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Wrapped Sujatha distribution: Properties and Applications

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Abstract: *In this article, we introduce a new circular distribution to be called as wrapped sujatha distribution and derive expressions for characteristic function, trigonometric moments, coefficients of skew ness, and kurtosis. Method of maximum likelihood estimation is used for the estimation of parameters, The proposed model is also applied to a real-life dataset, and its performance is compared with that of wrapped exponential distribution and Lindley distribution.*

Keywords: Circular variance, Circular standard deviation, sujatha distribution, Mean direction, Mean resultant length, Trigonometric moments, Wrapped distribution.

1. Introduction

Circular distributions are used to model directional data in two dimensions which arise quite frequently in many natural and physical sciences like Biology, Medicine, Ecology, Geology, etc. For example, the study of bird migrations records the flight directions of just-released birds as they disappear over the horizon, angle of knee flexion is measured to assess the recovery of orthopedic patients (Rao et al. 1986), study of paleocurrents to infer about the direction of flow of rivers in the past in Geology (Rao and Sengupta 1972) etc. Also, any periodic phenomenon with a known period, say a day, a month, or a year, can be represented on the circle where the circumference corresponds to this period for an individual. For instance, arrival times of patients say with heart attacks at a hospital over the day, the timing of breast cancer surgery within the menstrual cycle, or the occurrence of airplane accidents over the different seasons of the year. There are various ways to obtain a circular distribution, one of them is wrapping a linear distribution around the unit circle. Lévy (1939) introduced wrapped distributions, and since then a lot of work has been done in this field. To list a few, Rao and Kozubowski (2004) discussed circular distributions obtained by wrapping the classical exponential and Laplace distributions on the real line around the circle. Rao, Sarma, and Girija (2007) derived new circular models by wrapping the well-known life-testing models like lognormal, logistic, Weibull, and extreme-value distributions. Roy and Adnan (2012a) developed a new class of circular distributions namely wrapped weighted exponential distribution. In another work, Roy and Adnan (2012b) explored wrapped generalized Gompertz distribution and discussed its application to Ornithology. Recently, Jacob and Jayakumar (2013) proposed a new family of circular distributions by wrapping geometric distribution and studied its properties. Rao, Girija, and Devaraaj (2013) discussed the characteristics of wrapped gamma distribution. Adnan and Roy (2014) derived wrapped variance gamma distribution and showed its applicability to wind direction.

2- Wrapped sujatha distribution (WSD)

A circular distribution is a probability distribution whose total probability is concentrated on the circumference of a unit circle $\{(\cos\theta, \sin\theta)\}|\theta \in [0, 2\pi)\}$,The probability density function g(θ) for a circular random variable θ in a continuous circular distribution must satisfies the following properties:

1. $f(\theta) = \geq 0$ for all θ
2. $\int_0^{2\pi} f(\theta)d\theta = \int_{-\pi}^{\pi} f(\theta)d\theta = 1, -\infty < \theta < \infty$
3. $f(\theta) = (\theta + 2\pi)$

For any linear random variable X on the real line with probability density function f(x) we may define a circular random variable θ = X(mod 2π) with probability density function

Let X follow sujatha distribution, then the probability density function (PDF) of X is given by:-

$$g(\theta) = \sum_{m=0}^{\infty} f(\theta + 2\pi m) \; ; 0 < \theta < 2\pi \qquad (1)$$

sujatha distribution is one parameter lifetime distribution which is a mixture of exponential distribution with scale parameter θ and a gamma distribution with shape parameter 2 and scale parameter [10]. Let X have sujatha distribution with probability density function and cumulative distribution function defined as follows:

$$f(x, \beta) = \frac{\beta^3(1 + x + x^2)e^{-\beta x}}{(\beta^2 + \beta + 2)} \quad (2)$$

where β is the model parameter.

