the later sections.

<u>Def. 2.1 [15].</u>

An algebra$(\partial;*,0)$ is called **an RG-algebra** if the following axioms are satisfied: $\forall\ \rho,\sigma,\tau \in \partial$,

(i) $\rho * 0 = \rho$,

(ii) $\rho * \sigma = (\rho * \tau) * (\sigma * \tau)$,

(iii) $\rho * \sigma = \sigma * \rho = 0$ *imply* $\rho = \sigma$.

<u>Re. 2.2 [15].</u>

For $brevity\ we\ also\ call\ \partial$ **an RG-alge*bra***, $we\ can\ define\ a\ binary\ relation$ ($\leq$) by putting $\rho \leq\ \sigma\ if and\ only\ if\ \rho * \sigma = 0$.

Ex. 2.3 [8].

Let ∂ ={0, a, b, c} and (∂;$*$) be the pair $given\ by\ the\ table$:

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

$Then\ (\partial;* ,0)\ is\ an\ RG - algebra.$

Prop. 2.4 [15].

In any RG-algebra (∂;$*$,0), the following hold: $\forall\rho,\ \sigma \in \partial$,

i) $\rho * \rho = 0$,

ii) $0 * (0 * \rho) = \rho$,

iii) $\rho * (\rho * \sigma) = \sigma$,

iv) $\rho * \sigma = 0\ if\ and\ only\ if\ \sigma * \rho = 0$,

v) $\rho * 0 = 0\ implies\ \rho = 0$,

vi) $0 * (\sigma * \rho) = \rho * \sigma$.

Prop. 2.5 [15].

In $any\ RG - algebra$ (∂;$*$,0), the f$ollowing\ hold$: $\forall\ \rho, \sigma, \tau \in\ \partial$,

i) $(\rho * \sigma) * (0 * \sigma) = \big(\rho * (0 * \sigma)\big) * \sigma = \rho$,

ii) $\rho * (\rho * (\rho * \sigma)) = \rho * \sigma$,

iii) $(\rho * \sigma) * \tau = (\rho * \sigma) * \tau$.

iv) $\rho * \sigma = (\tau * \sigma) * (\tau * \rho)$,

v) $\big((\rho * \sigma) * (\rho * \tau)\big) * (\tau * \sigma) = 0$.

Prop. 2.6 [11].

$Every\ RG - algebra\ is\ a\ BCI - algebra.\ The\ converse\ of\ this\ Prop. may\ be\ not\ true.$

Ex. 2.7 [11].

Let $\partial\ = \{0, a, b, c\}\ in\ which\ *\ is\ defined$ by:

$*$	0	a	b	c
0	0	0	b	b
a	a	0	b	b
b	b	b	0	0

c	c	b	a	0

$Then\ (\partial;*,0)\ is\ a\ BCI-algebra, but\ it\ is\ not\ an\ RG-algebra\ because$

$0*(b*c)=0, c*b=a$, then $0*(b*c)\neq c*b$.

Def. 2.8 [15].

Let $(\partial;*,0)$ be $an\ RG-algebra\ and\ S\ be\ a\ nonempty$ subset of ∂ .Then $S\ is\ called$ a **subalgebra**of X if $(S;*,0)$ is itself an RG-algebra.

Def. 2.9 [15].

Let $(\partial;*,0)$ be $an\ RG-algebra, a\ nonempty\ subse$t Ͼ of ∂ is call$ed\ an$ $\boldsymbol{RG-ideal\ of}$ ∂ if: $\forall\rho,\sigma\in\partial$,

i) 0∈Ͼ,

ii) $\rho*\sigma\in$ Ͼ and $0*\rho\in$ Ͼ $imply$ $0*\sigma\in$ Ͼ .

Prop. 2.10 [15].

In $n\ RG-algebra$ $(\partial;*,0)$, ever$y\ RG-idea$l is a subalgebraof ∂.

Def. 2.11[11].

Let $(\partial;*,0)$ and $(\partial';*',0')$ be non$empty\ sets. The\ mapp$ing

$Q:(\partial;*,0)\rightarrow(\partial';*',0')$ is called **a** $\boldsymbol{homomorphism}$ $if\ it\ satisfies$: $Q(\rho*\sigma)=Q(\rho)*`Q(\sigma)$, for all $\forall\rho,\sigma\in\partial$,.

