

On The Ideals and q -ideals of RG-algebra with Homomorphism

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Abstract - The aim of this paper is introducing the notion of RG – ideals, ideals and q -ideal of RG – algebra, several theorems, properties are stated and proved. The relations on RG – algebras are also studied.

Keywords— RG – algebra, RG – ideal, ideal, fuzzy q – ideal, homomorphisms of RG – algebra, the cartesian product of fuzzy q – ideals.

1. INTRODUCTION

BCK – algebras form an important class of logical algebras introduced by K. Iseki [13] and was extensively investigated by several researchers. The class of all BCK – algebras is quasi variety. J. Meng and Y. B. Jun posed an interesting problem (solved in [16]) whether the class of all BCK – algebras is a variety. In connection with this problem, Komori introduced in [15] a notion of BCC algebras. W. A. Dudek (cf. [4]) redefined the notion of BCC – algebras by using a dual form of the ordinary definition in the sense of Y. Komori and studied ideals and congruences of BCC – algebras. In [20,21], C. Prabpayak and U. Leerawat introduced a new algebraic structure, which is called KU – algebra. They gave the concept of homomorphisms of KU – algebras and investigated some related properties. L. A. Zadeh [23] introduced the notion of fuzzy subsets. At present this concept has been applied to many mathematical branches, such as group, functional analysis, probability theory, topology, and so on. In 1991, O. G. Xi [22] applied this concept to BCK – algebras, and he introduced the notion of fuzzy sub – algebras (ideals) of the BCK – algebras with respect to minimum, and since then Jun et al studied fuzzy ideals (cf. [14]), and moreover several fuzzy structures in BCC – algebras are considered (cf. [17]). In [6], the anti – fuzzy AB – ideals of AB – algebras and in [11], the anti – fuzzy AT – ideals of AT – algebras were introduced. Several theorems are stated and proved. In 2013, S. M. Mostafa, and A. T. Hameed introduced On KUS – algebras. In 2014, R. A. K. Omar introduced on RG – algebra. In 2018, P. Patthanankoor have been given RG – homomorphism and Its Properties. In 2020, A. T. Hameed and N. J. Raheem introduced hyper SA – algebra. In this paper, the study of new algebraic structures was started with the introduction of the concept of RG-subalgebras, ideals and q –ideals of RG-algebra and we describe how to deal with the homomorphism of image and inverse image of them.

2. Preliminaries

Now, we give some definitions and preliminary results needed in the later sections.

Def. 2.1 [15].

An algebra $(\partial; *, 0)$ is called **an RG-algebra** if the following axioms are satisfied: $\forall \rho, \sigma, \tau \in \partial$,

(i) $\rho * 0 = \rho$,

(ii) $\rho * \sigma = (\rho * \tau) * (\sigma * \tau)$,

(iii) $\rho * \sigma = \sigma * \rho = 0$ imply $\rho = \sigma$.

Prop. 2.2 [15].

In any RG-algebra $(\partial; *, 0)$, the following hold: $\forall \rho, \sigma \in \partial$,

- i) $\rho * \rho = 0$,
- ii) $0 * (0 * \rho) = \rho$,
- iii) $\rho * (\rho * \sigma) = \sigma$,
- iv) $\rho * \sigma = 0$ if and only if $\sigma * \rho = 0$,
- v) $\rho * 0 = 0$ implies $\rho = 0$,
- vi) $0 * (\sigma * \rho) = \rho * \sigma$.

Prop. 2.3 [15].

In any RG – algebra $(\partial; *, 0)$, the following hold: $\forall \rho, \sigma, \tau \in \partial$,

- i) $(\rho * \sigma) * (0 * \sigma) = (\rho * (0 * \sigma)) * \sigma = \rho$,
- ii) $\rho * (\rho * (\rho * \sigma)) = \rho * \sigma$,
- iii) $(\rho * \sigma) * \tau = (\rho * \sigma) * \tau$.
- iv) $\rho * \sigma = (\tau * \sigma) * (\tau * \rho)$,
- v) $((\rho * \sigma) * (\rho * \tau)) * (\tau * \sigma) = 0$.

Prop. 2.6 [11].

Every RG – algebra is a BCI – algebra. The converse of this Prop. may be not true.

Def. 2.8 [15].

