International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 7 Issue 12, December - 2023, Pages: 93-98

Some Results of Fuzzy on BZ − algebra

Prof. Dr. Areej Tawfeeq Hameed[1] and Prof. Dr. Ahmed Hamzah Dbed[2]

[1]*College of Education for Pure Science − Ibn − Al Haitham, University of Baghdad, Iraq.*
E-mail: areej.t@ihcoedu.uobaghdad.edu.iq areej.tawfeeq@uokufa.edu.iq

[2]*College of Islamic Sciences, AL Iraqia University, Baghdad, Iraq.*
E-mail: ahmedh.abed@uokufa.edu.iq areej238@gmail.com

Abstract – *The aim of this paper is introducing the notion of fuzzy q-ideal of BZ-algebra, several theorems, properties are stated and proved. The fuzzy relations on BZ-algebras are also studied.*

Keywords – *BZ − algebra, fuzzy q − ideal, homomorphisms of BZ − algebras, image and pre − image of fuzzy q − ideals.*

2010 Mathematics Subject Classification: 06F35, 03G25, 08A72.

1- Introduction

Asawasamrit and Sudprasert have introduced the notion of BZ − algebras, q-ideals and studied the relations among them and gave the concept of homomorphism of BZ − algebras and investigated some related properties. Abed, studied the structure of BZ −algebras and its properties, studied the isomorphism of BZ − algebras and introduced on the special ideals in BZ − algebras. In this paper, the study of fuzzy algebraic structures was started with the introduction of the concept of fuzzy q −ideals of BZ − algebras and then we investigate several basic properties which are related to fuzzy q − ideals. We describe how to deal with the homomorphism of image and inverse image of fuzzy q − ideals.

2. Preliminaries

In this section we introduced an algebraic structure called a BZ-algebra .

Def. 2.1([9]). Let $(X;*,e)$ be an algebra with operation (*) and constant (e). X is called a ***BZ*-algebra**$(\boldsymbol{BZ-A})$, if it satisfies the following identities: for any $x,y,z \in X$,

$(BZ-1)$ $((x*z)*(y*z))*(x*y)=e$;
$(BZ-2)$ $x*e=x$;
$(BZ-3)$ $x*y=e$ and $y*x=e \Rightarrow x=y$.

Rem. 2.2. ([9-18]).

On BZ-algebra $(X,*,e)$, we defined a binary relation $\le$ on X by putting $x \le y \Leftrightarrow x*y=e$.

Prop. 2.3 ([7,9,18]). $(X;*,e)$ be a BZ-algebra, the following properties are true for a BZ -algebra. For any $x,y,z \in X$:

$(P1)$ $x*((x*y)*y)=e$;
$(P2)$ $x*x=e$;
$(P3)$ $x*(y*z)=y*(x*z)$;

Def. 2. 4. ([9]). A subset S of a $BZ-A$ X is called **subalgebra of *X*** $(\boldsymbol{SA})$,if $x*y \in S$ whenever $x,y \in S$.

Def. 2.5. ([1-3]). A nonempty subset I of a $BZ-algebra$ $(X,*,e)$ is called $\boldsymbol{q-ideal\ of\ X}$ $(q-\boldsymbol{IA})$, if it satisfies the following conditions: for any $x,y,z \in X$

$(I1)$ $e \in I$
$(I2)$ $(y*(x*z)) \in I$ and $x \in I \Rightarrow (y*z) \in I$.

Prop. 2.6 ([4,5]). Every IA of BZ-A $(X,*,e)$ is a SA of X.

Prop. 2.7 ([4,5]). Let $\{I_i \mid i\in\Lambda\}$ be a family of IAs of BZ-A $(X,*,e)$. The intersection of any set of IAs of X is also an IA of X.

Def. 2.8 ([10]). Let $(X,*,e)$and $(Y,*',e')$ be nonempty sets. The mapping $f:(X;*,e) \rightarrow (Y;*',e')$is called **a homo.(homo.)** if it satisfies:

$f(x*y)=f(x)*'f(y)$, for all x , $y \in X$. The set $\{x\in X \mid f(x)=e'\}$ is called **the kernel of $\boldsymbol{f}$** denoted by $\ker f$.