The set $\{\rho\in\partial \mid Q(\rho)=0'\}$is called **the** $\boldsymbol{kernel\ of}$ f $denot$ed by ker$Q$.

Th.2.12 [11].

Let $Q:(\partial;*,0)\rightarrow(\partial';*',0')$be a hom. of an RG-algebra ∂ into an RG-algebra ∂', then:

A. $Q(0)=0'$.

B. Q is in$jective\ if\ and\ only\ if\ ker\ Q=\{0\}$.

C. If $0*\rho=\rho$, $\forall\rho\in\partial$, then $Q(0)*'\sigma=\sigma$.

Th.2.13 [11].

Let $Q:(\partial;*,0)\rightarrow(\partial';*',0')$ be a hom. of an RG-algebra ∂ into an RG-algebra ∂', then:

(F_1) If S is a $subalgebra$ of ∂, then $Q(S)$ is a subalgebraof ∂' .

(F_2) If Ͼ is an RGideal of ∂, then Q (Ͼ) is an RGideal of ∂' .

(F_3) If S is a subalg $subalgebra$ ebra of ∂', then $Q^{-1}(S)$ is a subalgebraof ∂ .

(F_4) If Ͼ is an RGideal of ∂', then Q^{-1} (Ͼ) is an RGideal of ∂ .

(F_5) ker Q is an RGideal of ∂.

(F_6) Im(Q) is a subalgebraof Q'.

Prop. 2.14.[15].

Let $(\partial;\ *,0)$ be an RG-algebra. Then the following hold: for any $\rho,\sigma,\tau \in \partial$,,

1- $\rho \leq \sigma$ $imply$ $\tau * \sigma \leq \tau * \rho$,

2- $\rho \leq \sigma$ $implies$ $\rho * \tau \leq \sigma * \tau$,

3- $\rho * \sigma \leq \tau$ $imply$ $\rho * \tau \leq \sigma$.

3. On RG-subalgebras and Hom. of RG-algebra

In this s*ection, we will discu*ss a new n*otion calle*d RG-subalgebraof RG-*algebra and study several basic proper*ties of R$G - algeb$ra.

Def. 3.1[8].

A non*empty subse*t S of an RG-al*gebra* $(\partial;\ *,0)$ *is cal*led

an RG-subalgebraof *X* if it *satisfies the following conditions*: *for an*y $\rho,\sigma \in \partial$,, if $\rho * \sigma \in S$, *for any* $\rho, \sigma \in S$.

Ex.s 3.2[8].

Let ∂ = {0, a, b, c} *and let* $*$ *be defined by the table*:

$*$	0	a	b	c
0	0	0	c	c
a	a	0	c	b
b	b	c	0	a
c	c	c	0	0

Thus $(\partial;\ *,0)$ is an RG$-$*algebra. And we see that* C = {0, a} and C$'$ = {0, b, c} are RG-*subalg*ebras of ∂.

Prop. 3.3.

Let $\{S_i \mid i{\in}\Lambda\}$ be a family of RG-subalgebras of an RG-algebra

$(\partial;\ *,0)$. The intersection of any set of RG-subalgebras of ∂ is also RG-subalgebraof ∂.

Pr.:

Since $\{S_i| \ i{\in}\Lambda\}$ be a family of RG-subalgebras of RG-algebra ∂, for any $\rho,\sigma \in \partial$,, suppose $\rho \in \cap_{i \in\Lambda} S_i$ and $\sigma \in \cap_{i \in\Lambda} S_i$, then $\rho \in S_i$ and $\sigma \in S_i$, for all i $\in\Lambda$, but S_i is an RG-subalgebraof ∂, for all i $\in\Lambda$.

Then $(\rho * \sigma) \in S_i$, for all i $\in\Lambda$, therefore, $(\rho * \sigma) \in \cap_{i \in\Lambda} S_i$.

Hence $\bigcap_{i\in\Lambda} S_i$ is an RG-subalgebraof RG-algebra .⌂

Re. 3.4.