And

$$F(x, \beta) = 1 - \left[1 + \frac{\beta x(\beta x + \beta + 2)}{(\beta^2 + \beta + 2)}\right] e^{-\theta x} \quad (3)$$

Now, the circular (wrapped) sujatha random variable is defined as θ = X(mod 2π), such that for θ∈ [0, 2π), the PDF is given by

$$g(\theta) = \sum_{m=0}^{\infty} f(\theta + 2\pi m) \quad (4)$$

$$= \frac{\boldsymbol{\beta^3(1 + (\theta + 2\pi m) + (\theta + 2\pi m)^2)e^{-\beta(\theta+2\pi m)}}}{\boldsymbol{(\beta^2 + \beta + 2)}}$$

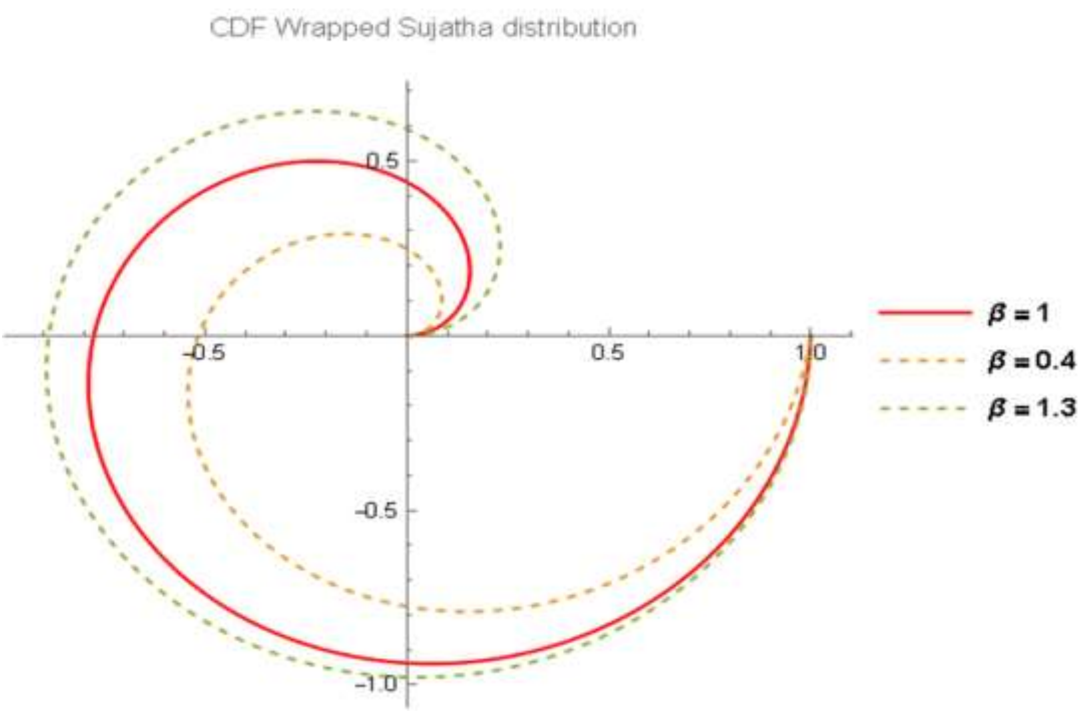
$$= \sum_{m=0}^{\infty} \frac{e^{-\beta(2m\pi+\theta)}\beta^3(1 + 2m\pi(1 + 2m\pi) + \theta + 4m\pi\theta + \theta^2)}{2 + \beta + \beta^2}$$

$$= \frac{e^{2\pi\beta-\beta\theta}\beta^3(1 - 2\pi + \theta + (-2\pi + \theta)^2 + e^{4\pi\beta}(1 + \theta + \theta^2) + 2e^{2\pi\beta}(-1 + \pi + 2\pi^2 + 2\pi\theta - \theta(1 + \theta)))}{(-1 + e^{2\pi\beta})^3(2 + \beta + \beta^2)} \quad (5);$$

$\boldsymbol{\theta} \in [\mathbf{0}, \mathbf{2\pi}], \beta > \mathbf{0}$

The random variable θ having wrapped sujatha distribution is denoted by θ ~ W**S** (θ).