Def. 2.9([19]). $(X,*,e)$ be a nonempty set, a fuzzy subset (FSS) μ of X is a mapping $\mu: X \rightarrow [e,1]$.

International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 7 Issue 12, December - 2023, Pages: 93-98

Def. 2.10 ([19]). Let X be a nonempty set and μ be a fuzzy subset of X, for $t \in [e,1]$, the set $\mu_t=\{x \in X \mid \mu(x) \geq t\}$ is called a level subset of μ.

Def. 2.11 ([16]).
Let $f: (X;*,e) \rightarrow (Y;*',e')$ be a mapping nonempty sets X and Y respectively. If μ is a fuzzy subset of X, then the fuzzy subset β of Y defined by:

$$f(\mu)(y) = \begin{cases} sup\{\mu(x): x \in f^{-1}(y)\} & if f^{-1}(y) = \{x \in X, f(x) = y\} \neq \emptyset \\ e & otherwise \end{cases}$$

is said to be the image of μ under f.
Similarly if β is a FSS of Y, then the FSS μ= (β o f) of X (i.e the fuzzy subset defined by $\mu(x) = \beta(f(x))$ for all $x \in X$) is called the pre-image of β under f.

Def. 2.12 ([16]).
A FSS μ of a set X **has sup property** if for any subset T of X, there exist $t_e \in T \ni \mu(t_e) = sup\{\mu(t) \mid t \in T\}$.

3. Fuzzy subalgebras and Homo. of BZ-algebra

Def. 3.1([1-3]). Let $(X,*,e)$ be an BZ-A, a FSS μ of X is called a **fuzzy subalgebra of X (FSA),** if for all x, y $\in X$, $\mu(x*y) \geq$ min $\{\mu(x), \mu(y)\}$.

Prop. 3.2.
Let μ be a FSS of BZ-A $(X,*,e)$. If μ is FSA of , then for any t $\in [0,1]$, μ_t is a SA of X.
Pr.: Assume that μ is a FSA of X ,let $x, y \in X$ be ∋ x $\in \mu_t$ and y $\in \mu_t$, then $\mu(x) \geq$ t and $\mu(y) \geq t$. Since μ is a FSA, it follows that $\mu(x*y) \geq$ min $\{\mu$ (x) , $\mu(y)\} \geq t$ and that $\mu(x*$y$) \geq t$ ⇒$(x*y) \in \mu_t$. ∴ μ_t is a SA of X, for any t $\in [0,1]$. ⌂

Prop. 3.3.
Let μ be a FSS of BZ-A $(X,*,e)$. If for all t $\in [e,1]$, μ_t is a SA of X, then μ is a FSA of X.
Pr.: Assume $\mu(x*y) \geq min\{\mu(x), \mu(y)\}$ is not true, then there exist
$x', y' \in X \ni \mu(x' * y') < min\{\mu(x'), \mu(y')\}$.
Putting $t'=(\mu(x'*y') + min\{\mu(x'), (y')\})/2$, then
$\mu(x') < t'$ and $e \leq t' < min\{\mu(x'), \mu(y')\} \leq 1$, ∴ $\mu(x') > t'$ and $\mu(y') >$ t', ⇒ $x' \in \mu_{t'}$ and $y' \in \mu_{t'}$, since $\mu_{t'}$ is a SA, it follows that $x' * y' \in \mu_{t'}$ and that $\mu(x' * y') \geq t'$, this is also a C!. ∴μ is a FSA of . ⌂

4. Fuzzy q-ideals and Homo. of BZ-algebra

Def. 4.1. $(X,*,e)$ be a $BZ-algebra$, a fuzzy subset μ of X is called ***a fuzzy q-ideal of X*** if it satisfies the following conditions: for all $x, y, z \in X$,

(F1) $\mu(e) \geq \mu(x)$,
(F2) $\mu(y*z) \geq$ min $\{\mu(y*(x*z)), \mu(x)\}$.