Not that the union of two RG-subalgebras of RG-algebra it is not necessarily RG-subalgebraof RG-algebra, as it is shown in the following Ex..

Ex. 3.5.

Let ∂= {0, a, b, c} and let $*$ be a binary operation defined by the table:

*	0	a	b	c
0	0	0	0	0
a	a	0	0	c
b	b	b	0	b
c	c	c	c	0

It is clear that $(\partial; *, 0)$ is an RG-algebra. Let $C= \{0, b\}$ and $C' = \{0, a\}$ be two RG-subalgebras of ∂, but $(C \cup C') = \{0, a, b\}$ is not RG-subalgebraof ∂.

Prop. 3.6.

Let $\{S_i \mid i\in\Lambda\}$ be $a\ family\ of\ RG-subalgebra$ of RG-algebra $(\partial;*,0)$, then $\bigcup_{i\in\Lambda} S_i$ $is\ a\ RG-sub$algebraof X, where $S_i \subseteq S_{i+1}$, for all i $\in \Lambda$.

Pr.:

Since $\{S_i \mid i\in\Lambda\}$ be a $family\ of\ RG-subalgebrasof$ RG-alge$bra\ \partial\ , for\ any\ \ \rho, \sigma \in\ \partial, supp$ose $\rho \in \bigcup_{i\in\Lambda} S_i$ and $\sigma \in \bigcup_{i\in\Lambda} S_i$, then $\rho \in S_i$ and

$\sigma \in S_j$ for some i, j $\in\Lambda$. By assu$mption$ $S_i \subseteq S_k$ and $S_j \subseteq S_k$.

$Therefore$, $(\rho * \sigma) \in S_k$, but S_k is an RG-subalgebraof ∂, then

$(\rho * \sigma) \in S_k$, therefore, $(\rho * \sigma) \in \bigcup_{i\in\Lambda} S_i$.

Hence $\bigcup_{i\in\Lambda} S_i$ is an RG-subalgebraof .⌂

Prop. 3.7.

Let Q: $(\partial;*,0)\rightarrow(\partial';*',0')$ be a hom. of an RG-$algebra\ \partial\ into\ an\ RG-algebra\ \partial'$, then :

A. $Q(0) = 0'$, where 0 and $0'$ are $he\ constants\ of\ \ \partial\ and\ \ \partial'\ respectively$, where Q is onto.

B. Q is $injective\ if\ and\ only\ if\ \ ker\ Q$ ={0}.

C. $\rho \leq \sigma$ implies $Q(\rho) \leq Q(\sigma)$.

Prop. 3.8.

If Q: $(\partial;*,0) \rightarrow (\partial';*',0')$ is an epimorphism from RG-algebra $(\partial;\ *, 0)$ into RG-algebra $(\partial';\ *', 0')$, then the image of RG-subalgebrais an RG-subalgebra.

Pr.:

Let Q: $(\partial;*,0)\rightarrow(\partial';*',0')$ be an e$pimorphism\ from\ \partial\ into\ \partial'\ and\ let\ S\ be\ a\ RG-sub$algebraof ∂, then $0 \in A$, so $Q(0) = 0' \in Q(S)$, by Prop. (3.7).

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Now, let $(\rho) \in Q(S) and\ \ \sigma \in\ Q(S)$, then $Q^{-1}(\rho) \in S$

and $Q^{-1}(\ \sigma) \in S$, since Q is onto.

But $Q^{-1}(\rho *' \sigma) = \ (Q^{-1}(\rho *' \sigma)) = \ (Q^{-1}(\rho\ ) *\ Q^{-1}(\ \sigma))$, by Prop. (3.7).

Therefore, $(Q^{-1}(\rho\ ) \in\ S\ and\ Q^{-1}(\ \sigma)\ \in\ S$. Since S is a RG-subalgebraof ∂, then $Q^{-1}(\rho *' \sigma)\ \in\ S$, thus $\rho *' \sigma\ \in\ Q(S)$, that means $Q(S)$ is a RG-subalgebraof ∂'. ⌂

Prop. 3.9.