Figure 1 shows the probability density function (PDF) of the wrapped sujatha distribution for different values of β.



The cumulative distribution function (CDF) of the wrapped sujatha distribution is obtained as follows:

$$G(\theta) = \sum_{m=0}^{\infty} F(\theta + 2\pi m) - F(2\pi m)$$

$$G(\theta) = \sum_{m=0}^{\infty} F(\theta + 2\pi m) - \sum_{m=0}^{\infty} F(2\pi m)$$

$$F(\theta, \boldsymbol{\beta}) = \sum_{m=0}^{\infty} 1 - \left[1 + \frac{\boldsymbol{\beta}(\theta + 2\pi m)(\boldsymbol{\beta}(\theta + 2\pi m) + \boldsymbol{\beta} + 2)}{(\boldsymbol{\beta^2} + \boldsymbol{\beta} + \mathbf{2})}\right] \mathbf{e}^{-\boldsymbol{\theta}(\boldsymbol{\theta}+\mathbf{2\pi m})}$$
$$- \sum_{m=0}^{\infty} 1 - \left[1 + \frac{\boldsymbol{\beta}(2\pi m)(\boldsymbol{\beta}(2\pi m) + \boldsymbol{\beta} + 2)}{(\boldsymbol{\beta^2} + \boldsymbol{\beta} + \mathbf{2})}\right] \mathbf{e}^{-\boldsymbol{\theta}(2\pi m)}$$

$$c1 = 1 - \left[1 + \frac{\beta(\theta + 2\pi m)(\beta(\theta + 2\pi m) + \beta + 2)}{(\beta^2 + \beta + 2)}\right] e^{-\theta(\theta+2\pi m)}$$

$$c1 = \frac{e^{-\beta(2m\pi+\theta)}(-2 + e^{\beta(2m\pi+\theta)}(2 + \beta + \beta^2) - \beta(1 + \beta + 4m^2\pi^2\beta + 2\theta + \beta\theta(1 + \theta) + 2m\pi(2 + \beta + 2\beta\theta)))}{2 + \beta + \beta^2}$$

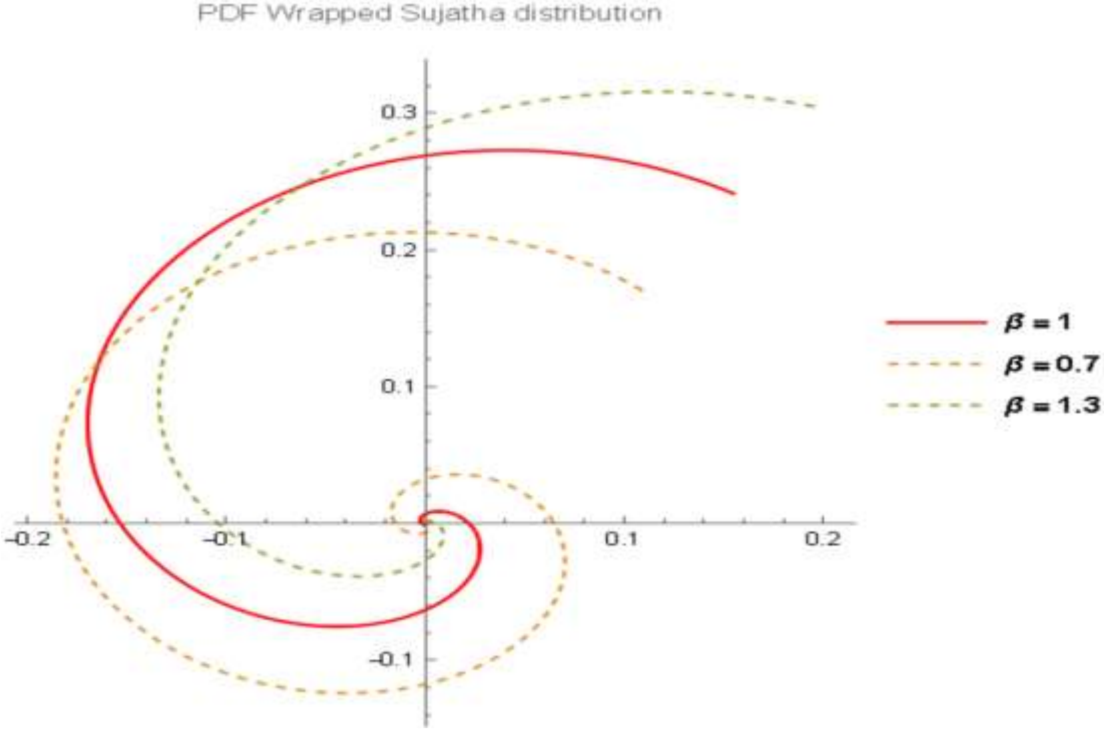
$$c2 = 1 - \left[1 + \frac{\boldsymbol{\beta}(2\pi m)(\boldsymbol{\beta}(2\pi m) + \boldsymbol{\beta} + 2)}{(\boldsymbol{\beta^2} + \boldsymbol{\beta} + \mathbf{2})}\right] \mathbf{e}^{-\boldsymbol{\theta}(2\pi m)}$$