Ex. 4.2.
Let X ={e, 1, 2, 3} in which $(*)$ is defined by the following table:

*	e	1	2	3
e	e	e	3	3
1	1	e	3	2
2	2	3	e	1
3	3	3	e	e

Then $(X,*,e)$ is $BZ-algebra$. Define a fuzzy subset $\mu: X \rightarrow [0,1]$ by

$$\mu(x) = \begin{cases} 0.7 & if \quad x \in \{e,1\} \\ 0.3 & otherwise \end{cases}$$

I_1 = {e, 1} is $q-ideal$ of X. Routine calculation gives that μ is a fuzzy $q-ideal$ of X.

Le. 4.3.
Let μ be a fuzzy $q-ideal$ of $BZ-algebra$ $(X,*,e)$ and if $x \leq y$, then $\mu(y) \geq \mu(x)$, for all $x, y \in X$.
Pr.: Assume that $x \leq y \Rightarrow (x*y)=e$, and
$\mu(y) \geq min\{\mu(x*y), \mu(x)\} = \mu(x) \Rightarrow \mu(y) \geq \mu(x)$.⌂

Prop. 4.4.

Let μ be a fuzzy $q-$ ideal of $BZ-$algebra $(X,*,e)$. μ is a fuzzy $q-$ ideal of X, then for any $t \in [e,1]$, μ_t is an q−ideal of X.

Pr.: Assume that μ is a fuzzy q−ideal of X, by $(F1)$, we have $\mu(e) \geq \mu(x)$ for all $x \in X$ therefore $\mu(e) \geq \mu(x) \geq t$ for $x \in \mu_t$ and so $e \in \mu_t$.

Let $x, y, z \in X$ be such that $(y*(x*z)) \in \mu_t$ and $x \in \mu_t$, then $\mu(y*(x*z)) \geq t$ and $\mu(x) \geq t$, since μ is a fuzzy q-ideal, it follows that

$\mu(y*z) \geq min\{\mu(y*(x*z)), \mu(x)\} \geq t$ and we have that $(y*z) \in \mu_t$. Hence μ_t is an $q-$ ideal of X, for any $t \in [0,1]$. ⌂

Prop. 4.5.

Let μ be a fuzzy q−ideal of $BZ-$ algebra $(X,*,e)$. If for all $t \in [e,1]$, μ_t is an q−ideal of X, ⇒μ is a fuzzy q−ideal of X.

Pr.: We only need to show that $(F1)$ and $(F2)$ are true. If $(F1)$ is false, ⇒there exist $x' \in X$ such that $\mu(e) < \mu(x')$. If we take $t' = (\mu(x') + \mu(e))/2$, then $\mu(e) < t'$ and $e \leq t' < \mu(x') \leq 1$, then $x' \in \mu$ and $\mu \neq \emptyset$. As $\mu_{t'}$ is an $q-$ideal of , we have $e \in \mu_{t'}$ and so $\mu(e) \geq t'$. This is a contradiction.

Now, assume (F_2) is not true, then there exist $x', y', z' \in X$ such that,

$\mu(y'*z') < min\{\mu(y'*(x'*z')), \mu(x')\}$.

Putting $t' = (\mu(y'*z') + min\{\mu(y'*(x'*z')), \mu(x')\}/2$, ⇒

$\mu(y'*z') < t'$ and $e \leq t' < min\{\mu(y'*(x'*z')), \mu(x')\} \leq 1$, ⇒

$\mu(y'*(x'*z')) > t'$ and $\mu(x') > t'$, which imply that $(y'*(x'*z')) \in \mu_{t'}$ and $x' \in \mu_{t'}$. Since $\mu_{t'}$ is an $q-$ideal, it follows that $y'*z' \in \mu_{t'}$, and that $\mu(y'*z') \geq t'$, this is also a contradiction. Hence μ is a fuzzy q-ideal of X. ⌂

Coro. 4.6.