Let $(\partial; *, 0)$ be an RG – algebra and S be a nonempty subset of ∂ . Then S is called a **subalgebra** of X if $(S; *, 0)$ is itself an RG-algebra.

Def. 2.9 [15].

Let $(\partial; *, 0)$ be an RG – algebra, a nonempty subset C of ∂ is called an **RG – ideal** of ∂ if: $\forall \rho, \sigma \in \partial$,

- i) $0 \in C$,
- ii) $\rho * \sigma \in C$ and $0 * \rho \in C$ imply $0 * \sigma \in C$.

Prop. 2.10 [15].

In an RG – algebra $(\partial; *, 0)$, every RG – ideal is a subalgebra of ∂ .

Def. 2.11 [11].

Let $(\partial; *, 0)$ and $(\partial'; *', 0')$ be nonempty sets. The mapping

$Q: (\partial; *, 0) \rightarrow (\partial'; *', 0')$ is called a **homomorphism** if it satisfies: $Q(\rho * \sigma) = Q(\rho) *' Q(\sigma)$, for all $\forall \rho, \sigma \in \partial$.

The set $\{\rho \in \partial \mid Q(\rho) = 0'\}$ is called **the kernel of f** denoted by $\ker Q$.

Th.2.12 [11].

Let $Q: (\partial; *, 0) \rightarrow (\partial'; *', 0')$ be a hom. of an RG-algebra ∂ into an RG-algebra ∂' , then:

- A. $Q(0) = 0'$.
- B. Q is injective if and only if $\ker Q = \{0\}$.
- C. If $0 * \rho = \rho, \forall \rho \in \partial$, then $Q(0) *' \sigma = \sigma$.

Th.2.13 [11].

Let $Q: (\partial; *, 0) \rightarrow (\partial'; *', 0')$ be a hom. of an RG-algebra ∂ into an RG-algebra ∂' , then:

- (F₁) If S is a subalgebra of ∂ , then $Q(S)$ is a subalgebra of ∂' .
- (F₂) If C is an RGideal of ∂ , then $Q(C)$ is an RGideal of ∂' .
- (F₃) If S is a subalgebra of ∂' , then $Q^{-1}(S)$ is a subalgebra of ∂ .
- (F₄) If C is an RGideal of ∂' , then $Q^{-1}(C)$ is an RGideal of ∂ .
- (F₅) $\ker Q$ is an RGideal of ∂ .
- (F₆) $\text{Im}(Q)$ is a subalgebra of ∂' .

Prop. 2.14.[15].

Let $(\partial; *, 0)$ be an RG-algebra. Then the following hold: for any $\rho, \sigma, \tau \in \partial$,

- 1- $\rho \leq \sigma$ imply $\tau * \sigma \leq \tau * \rho$,
- 2- $\rho \leq \sigma$ implies $\rho * \tau \leq \sigma * \tau$,
- 3- $\rho * \sigma \leq \tau$ imply $\rho * \tau \leq \sigma$.

3. The RG-subalgebras of RG-algebra

In this section, we will discuss a new notion called RG-subalgebra of RG-algebra and study several basic properties of RG-algebra.

Def. 3.1[8].

A nonempty subset S of an RG-algebra $(\partial; *, 0)$ is called

an RG-subalgebra of X if it satisfies the following conditions: for any $\rho, \sigma \in \partial$, if $\rho * \sigma \in S$, for any $\rho, \sigma \in S$.

Prop. 3.2.

Let $\{S_i \mid i \in \Lambda\}$ be a family of RG-subalgebras of an RG-algebra

$(\partial; *, 0)$. The intersection of any set of RG-subalgebras of ∂ is also RG-subalgebra of ∂ .

Prop. 3.3.

Let $\{S_i \mid i \in \Lambda\}$ be a family of RG-subalgebra of RG-algebra $(\partial; *, 0)$, then $\bigcup_{i \in \Lambda} S_i$ is a RG-subalgebra of X , where $S_i \subseteq S_{i+1}$, for all $i \in \Lambda$.

Prop. 3.4.