$$c2 = 1 - e^{-2m\pi\beta} - \frac{4e^{-2m\pi\beta}m\pi\beta}{2 + \beta + \beta^2} - \frac{2e^{-2m\pi\beta}m\pi\beta^2}{2 + \beta + \beta^2} - \frac{4e^{-2m\pi\beta}m^2\pi^2\beta^2}{2 + \beta + \beta^2}$$

$$G(\theta) = \sum_{m=0}^{\infty} (c1 - c2)$$

$$G(\theta) = \left(\sum_{m=0}^{\infty} \frac{e^{-\beta(2m\pi+\theta)}(-2+e^{\beta(2m\pi+\theta)}(2+\beta+\beta^2)-\beta(1+\beta+4m^2\pi^2\beta+2\theta+\beta\theta(1+\theta)+2m\pi(2+\beta+2\beta\theta)))}{2+\beta+\beta^2} - c2\right) \quad (6)$$

Figure 2 shows the cumulative distribution function of the wrapped sujatha distribution with different values of β.



3 − Characterization of the density

The trigonometric moment of order p for a wrapped circular distribution corresponds to the value of the characteristic function of the unwrapped.

$$\varphi(p) = \varphi x(t) \quad (7)$$

On the real line, the characteristic function of the sujatha distribution is

$$\varphi x(t) = E(e^{xti}) \quad (8)$$

$$(e^{xti}) = \left[\frac{\beta^3}{(\beta^2+\beta+2)}\right]\int_0^\infty e^{xti}\,(1+x+x^2)e^{-\beta x}.\,dx$$

$$= \left[\frac{\beta^3}{(\beta^2+\beta+2)}\right]\int_0^\infty (1+x+x^2)e^{-\beta x+xti}.\,dx$$

$$= \left[\frac{\beta^3}{(\beta^2+\beta+2)}\right]\int_0^\infty \left(e^{-\beta x+xti}+xe^{-\beta x+xti}+x^2e^{-\beta x+xti}\right).\,\mathbf{dx}$$

$$= \left[\frac{\beta^3}{(\beta^2+\beta+2)}\right]\int_0^\infty e^{-x(\beta-ti)}.\,\mathbf{dx}+\int_0^\infty xe^{-x(\beta-ti)}.\,\mathbf{dx}+\int_0^\infty x^2e^{-x(\beta-ti)}.\,\mathbf{dx}$$