Let μ be a fuzzy subset of BZ-algebra $(X,*,e)$. If μ is a fuzzy q-ideal of , then for every $t \in Im(\mu)$, μ_t is an q-ideal of X, when $\mu_t \neq \emptyset$

Prop. 4.7.

Every fuzzy q-ideal of BZ-algebra $(X,*,e)$ is a fuzzy subalgebra of X.

Pr.:

Since μ is fuzzy q-ideal of a BZ-algebra $(X,*,e)$, ⇒by Prop. (4.4), for any $t \in [0,1]$, μ_t is q-ideal of X. By Prop. (2.8), for any $t \in [0,1]$, μ_t is a subalgebra of X ⇒ μ is a fuzzy subalgebra of X by Prop. (3.3). ⌂

Prop. 4.8.

Let I be an q-ideal of BZ-algebra $(X,*,e)$. For any fixed number t in an open interval (e,1), there exists a fuzzy q-ideal μ of X such that $\mu_t = I$.

Pr.: Define $\mu: X \to [0,1]$ by $\mu(x) = \begin{cases} t & if \;\; x \in I \\ e & otherwise \end{cases}$. Where t is a fixed number in (e, 1). Clearly, $\mu(e) \geq \mu(x)$, for all $x \in X$.

Let $x, y, z \in X$.

If $y \notin I$, then $\mu(y) = e$ and so $\mu(y*z) \geq e = min\{\mu(y*(x*z)), \mu(x)\}$.

If $(y*(x*z))I$, then clearly $\mu(y*z) \geq min\{\mu(y*(x*z)), \mu(x)\}$.

If $y*z \notin I, y \in I$, ⇒$(y*(x*z)) \notin I$, since I is an q- ideal. Thus

$$\mu(y*z) = e = min\{\mu(y*(x*z)), \mu(x)\}.$$

⇒ μ is a fuzzy q-ideal of X. It is clear that $\mu_t = I$. ⌂

Th. 4.9. Let I be a nonempty subset of a BZ-algebra $(X,*,e)$ and μ be a fuzzy subset of X such that μ is into {0, 1}, so that μ is the characteristic function of I, ⇒ μ is a fuzzy q-ideal of X ⇔ I is an q-ideal of X.

Pr.:

Assume that μ is a fuzzy q-ideal of X, since $\mu(e) \geq \mu(x)$ for all $x \in X$, clearly, we have $\mu(e) = 1$, and so $e \in I$.

Let $x, y, z \in X$ be such that $(y*(x*z)) \in I$ and $x \in I$, since μ is a fuzzy q-ideal of X, it follows that $\mu(y*z) \geq min\{\mu(y*(x*z)), \mu(x)\} = 1$, and $\mu(x) = 1$. This ⇒that $(y*z) \in I$, i.e., I is an q-ideal of X.

Conversely, suppose I is an q-ideal of X, since $e \in I$, $\mu(e) = 1 \geq \mu(x)$, for all $x \in X$. Let $x, y, z \in X$,

If $x \notin I$, ⇒ $\mu(y) = e$ and so $\mu(y*z) \geq e = min\{\mu(y*(x*z)), \mu(x)\}$.

If $((x*y)*z) \in I$, ⇒clearly $\mu(y*z) \geq min\{\mu((x*y)*z), \mu(x)\}$.

If $x*z \notin I, x \in I$, ⇒$(y*(x*z)) \notin I$, since I is an q-ideal. Thus

$$\mu(y*z) = e = min\{\mu(y*(x*z)), \mu(x)\}.$$

⇒ μ is a fuzzy q-ideal of X. ⌂

Prop. 4.10.

The intersection of any set of fuzzy q-ideals of BZ-algebra $(X,*,e)$ is also fuzzy q-ideal of X.