Let $Q: (\partial; *, 0) \rightarrow (\partial'; *', 0')$ be a hom. of an RG-algebra ∂ into an RG-algebra ∂' , then :

- A. $Q(0) = 0'$, where 0 and $0'$ are the constants of ∂ and ∂' respectively, where Q is onto.
- B. Q is injective if and only if $\ker Q = \{0\}$.
- C. $\rho \leq \sigma$ implies $Q(\rho) \leq Q(\sigma)$.

Prop. 3.5.

If $Q: (\partial; *, 0) \rightarrow (\partial'; *', 0')$ is an epimorphism from RG-algebra $(\partial; *, 0)$ into RG-algebra $(\partial'; *', 0')$, then the image of RG-subalgebra is an RG-subalgebra.

Prop. 3.6.

If $Q: (\partial; *, 0) \rightarrow (\partial'; *', 0')$ is a hom. from RG-algebra ∂ into RG-algebra ∂' , then the inverse image of RG-subalgebra is a RG-subalgebra.

4. On The Ideals and Hom. of RG-algebra

In this section, we will discuss a new notion called ideal of RG-algebra and study several basic properties which are related of RG-algebra.

Def. 4.1[8].

A nonempty subset I of an RG-algebra $(\partial; *, 0)$ is called

an ideal of ∂ if it satisfies the following conditions: for any $\rho, \sigma \in \partial$,

$(I_1) 0 \in I,$

$(I_2) \rho * \sigma \in I \text{ and } \rho \in I \text{ imply } \sigma \in I.$

Ex.s 4.2.

Let $\partial = \{0, 1, 2, 3\}$ and let $*$ be defined by the table:

*	0	1	2	3
0	0	0	3	3
1	1	0	2	2
2	2	3	0	0
3	3	3	0	0

Thus $(\partial; *, 0)$ is an RG – algebra and we see that $C = \{0, 1\}$ and $C' = \{0, 2, 3\}$ are ideals of ∂ .

Prop. 4.3.

Let $\{C_i | i \in \Lambda\}$ be a family of ideals of an RG – algebra $(\partial; *, 0)$. The intersection of any set of ideals of ∂ is also ideal of ∂ .

Pr.:

Since $\{C_i | i \in \Lambda\}$ be a family of ideals of RG – algebra ∂ , then $0 \in C_i$, for all $i \in \Lambda$, then $0 \in \bigcap_{i \in \Lambda} C_i$.

For any $\rho, \sigma \in \partial$, suppose $(\rho * \sigma) \in \bigcap_{i \in \Lambda} C_i$ and $\rho \in \bigcap_{i \in \Lambda} C_i$, then $(\rho * \sigma) \in C_i$ and $\rho \in C_i$, for all $i \in \Lambda$, but C_i is an ideal of ∂ , for all $i \in \Lambda$.

Then $\sigma \in C_i$, for all $i \in \Lambda$, therefore, $\sigma \in \bigcap_{i \in \Lambda} C_i$. Hence $\bigcap_{i \in \Lambda} C_i$ is an ideal of RG -algebra ∂ . Δ

Re. 4.4.

Not that the union of two ideals of RG – algebra it is not necessarily ideal of RG – algebra, as it is shown in the following Ex..

Ex. 4.5.

Let $\partial = \{0, a, b, c\}$ and let $*$ be a binary operation defined by the table:

*	0	a	b	c
0	0	0	0	0
a	a	0	0	c
b	b	b	0	b
c	c	c	c	0

It is clear that $(\partial; *, 0)$ is an RG – algebra. Let $C = \{0, b\}$ and $C' = \{0, a\}$ be two ideals of ∂ , but $(C \cup C') = \{0, a, b\}$ is not ideal of ∂ .

Prop. 4.6.

Let $\{C_i | i \in \Lambda\}$ be a family of ideals of RG – algebra $(\partial; *, 0)$, then $\bigcup_{i \in \Lambda} C_i$ is an ideal of ∂ , where $C_i \subseteq C_{i+1}$, for all $i \in \Lambda$.

Pr.:

Since $\{C_i | i \in \Lambda\}$ be a family of ideals of RG – algebra ∂ , then $0 \in C_i$, for some $i \in \Lambda$, and $C_i \subseteq C_{i+1}$, then $0 \in \bigcup_{i \in \Lambda} C_i$.