If $Q: (\partial;* ,0) \rightarrow\ (\partial';*', 0')$ is a hom. from RG-algebra ∂ into RG-algebra ∂', then the inverse image of RG-subalgebrais a RG-subalgebra.

Pr.:

Let $Q: (\partial;* ,0) \rightarrow\ (\partial';*', 0')$ be an hom. from ∂ into ∂ and let D be a RG-subalgebraof ∂' . Then $0' \in D$, so $Q(\ 0\ ) =\ 0' \in D$, by Prop. (3.7), thus $0' = \partial^{-1}(\ 0\ ) \in \partial^{-1}(D)$.

Now, let $\rho \in Q^{-1}(D)\ and\ \ \sigma\ \in\ f^{-1}(D)$, so $Q(\rho)\ \in D$

and $Q(\ \sigma) \in D$. But $Q(\rho *\ \sigma) = (Q(\rho\ ) *' Q(\ \sigma))$, by Prop. (3.7). Therefore $Q(\rho) \in\ D\ and\ Q(\ \sigma)\ \in\ D$, then $Q(\rho *\ \sigma)\ \in\ D$, since D is a RG-subalgebra, thus $\rho *\ \sigma\ \in\ Q^{-1}(D)$, that means $Q^{-1}(D)$ is a RG-subalgebraof ∂.⌂

4. On The Ideals and Hom. of RG-algebra

In this $section, we\ will\ discuss\ a\ new\ notion\ called$ ideal $of\ RG - algebra\ a$
n$d\ study$ sever$al\ basic\ properties\ which\ are\ rela$ted of RG-algebra.

Def. 4.1[8].

A nonempty subset I of an RG-algebra $(\partial;\ *, 0)$ is called

an ideal of ∂ if it satisfies the following conditions: for any $\rho, \sigma \in\ \partial$,,

(I_1) 0 ∈C,

(I_2) $\rho *\ \sigma \in$ C $and\ \rho\ \in$ C $imply\ \ \sigma\ \in$ C.

Ex.s 4.2.

Let ∂ = {0, 1, 2, 3} and $let\ *\ be\ defined\ by\ the\ tab$le:

*	0	1	2	3
0	0	0	3	3
1	1	0	2	2
2	2	3	0	0
3	3	3	0	0

Thus $(\partial;\ *, 0)$ is an $RG - algebra\ and\ we\ see\ that$ C = {0, 1} and C$'$ = $\{0, 2, 3\}$ $are\ ideals$ of ∂.

Prop. 4.3.

Let $\{C_i \mid i\in\Lambda\}$ be a family of ideals of an $RG-$algebra $(\partial;\ *,0)$. The intersection of any set of ideals of ∂ is also ideal of ∂.

Pr.:

Since $\{C_i / i\in\Lambda\}$ be a family of ideals of $RG-$algebra ∂, then $0\in C_i$, for all $i\in\Lambda$, then $0\in\bigcap_{i\in\Lambda} C_i$.

For any $\rho,\sigma\in\partial$,, suppose $(\rho*\sigma)\in\cap_{i\in\Lambda} C_i$ and $\rho\in\cap_{i\in\Lambda} C_i$, then $(\rho*\sigma)\in C_i$ and $\rho\in C_i$, for all $i\in\Lambda$, but C_i is an ideal of ∂, for all $i\in\Lambda$.

Then $\sigma\in C_i$, for all $i\in\Lambda$, therefore, $\sigma\in\cap_{i\in\Lambda} C_i$. Hence $\bigcap_{i\in\Lambda} C_i$ is an ideal of RG-algebra .⌂

Re. 4.4.

Not that the union of two ideals of $RG-$algebra it is not necessarily ideal of $RG-$algebra, as it is shown in the following Ex..

Ex. 4.5.

Let ∂= {0, a, b, c} and let $*$ be a binary operation defined by the table:

*	0	a	b	c
0	0	0	0	0
a	a	0	0	c
b	b	b	0	b
c	c	c	c	0

It is clear that $(\partial;*,0)$ is an $RG-$algebra. Let $C=\{0,b\}$ and $C'=\{0,a\}$ be two ideals of ∂, but $(C\cup C')=\{0,a,b\}$ is not ideal of ∂.