Pr.:

Let $\{\mu_i | i \in \Lambda\}$ be a family of fuzzy q-ideals of BZ-algebra $(X,*,e)$, $\Rightarrow$ for any $x, y, z \in X$, $i \in \Lambda$,

$(\bigcap_{i\in\Lambda} \mu_i)(e) = \inf(\mu_i(e)) \geq \inf(\mu_i(x)) = (\bigcap_{i\in\Lambda} \mu_i)(x)$ and

$(\bigcap_{i\in\Lambda} \mu_i)(y * z) = \inf(\mu_i(y * z)) \geq \inf(\min\{\mu_i(y * (x * z)), \mu_i(x)\})$

$= \min\{\inf(\mu_i(y * (x * z)), \inf(\mu_i(x)\}$

$= \min\{(\bigcap_{i\in\Lambda} \mu_i)(y * (x * z)), (\bigcap_{i\in\Lambda} \mu_i)(x)\}$.

$\Rightarrow$, $(\bigcup_{i\in\Lambda} \mu_i)$ is fuzzy q-ideal of X. This completes the Pr.. ⌂

Th. 4.11.

A homomorphic pre-image of a fuzzy q-ideal of BZ-algebra $(X,*,e)$ is also a fuzzy q-ideal of X.

Pr.:

Let $f: (X;*,e) \to (Y;*',e')$ be a of BZ-algebras, β a fuzzy q-ideal of Y and μ the pre-image of β under f, $\Rightarrow \beta(f(x)) = \mu(x)$, for all $x \in X$.

Since $f(x) \in Y$ and β is a fuzzy q-ideal of Y, it follows that $\beta(e') \geq \beta(f(x)) = \mu(x)$, for every $x \in X$, where e' is the zero element of Y.

But $\beta(e') = \beta(f(e)) = \mu(e)$ and so $\mu(e) \geq \mu(x)$, for any $x \in X$.

Now, let $x, y, z \in X$, $\Rightarrow$ we get

$\mu(y * z) = \beta(f(y * z))$

$\geq \min\{\beta(f(y * (x * z))), \beta(f(x))\}$

$= \min\{\beta f(y) *' (f(x) *' f(z))), \beta(f(x))\}$

$= \min\{\beta(f(y * (x * z))), \beta(f(x))\}$

$= \min\{\mu(y * (x * z))\mu(x)\}$

i.e., $\mu(y*z) \geq \min\{\mu(y * (x * z)), \mu(x)\}$, for all $x, y, z \in X$. ⌂

Th. 4.12.

Let $f: (X;*,e) \to (Y;*',e')$ be an epimorphism between BZ-algebras X and Y respectively. For every fuzzy q-ideal μ of X with sup property, $f(\mu)$ is a fuzzy BZ-ideal of Y.

Pr.:

By Def. $\beta(y') = f(\mu)(y') = \sup\{\mu(x) | x \in f^{-1}(y')\}$, for all $y' \in Y$ ($\sup \emptyset = e$).

We have to prove that $\beta(x' *' z') \geq \min\{\beta((x' *' y') *' z'), \beta(y')\}$, for all $x', y', z' \in Y$.

Let $f: (X;*,e) \to (Y;*',e')$ be an epimorphism of BZ-algebras, μ is a fuzzy BZ-ideal of X with sup property and β the image of μ under f.

Since μ is a fuzzy q-ideal of , we have $\mu(e) \geq \mu(x)$, for all $x \in X$.

Note that $e \in f^{-1}(e')$, where e and e' are the zero elements of X and Y respectively. Thus $\beta(0') = \sup_{t\in f^{-1}(x')} \mu(t) = \mu(0) \geq \mu(x)$

$= \mu(e) \geq \mu(x)$ for all $x \in X$, which $\Rightarrow$ $\beta(0') \geq \sup_{t\in f^{-1}(x')} \mu(t) = \beta(x')$, for any $x' \in Y$

For any $x', y', z' \in Y$, let $x_e \in f^{-1}(x')$, $y_e \in f^{-1}(y')$, $z_e \in f^{-1}(z')$ be such that:

$\mu(x_0 * y_0) = \beta[f(x_0 * y_0)] = \beta[f(x' * y')] = \sup_{x_0 * y_0 \in f^{-1}(x' * y')} \mu(x_0 * y_0)$