$$E(e^{xti}) = \emptyset_{x(t)} = \frac{\left(2+(ti-1)ti-2ti\beta+\beta+\beta^2\right)\beta^3}{(\beta^2+\beta+2)(ti-\beta)^3}$$

$$\emptyset_{x(t)} = \frac{\left(2+(ti-1)ti-2ti\beta+\beta+\beta^2\right)\beta^3}{(\beta^2+\beta+2)(ti-\beta)^3}$$

$$\emptyset_{(p)} = \left(\frac{\beta^3}{(\beta^2+\beta+2)}\right)\frac{2+(pi-1)pi-2pi\beta+\beta+\beta^2}{(pi-\beta)^3}$$

$$\emptyset_{(p)} = \left(\frac{\beta^3}{(\beta^2+\beta+2)}\right)\frac{2+\beta+\beta^2+pi\left((pi-1)-2\beta\right)}{(pi-\beta)^3}$$

$$\emptyset_{(p)} = \left(\frac{\beta^3}{(\beta^2+\beta+2)}\right)\frac{2+\beta+\beta^2+pi\left((pi-1)-2\beta\right)}{(pi-\beta)^3}$$

$$\emptyset_{(p)} = \left(\frac{\beta^3}{(\beta^2+\beta+2)}\right)\left(2+\beta+\beta^2+pi\left(pi-1-2\beta\right)\right)(pi-\beta)^{-3}$$

$$(a+ip)^r = (a^2+p^2)^{\frac{-r}{2}} e^{ir\arctan\left(\frac{a}{p}\right)}$$

$$(-\beta+pi)^{-3} = (\beta^2+p^2)^{\frac{-3}{2}} e^{3i\arctan\left(\frac{p}{\beta}\right)}$$

$$\left(2+\beta+\beta^2+pi\left(pi-1-2\beta\right)\right) = \left(\left(2+\beta+\beta^2\right)^2+\left(pi-1-2\beta\right)^2\right)^{\frac{-1}{2}} e^{-i\arctan\left(\frac{(p-1-2\beta)}{(2+\beta+\beta^2)}\right)}$$

$$\emptyset_{(p)} = \left(\frac{\beta^3}{(\beta^2+\beta+2)}\right)\left(\left(2+\beta+\beta^2\right)^2+\left(pi-1-2\beta\right)^2\right)^{\frac{-1}{2}} e^{-i\arctan\left(\frac{(p-1-2\beta)}{(2+\beta+\beta^2)}\right)}(\beta^2+p^2)^{\frac{-3}{2}} e^{3i\arctan\left(\frac{p}{\beta}\right)}$$

$$\emptyset_{(p)} = \left(\frac{\beta^3}{(\beta^2+\beta+2)}\right)\left(\left(2+\beta+\beta^2\right)^2+\left(pi-1-2\beta\right)^2\right)^{\frac{-1}{2}} e^{-i\arctan\left(\frac{(p-1-2\beta)}{(2+\beta+\beta^2)}\right)+3i\arctan\left(\frac{p}{\beta}\right)}(\beta^2+p^2)^{\frac{-3}{2}}$$

$$\emptyset_{(p)} = \left(\frac{\beta^3\left(\left(2+\beta+\beta^2\right)^2+\left(pi-1-2\beta\right)^2\right)^{\frac{-1}{2}}(\beta^2+p^2)^{\frac{-3}{2}}}{(\beta^2+\beta+2)}\right) e^{-i\arctan\left(\frac{(p-1-2\beta)}{(2+\beta+\beta^2)}\right)+3i\arctan\left(\frac{p}{\beta}\right)} \qquad (9)$$

$$\emptyset_{(p)} = P_p e^{-i\mu p} \qquad (10)$$

$$P_p = \left(\frac{\beta^3\left(\left(2+\beta+\beta^2\right)^2+\left(p-1-2\beta\right)^2\right)^{\frac{-1}{2}}(\beta^2+p^2)^{\frac{-3}{2}}}{(\beta^2+\beta+2)}\right)$$

$$\mu p = 3\arctan\left(\frac{p}{\beta}\right) - \arctan\left(\frac{(p-1-2\beta)}{(2+\beta+\beta^2)}\right)$$