For any $\rho, \sigma \in \partial$, suppose $(\rho * \sigma) \in \bigcup_{i \in \Lambda} C_i$ and $\rho \in \bigcup_{i \in \Lambda} C_i$, then $(\rho * \sigma) \in C_i$ and $\rho \in C_j$ for some $i, j \in \Lambda$. By assumption $C_i \subseteq C_k$ and $C_j \subseteq C_k$.

Hence $\sigma \in C_k$, but C_k is an ideal of RG -algebra ∂ , then $\sigma \in C_k$, therefore, $\sigma \in \bigcup_{i \in \Lambda} C_i$.

Hence $\bigcup_{i \in \Lambda} C_i$ is an ideal of RG -algebra ∂ . Δ

Th.4.7.

An ideal of RG -algebra $(\partial; *, 0)$ is a RG -subalgebra.

Pr.:

Suppose that C is an ideal of ∂ and let $(\rho * \sigma) \in C$ and $\rho \in C$ implies that $\sigma \in C$. It follows that $\rho \in C$ imply $\sigma \in C$.

Thus $(\rho * \sigma) \in C$. \triangle

Prop. 4.8.

If $Q: (\partial; *, 0) \rightarrow (\partial'; *', 0')$ is an epimorphism from RG-algebra $(\partial; *, 0)$ into RG-algebra $(\partial'; *', 0')$, then the image of ideal is an ideal.

Pr.:

Let $Q: (\partial; *, 0) \rightarrow (\partial'; *', 0')$ be an epimorphism from ∂ into ∂' and let C be an ideal of ∂ , then $0 \in A$, so $Q(0) = 0' \in Q(C)$, by Prop. (3.7).

Now, let $(\rho *' \sigma) \in Q(C)$ and $\rho \in Q(C)$, then $Q^{-1}(\rho *' \sigma) \in C$

and $C^{-1}(\rho) \in C$, since Q is onto.

But $Q^{-1}(\rho *' \sigma) = (Q^{-1}(\rho * \sigma)) = (Q^{-1}(\rho) * Q^{-1}(\sigma))$, by Prop. (3.7).

Therefore, $(Q^{-1}(\rho) * Q^{-1}(\sigma)) \in C$ and $Q^{-1}(\rho) \in C$. Since C is an ideal of ∂ , then $Q^{-1}(\sigma) \in C$, thus $\sigma \in Q(C)$, that means $Q(C)$ is an ideal of ∂' . \triangle

Prop. 4.9.

If $Q: (\partial; *, 0) \rightarrow (\partial'; *', 0')$ is a hom. from RG-algebra ∂ into RG-algebra ∂' , then the inverse image of ideal is an ideal.

Pr.:

Let $Q: (\partial; *, 0) \rightarrow (\partial'; *', 0')$ be a hom. from ∂ into ∂' and let C be an ideal of ∂' , then $0' \in C$, so $Q(0) = 0' \in C$, by Prop. (3.7), thus $0' = Q^{-1}(0) \in Q^{-1}(C)$.

Now, let $(\rho * \sigma) \in Q^{-1}(C)$ and $\rho \in Q^{-1}(C)$, so $Q(\rho * \sigma) \in C$

and $Q(x) \in C$. But $Q(\rho * \sigma) = Q(\rho) *' Q(\sigma)$, by Prop. (3.7). Therefore $(Q(\rho) *' Q(\sigma)) \in C$ and $Q(x) \in C$, then $Q(\sigma) \in C$, since C is an ideal, thus $\sigma \in Q^{-1}(C)$, that means $Q^{-1}(C)$ is an ideal of ∂ . \triangle

5. An q-ideals and Homomorphism of RG-algebra

In this section, we will discuss a new notion called q – ideal of RG – algebra and study several basic properties which are related to q – ideal and ideal of RG – algebra.

Def. 5.1.

A nonempty subset I of an RG – algebra $(\partial; *, 0)$ is called **an q – ideal of X** if it satisfies the following conditions: for any $\forall \rho, \sigma, \tau \in \partial$,

(I₁) $0 \in C$,

(I₂) $(\rho * \sigma) * \tau \in C$ and $\sigma \in I$ imply $\rho * \tau \in C$.

Ex. 5.2.

Let $\partial = \{0, 1, 2, 3\}$ and let $*$ be defined by the table:

*	0	1	2	3
0	0	0	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Thus $(\partial; *, 0)$ is an RG – algebra. And we see that $C = \{0, 1\}$ and $C' = \{0, 2\}$ are q – ideals of ∂ .