$\mu(x_0) = \beta[f(x_0)] = \beta[f(x')] = \sup_{x_0 \in f^{-1}(x)} \mu(x_0)$, then

$\beta(y' *' z') = \sup \mu(t) = \mu_{t\in f^{-1}(y' *' z')}(y_e * z_e)$

$\geq \min\{\mu(y_e * (x_e * z_e)), \mu(x_e)\}$

$= \min\{\sup \mu_{t\in f^{-1}(y' * (x' * z'))}(t), \sup \mu_{t\in f^{-1}(x')}(t)\}$

$= \min\{\beta(y' * (x' * z')), \beta(x')\}$

$\Rightarrow$ β is a fuzzy q-ideal of Y. ⌂

I. DISCUSSION

A specific set of algebraic properties of a specific type of RG-algebra, namely algebra, has been studied, and these properties and theorems were also related to fuzzy q-ideals of RG-algebra.

II. CONCLUSION

We have reached many results related to fuzzy q-ideals, the most important of which is studying the image and the inverse image of fuzzy q-ideals of RG-algebra.

REFERENCES

[1] A.H. Abed, A Structure of BZ -Algebras and its Properties, International Journal of Engineering and Information Systems (IJEAIS), vol. 4, Issue 8, (2020), pp:110-115.

[2] A.H. Abed, Isomorphism of BZ -algebras, International Journal of Engineering and Information Systems (IJEAIS), vol. 4, Issue 8, (2020), pp:116-120.

[3] A.H. Abed, On the Special Ideals in BZ -algebras, International Journal of Engineering and Information Systems (IJEAIS), vol. 4, Issue 8, (2020), pp:121-126.

[4] S. Asawasamrit and A. Sudprasert, A structure of BZ-algebras and its properties, Int. Journal of Math. Analysis, vol.6, no. 21 (2012), pp:1035-1044.

[5] S. Asawasamrit, BZ-isomorphism and Its Properties, International Journal of Pure and Applied Mathematics, vol. 78, no. 1 (2012), pp:65-73.

[6] S. Asawasamrit and A. Sudprasert, On The Special Ideals in BZ-algebras, International Journal of Pure and Applied Mathematics, vol. 82, no. 4 (2013), pp:605-613.

[7] S. Asawasamrit, and A. Sudprasert, On P-semisimple in BZ-algebras, International Journal of Pure and Applied Mathematics, vol. 98, no. 1 (2015), pp:23-32.

[8] W. A. Dudek, X. Zhang and Y. Wang, Ideals and Atom of BZ-algebras, J. East China Univ. Sci. Technology, vol.59 (4) (1993), pp:387–404. DOI: 10.2478/s12175-00-0135-2

[9] J. M. Xu and X. H. Zhang, On Anti-grouped Ideals in BZ-algebras, Pure Appl. Math. , vol.14 (1998), pp:101–102.

[10] A.S. abed and A.T. Hameed, (2023), **The Fuzzy Results on SA-algebra,** Journal of Interdisciplinary Mathematics, pp:1-8, DOI : 10.47974/JIM-1625.

[11] A.T. Hameed and E.K. Kadhim, **Interval-valued IFAT-ideals of AT-algebra**, Journal of Physics: Conference Series (IOP Publishing), 2020, pp:1-5.

[12] A.T. Hameed and N.H. Malik, (2021), **(β, α)-Fuzzy Magnified Translations of AT-algebra**, Journal of Physics: Conference Series (IOP Publishing), 2021, pp:1-13.

[13] A.T. Hameed and N.H. Malik, (2021), **(β, α)-Fuzzy Magnified Translations of AT-algebra,** Journal of Physics: Conference Series (IOP Publishing), 2021, pp:1-13.

[14] A.T. Hameed and N.H. Malik, (2021), **Magnified translation of intuitionistic fuzzy AT-q-ideals on AT-algebra**, Journal of Discrete Mathematical Sciences and Cryptography, (2021), pp:1-7.

[15] A.T. Hameed and N.J. Raheem, (2021**), Interval-valued Fuzzy SA-ideals with Degree (λ,κ) of SA-algebra**, Journal of Physics: Conference Series (IOP Publishing), 2021, pp:1-13.