4-Trigonometric moments and some related parameters

showed that the p th trigonometric moment for a wrapped circular distribution is equal to the value of the characteristic function of the unwrapped random variable at the integer value p. Then the p th trigonometric moment for wrapped **sujatha** distribution is given by**:**

$$\varphi p = \alpha p + i\beta p;\ p = \pm 1, \pm 2, \ldots \qquad (11)$$

and, therefore, the non-central trigonometric moments of the respective distribution are

$$\alpha_p = P_p \cos(\mu p)$$

$\beta_p = P_p \sin(\mu p)$

$$\alpha_p = \left(\frac{\beta^3\left(\left(2+\beta+\beta^2\right)^2+\left(pi-1-2\beta\right)^2\right)^{\frac{-1}{2}}(\beta^2+p^2)^{\frac{-3}{2}}}{(\beta^2+\beta+2)}\right)\cos\left(3\arctan\left(\frac{p}{\beta}\right)-\arctan\left(\frac{(p-1-2\beta)}{(2+\beta+\beta^2)}\right)\right)$$

$$\beta_p = \left(\frac{\beta^3\left(\left(2+\beta+\beta^2\right)^2+\left(pi-1-2\beta\right)^2\right)^{\frac{-1}{2}}(\beta^2+p^2)^{\frac{-3}{2}}}{(\beta^2+\beta+2)}\right)\sin\left(3\arctan\left(\frac{p}{\beta}\right)-\arctan\left(\frac{(p-1-2\beta)}{(2+\beta+\beta^2)}\right)\right)$$

Now, the central trigonometric moments are

$\overline{\alpha}_p = P_p\cos(\mu p - p\mu 1)$ and $\overline{\beta}_p = P_p\sin(\mu p - p\mu 1)$

Thus the central trigonometric moments of the same distribution will be

$$\bar{\alpha}_p = \left(\frac{\beta^3\left(\left(2+\beta+\beta^2\right)^2+\left(pi-1-2\beta\right)^2\right)^{\frac{-1}{2}}(\beta^2+p^2)^{\frac{-3}{2}}}{(\beta^2+\beta+2)}\right)\cos(3\arctan\left(\frac{p}{\beta}\right)-\arctan\left(\frac{(p-1-2\beta)}{(2+\beta+\beta^2)}\right)-p\left(3\arctan\left(\frac{p}{\beta}\right)-\arctan\left(\frac{(p-1-2\beta)}{(2+\beta+\beta^2)}\right)\right))$$

$$\overline{\beta}_p = \left(\frac{\beta^3\left(\left(2+\beta+\beta^2\right)^2+\left(pi-1-2\beta\right)^2\right)^{\frac{-1}{2}}(\beta^2+p^2)^{\frac{-3}{2}}}{(\beta^2+\beta+2)}\right)\sin(3\arctan\left(\frac{p}{\beta}\right)-\arctan\left(\frac{(p-1-2\beta)}{(2+\beta+\beta^2)}\right)-p3\arctan\left(\frac{p}{\beta}\right)-\arctan\left(\frac{(p-1-2\beta)}{(2+\beta+\beta^2)}\right)$$

1-4the Circular mean

The mean direction is given by:

$\mu = \mu_1$

$$\mu = 3\arctan\left(\frac{p}{\beta}\right)-\arctan\left(\frac{(p-1-2\beta)}{(2+\beta+\beta^2)}\right) \qquad (12)$$

2-4the circular variance

The circular variance is given by

$V_\circ = 1 - P_1$

. Hence

$$V_\circ = 1-\left(\frac{\beta^3\left(\left(2+\beta+\beta^2\right)^2+(-2\beta)^2\right)^{\frac{-1}{2}}(\beta^2+1)^{\frac{-3}{2}}}{(\beta^2+\beta+2)}\right) \qquad (13)$$