Prop. 5.3.

Let $\{C_i \mid i \in \Lambda\}$ be a family of q – ideals of an RG – algebra $(\partial; *, 0)$. The intersection of any set of q – ideals of ∂ is also q –ideal of ∂ .

Pr:

Since $\{C_i \mid i \in \Lambda\}$ be a family of q – ideals of RG – algebra ∂ , then $0 \in C_i$, for all $i \in \Lambda$, then $0 \in \bigcap_{i \in \Lambda} C_i$.

For any $\rho, \sigma, \tau \in \partial$, suppose $((\rho * \sigma) * \tau) \in \bigcap_{i \in \Lambda} C_i$ and $\sigma \in \bigcap_{i \in \Lambda} C_i$, then $((\rho * \sigma) * \tau) \in C_i$ and $\sigma \in C_i$, for all $i \in \Lambda$, but C_i is an q –ideal of RG – algebra ∂ , for all $i \in \Lambda$. Then $\rho * \tau \in C_i$, for all $i \in \Lambda$, therefore, $\rho * \tau \in \bigcap_{i \in \Lambda} C_i$. Hence $\bigcap_{i \in \Lambda} C_i$ is an q -ideal of RG-algebra ∂ . \triangle

Re. 5.4.

Not that the union of two q – ideals of RG – algebra it is not necessarily q -ideal of RG – algebra, as it is shown in the following example.

Ex. 5.5.

Let $\partial = \{0, a, b, c\}$ and let $*$ be a binary operation defined by the table:

*	0	a	b	c
0	0	0	0	0
a	a	0	0	0
b	b	b	0	0
c	c	c	c	0

It is clear that $(X; *, 0)$ is an RG – algebra. Let $I = \{0, b\}$ and $J = \{0, a\}$ be two q – ideals of X , but $(I \cup J) = \{0, a, b\}$ is not q -ideal of X .

Prop. 5.6.

Let $\{C_i \mid i \in \Lambda\}$ be a family of q – ideals of RG – algebra $(\partial; *, 0)$, then $\bigcup_{i \in \Lambda} C_i$ is an q –ideal of ∂ , where $C_i \subseteq C_{i+1}$, for all $i \in \Lambda$.

Pr.:

Since $\{C_i \mid i \in \Lambda\}$ be a family of q – ideals of RG – algebra ∂ , then $0 \in C_i$, for some $i \in \Lambda$, and $C_i \subseteq C_{i+1}$, then $0 \in \bigcup_{i \in \Lambda} C_i$.

For any $\rho, \sigma, \tau \in \partial$, suppose $((\rho * \sigma) * \tau) \in \bigcup_{i \in \Lambda} C_i$ and $\sigma \in \bigcup_{i \in \Lambda} C_i$,

then $((\rho * \sigma) * \tau) \in C_i$ and $\sigma \in C_j$ for some $i, j \in \Lambda$. By assumption

$C_i \subseteq C_k$ and $C_j \subseteq C_k$.

Hence $\rho * \tau \in C_k$, but C_k is an q -ideal of RG -algebra ∂ , then

$\rho * \tau \in C_k$, therefore, $\rho * \tau \in \bigcup_{i \in \Lambda} C_i$.

Hence $\bigcup_{i \in \Lambda} C_i$ is an q -ideal of RG -algebra ∂ . \triangle

Prop. 5.7.

A q -ideal of RG -algebra $(\partial; *, 0)$ is an ideal.

Pr.:

Suppose that C is an q -ideal of ∂ and let $((\rho * \sigma) * \tau) \in C$ and $\sigma \in C$. It follows that $(0 * \rho) * \sigma \in C$ imply $0 * \rho \in C$. Thus $\rho \in C$. \triangle

Re. 5.8.

The converse of Proposition (5.7) needs not be true in general as in the Example.

Ex. 5.9.

Let $\partial = \{0, a, b, c\}$ and let $*$ be a binary operation defined by the table:

*	0	a	b	c
0	0	0	0	0
a	a	0	0	0
b	b	a	0	0
c	c	b	a	0

Then $(\partial; *, 0)$ is an RG -algebra with $C = \{0, b\}$ is an ideal of ∂ , but not a q -ideal of ∂ , since $(c * b) * b = a * b = 0 \in \{0, b\}$ and $b \in \{0, b\}$, but $c * b = a \notin \{0, b\}$.