Prop. 4.6.

Let $\{C_i \mid i\in\Lambda\}$ be a family of ideals of $RG-$algebra $(\partial;*,0)$, then $\bigcup_{i\in\Lambda} C_i$ is an ideal of ∂, where $C_i\subseteq C_{i+1}$, for all $i\in\Lambda$.

Pr.:

Since $\{C_i \mid i\in\Lambda\}$ be a family of ideals of $RG-$algebra , then $0\in C_i$, for some $i\in\Lambda$, and $C_i\subseteq C_{i+1}$, then $0\in\bigcup_{i\in\Lambda} C_i$.

For any $\rho,\sigma\in\partial$,, suppose $(\rho*\sigma)\in\bigcup_{i\in\Lambda} C_i$ and $\rho\in\bigcup_{i\in\Lambda} C_i$, then $(\rho*\sigma)\in C_i$ and $\rho\in C_j$ for some $i,j\in\Lambda$. By assumption $C_i\subseteq C_k$ and $C_j\subseteq C_k$.

Hence $\sigma\in C_k$, but C_k is an ideal of RG-algebra ∂, then $\sigma\in C_k$, therefore, $\sigma\in\bigcup_{i\in\Lambda} C_i$.

Hence $\bigcup_{i\in\Lambda} C_i$ is an ideal of RG-algebra .⌂

Th.4.7.

An ideal of RG-algebra $(\partial;\ *,0)$ is a RG-subalgebra.

Pr.:

Suppose that C is an ideal of ∂ and let $(\rho*\sigma)\in C$ and $\rho\in C$ implies that $\sigma\in C$. It follows that $\rho\in C$ imply $\sigma\in C$.

Thus $(\rho * \sigma) \in C$. ⌂

Prop. 4.8.

If $Q: (\partial;*,0) \to (\partial';*',0')$ is an epimorphism from RG-algebra $(\partial;\ *,0)$ into RG-algebra$(\partial';*',0')$, then the image of ideal is an ideal.

Pr.:

Let $Q: (\partial;*,0) \to (\partial';*',0')$ be an epimorphism from ∂ into ∂' and let C be an ideal of ∂, then $0 \in A$, so $Q(0) = 0' \in Q(C)$, by Prop. (3.7).

Now, let $(\rho *' \sigma) \in Q(C)\ and\ \rho \in Q(C)$, then $Q^{-1}(\rho *' \sigma) \in C$

and $C^{-1}(\rho) \in C$, since Q is onto.

But $Q^{-1}(\rho *' \sigma) = (Q^{-1}(\rho * \sigma)) = (Q^{-1}(\rho) * Q^{-1}(\sigma))$, by Prop. (3.7).

Therefore, $(Q^{-1}(\rho) * Q^{-1}(\sigma)) \in C\ and\ Q^{-1}(\rho) \in C$. Since C is an ideal of ∂, then $Q^{-1}(\sigma) \in C$, thus $\sigma \in Q(C)$, that means $Q(C)$ is an ideal of ∂'. ⌂

Prop. 4.9.

If $Q: (\partial;*,0) \to (\partial';*',0')$ is a hom. from RG-algebra ∂ into RG-algebra ∂', then the inverse image of ideal is an ideal.

Pr.:

Let $Q: (\partial;*,0) \to (\partial';*',0')$ be a hom. from ∂ into ∂' and let C be an ideal of ∂', then $0' \in C$, so $Q(0) = 0' \in C$, by Prop. (3.7), thus $0' = Q^{-1}(0) \in Q^{-1}(C)$.

Now, let $(\rho * \sigma) \in Q^{-1}(C)\ and\ \rho \in Q^{-1}(C)$, so $Q(\rho * \sigma) \in C$

and $Q(x) \in C$. But $Q(\rho * \sigma) = Q(\rho) *' Q(\sigma))$, by Prop. (3.7). Therefore $\left(Q(\rho) *' Q(\sigma)\right) \in C\ and\ Q(x) \in C$, then $Q(\sigma) \in C$, since C is an ideal, thus $\sigma \in Q^{-1}(C)$, that means $Q^{-1}(C)$ is an ideal of ∂.⌂

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