3-4the circular Standard deviation

The circular deviation is given by

$\sigma_\circ = \sqrt{-2\mathrm{Log}(V_\circ)}$

$$\sigma_\circ = \sqrt{-2Log\left(1-\left(\frac{\beta^3\left(\left(2+\beta+\beta^2\right)^2+(-2\beta)^2\right)^{\frac{-1}{2}}(\beta^2+1)^{\frac{-3}{2}}}{(\beta^2+\beta+2)}\right)\right)} \qquad (14)$$

4-4the Circular skewness

The circular The coefficient of skewness is given by:

$$\text{coefficient of skewness } = \frac{\bar{\beta}_2}{(V_\circ)^{\frac{3}{2}}}$$

$$= \frac{\left(\frac{\beta^3\left((2+\beta+\beta^2)^2+(pi-1-2\beta)^2\right)^{\frac{-1}{2}}(\beta^2+p^2)^{\frac{-3}{2}}}{(\beta^2+\beta+2)}\right)\sin\left(3\arctan\left(\frac{2}{\beta}\right)-\arctan\left(\frac{(1-2\beta)}{(2+\beta+\beta^2)}\right)-2(3\arctan\left(\frac{2}{\beta}\right)-\arctan\left(\frac{(1-2\beta)}{(2+\beta+\beta^2)}\right)\right)}{\left(1-\left(\frac{\beta^3\left((2+\beta+\beta^2)^2+(-2\beta)^2\right)^{\frac{-1}{2}}(\beta^2+1)^{\frac{-3}{2}}}{(\beta^2+\beta+2)}\right)\right)^{\frac{3}{2}}} \qquad (15)$$

5-4Coefficient of Kurtosis

The circular The coefficient of skewness is given by:

$$\text{The coefficient of kurtosis } = \frac{\overline{\alpha_2}-(1-V_\circ)^4}{(V_\circ)^2}$$

$$= \frac{\left(\frac{\beta^3\left((2+\beta+\beta^2)^2+(pi-1-2\beta)^2\right)^{\frac{-1}{2}}(\beta^2+p^2)^{\frac{-3}{2}}}{(\beta^2+\beta+2)}\right)\sin\left(3\arctan\left(\frac{2}{\beta}\right)-\arctan\left(\frac{(1-2\beta)}{(2+\beta+\beta^2)}\right)-2(3\arctan\left(\frac{2}{\beta}\right)-\arctan\left(\frac{(1-2\beta)}{(2+\beta+\beta^2)}\right)\right) \left(\frac{\beta^3\left((2+\beta+\beta^2)^2+(-2\beta)^2\right)^{\frac{-1}{2}}(\beta^2+1)^{\frac{-3}{2}}}{(\beta^2+\beta+2)}\right)^4}{\left(1-\left(\frac{\beta^3\left((2+\beta+\beta^2)^2+(-2\beta)^2\right)^{\frac{-1}{2}}(\beta^2+1)^{\frac{-3}{2}}}{(\beta^2+\beta+2)}\right)_\circ\right)^2} \qquad (16)$$

5-PARAMETER ESTIMATION

In this section, we discuss the method of maximum likelihood estimation to estimate the model parameter β. Let $\theta 1, \theta 2, \ldots, \theta n$ be a random sample of size n from WS(β). Then, the log-likelihood function is given by:

$$Lf(\theta_1.\theta_2.\ldots.\theta_n) = \prod_{i=1}^{n} f(\beta,\theta_i) \qquad (17)$$

$$= \prod_{i=1}^{n}\left[\frac{e^{2\pi\beta-\beta\theta}\beta^3(1-2\pi+\theta+(-2\pi+\theta)^2+e^{4\pi\beta}(1+\theta+\theta^2)+2e^{2\pi\beta}(-1+\pi+2\pi^2+2\pi\theta-\theta(1+\theta)))}{(-1+e^{2\pi\beta})^3(2+\beta+\beta^2)}\right]$$