Prop. 5.10.

A q -ideal of RG -algebra $(\partial; *, 0)$ is an RG -subalgebra.

Pr.:

Suppose that C is an q -ideal of ∂ , then C is an ideal of ∂ by Proposition (5.7). C is a RG -subalgebra by Proposition (4.7). \triangle

Re. 5.11.

The converse of Proposition (5.10) needs not be true in general as in the Example.

Ex. 5.12.

Let $\partial = \{0, a, b, c\}$ and let $*$ be a binary operation defined by the table:

*	0	a	b	c
0	0	0	0	0
a	a	0	0	0
b	b	a	0	0
c	c	b	a	0

Then $(\partial; *, 0)$ is an RG – algebra with $C = \{0, c\}$ is a RG-subalgebra of ∂ , but not a q – ideal of ∂ , since $(c * b) * b = a * b = 0 \in \{0, c\}$ and $c \in \{0, c\}$, but $c * b = a \notin \{0, c\}$.

Prop. 5.13.

If $Q: (\partial; *, 0) \rightarrow (\partial'; *', 0')$ is an epimorphism from RG – algebra $(\partial; *, 0)$ into RG – algebra $(\partial'; *', 0')$, then the image of q – ideal is an q – ideal.

Pr.:

Let $Q: (\partial; *, 0) \rightarrow (\partial'; *', 0')$ be an epimorphism from ∂ into ∂' and let C be an q – ideal of ∂ , then $0 \in C$, so $Q(0) = 0' \in Q(C)$, by Proposition (5.10).

Now, let $((\rho *' \sigma) *' \tau) \in Q(C)$ and $\sigma \in Q(C)$, then

$$Q^{-1}((\rho *' \sigma) *' \tau) \in C \text{ and } Q^{-1}(\sigma) \in C, \text{ since } Q \text{ is onto. But } Q^{-1}((\rho *' \sigma) *' \tau) = (Q^{-1}(\rho *' \sigma) * Q^{-1}(\tau)) \\ = ((Q^{-1}(\rho) * Q^{-1}(\sigma)) * Q^{-1}(\tau)), \text{ by Proposition (5.8).}$$

Therefore, $((Q^{-1}(\rho) * Q^{-1}(\sigma)) * Q^{-1}(\tau)) \in C$ and $Q^{-1}(\sigma) \in C$. Since C is an ideal of ∂ , then $Q^{-1}(\rho *' \tau) \in C$, thus $\rho *' \tau \in Q(C)$, that means $Q(C)$ is an ideal of ∂' . \triangle

Prop. 5.14.

If $Q: (\partial; *, 0) \rightarrow (\partial'; *', 0')$ is a homomorphism from RG – algebra ∂ into RG-algebra ∂' , then the inverse image of q – ideal is an q – ideal.

Pr.:

Let $Q: (\partial; *, 0) \rightarrow (\partial'; *', 0')$ be an homomorphism from X into Y and let C be an q – ideal of ∂' . Then $0' \in C$, so $Q(0) = 0' \in C$, by Proposition (3.7), thus $0 = Q^{-1}(0') \in Q^{-1}(C)$.

Now, let $((\rho * \sigma) * \tau) \in Q^{-1}(C)$ and $\sigma \in Q^{-1}(C)$, so

$$Q((\rho * \sigma) * \tau) \in C \text{ and } Q(\sigma) \in C.$$

But $Q((\rho * \sigma) * \tau) = (Q(\rho * \sigma) *' Q(\tau)) = ((Q(\rho) *' Q(\sigma)) *' Q(\tau))$, by Proposition (3.7). Therefore,

$$((Q(\rho) *' Q(\sigma)) *' Q(\tau)) \in C \text{ and } Q(\sigma) \in C, \text{ then}$$

$$(Q(\rho) *' Q(\tau)) = Q(\rho * \tau) \in C, \text{ since } C \text{ is an q-ideal, thus}$$

$$\rho * \tau \in Q^{-1}(C), \text{ that means } Q^{-1}(C) \text{ is an q – ideal of } \partial. \triangle$$

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