[16] A.T. Hameed and N.M. Hussein, (2023), **A Structure of p-Algebra and its Properties,** AIP Conference Proceedings, Volume 2414, Issue 1(2023), Doi.org/1e.1e63/5.e115656 .

[17] A.T. Hameed and N.Z. Mohammed, **Fuzzy Translation and Fuzzy multiplication of CI-algebras**, Journal of Karbala University, vol.15, no.1(2017), pp:110-123.

[18] A.T. Hameed and S.M. Abrahem, **Intuitionistic Fuzzy RG-ideals of RG-A**, (2022), Mathematical Statistician and Engineering Applications, vol. 71, no. 3s3 (2022), pp: 169- 184.

[19] A.T. Hameed, A.A. Abboodi Al-Koofee and A.H. Abed, **α-translation Fuzzy q-ideals of KK-algebra**, International Journal of Engineering and Information Systems (IJEAIS), vol. 5, Issue 12, (2021), pp:68-81.

[20] A.T. Hameed, A.H. Abed and A.A. Aalfatlawi, (2019), **Magnified translation fuzzy ideals of KUS-algebras,** Journal of Advanced Research in Dynamical and Control Systems, vol.11, no.2 (2019), pp:169-181.

[21] A.T. Hameed, I.H. Ghazi and A.H. Abed, (2020), **Fuzzy α-translation AB-ideal of AB-algebras**, Journal of Physics: Conference Series (IOP Publishing), 2020, pp:1-19.

[22] A.T. Hameed, I.H. Ghazi and A.H. Abed, (2020), **Fuzzy α-translation AB-ideal of AB-algebras**, Journal of Physics: Conference Series (IOP Publishing), 2020, pp:1-19.

[23] A.T. Hameed, I.H. Ghazi and A.H. Abed, (2020), **Fuzzy α-translation AB-ideal of AB-algebras**, Journal of Physics: Conference Series (IOP Publishing), 2020, pp:1-19;

[24] A.T. Hameed, M.S. Mahadi and N.H. Malik, (2021), **Magnified translation of intuitionistic fuzzy AT-ideals on AT-algebra,** Journal of Discrete Mathematical Sciences & Cryptography, (2021), pp:1645-1650.

[25] A.T. Hameed, N.J. Raheem and A.H. Abed, (2021), **Anti-fuzzy SA-ideals with Degree (λ,κ) of SA-algebra**, Journal of Physics: Conference Series (IOP Publishing), 2021, pp:1-16.

[26] A.T. Hameed, S.H. Ali and , R.A. Flayyih, **The Bipolar-valued of Fuzzy Ideals on AT-algebra**, Journal of Physics: Conference Series (IOP Publishing), 2021, pp:1-9.

[27] AT. Hameed, S.M. Abrahem and A.H. Abed, **Fuzzy RG-ideals of RG-algebra,** Journal of Discrete Mathematical Sciences & Cryptography, (2023), pp:1-4, DOI : 10.47974/JDMSC-1552.
[28] AT. Hameed, S.M. Abrahem and A.H. Abed, **Intuitionistic fuzzy KUS-ideals of KUS-algebras,** Jour of Adv Research in Dynamical & Control Systems, Vol. 10, 11-Special Issue, 2018.
[29] H.K. Abdullah, A.T. Hameed and E.J. Abed, (2023), **Spectrum of a Prime Filter on p-Algebra,** AIP Conference Proceedings, Volume 2414, Issue 1(2023), Doi.org/10.1063/5.0114882.
[30] K. B. Lee, Y.B. Jun and M. I. Doh, **Fuzzy Translations and Fuzzy Multiplications of BCK/BCI-algebras**, Commum. Korean Math. Sco., vol.24 (2009), pp:353–360.
[31] N.H. Jaber and A.T. Hameed, (2023), **On the φ-algebra,** Journal of Interdisciplinary Mathematics, pp:1-8, DOI : 10.47974/JIM-1626.