$$= \left[\frac{e^{2\pi\beta-\beta\theta}\beta^3(1-2\pi+\theta+(-2\pi+\theta)^2+e^{4\pi\beta}(1+\theta+\theta^2)+2e^{2\pi\beta}(-1+\pi+2\pi^2+2\pi\theta-\theta(1+\theta)))}{(-1+e^{2\pi\beta})^3(2+\beta+\beta^2)}\right]$$

$$\log Lf(f(\theta_1.\theta_2.\ldots.\theta_n)) = \log\left(\prod_{i=1}^{n} f(\beta,\theta_i)\right)$$

$$\frac{\log Lf(f(\theta_1.\theta_2.\dots.\theta_n))}{d\beta} = \left[\log\left(\prod_{i=1}^{n} [f(\beta,\theta_i)]\right).d\beta\right] = 0 \qquad (18)$$

Equations (18)cannot be solved by the usual analytical methods because they are non-linear equations and therefore they were solved using the numerical method (Nelder-Mead) to obtain the estimations of the greatest possibility method.

6. Data analysis

TABLE II

This section encompasses the application of proposed model to a real-life dataset. Corneal extremities The posterior cornea of the eyes (100) is diseased.

1.77	1.6	2.2	2.19	2.8	3.1	3.4	3.8	4.2	5
1.60	1.6	2.2	1.12	2.8	3.1	3.5	3.8	4.3	1.57
1.21	1.7	2.3	1.33	2.8	3.2	3.5	3.9	4.4	2.11
1.47	1.7	2.3	0.56	2.9	3.3	3.5	4	4.5	0.47
2.10	1.8	2.3	0.67	2.9	3.3	3.6	4	4.5	1.44
1.40	1.8	2.3	2.21	3	3.3	3.6	4	4.6	1.35
1.82	1.8	2.3	2.22	3	3.4	3.6	4	4.7	1.88
1.57	1.9	2.4	1.99	3	3.4	3.6	4	4.8	2.14
1.56	2	2.4	1.45	3	3.4	3.6	4	4.8	1.11
1.85	2	2.4	1.39	3.1	3.4	3.6	4.2	4.9	2.00

TABLE II

ML Estimates Wrapped sujatha distribution, wrapped exponential, wrapped Lindley,andAkaike Information Criterion (AIC), Akaike Information Criterion Corrected (AICC) and -2lnL

Distribution	MLE	-2Ln log	AIC	AIC_C	BIC	RANK
Wrapped sujatha	$\hat{\beta} = 1.334$	**321.345**	**329.345**	**321.353**	**329.415**	**1**
wrapped exponential	$\hat{\alpha} = 2.7432$	**387.332**	**383.34**	**383.58**	**383.39**	**2**
wrapped Lindley	$\hat{\theta} = 1.4567$	**432.42**	**396.565**	**390.125**	**396.615**	**3**

6-Conclusions

In this article, a new wrapped distribution is introduced, namely Wrapped sujatha distribution. The PDF and CDF of the proposed distribution are derived and their behaviour for characteristic function, trigonometric moments, and other parameters are obtained. To estimate the model parameter, the method of maximum likelihood estimation is used and a simulation study is carried out to show the consistency of the MLE. Last, to show the applicability of the model, we apply the proposed model to a real dataset on orientations of 100 turtles after laying eggs. The proposed model's performance is compared with that of wrapped exponential distribution and wrapped Lindley distribution using log-likelihood, AIC, BIC, On the basis of the results obtained, it is concluded that the Wrapped sujatha distribution is a better model for the given dataset than the wrapped exponential distribution and wrapped Lindley . The authors propose to carry out a change-point study with respect to Wrapped sujatha distribution in their future work .

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