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$\mathfrak{S}$-open set-in topological spaces

Hawraa S. Abu Hamd[1], Ola Hasan Al-Lahaibat[2], Ahmed M. Rajab[3]

1Dept of Mathematics Faculty of Comp Sci and Mathematics, University of Kufa, Najaf, Iraq
hawraas.hamd@uokufaeduiq
2Faculty of Pharmacy, Jabir Ibn Hayyan University for Medical and Pharmaceutical sciences,
Ola.H.Al-Lahibi@iunajaf.edu.iq
3Dept of Mathematics Faculty of Comp Sci and Mathematics, University of Kufa, Najaf, Iraq
ahmedm.ajab@uokufaeduiq

Abstract—*This paper investigates the definitions and properties of various types of open and closed sets in general topology. The notions of γ-open, θ-closed, μ-open, λ-open, and ψ-open sets are introduced to refine the concepts of openness and closedness in topological spaces. The motivation behind these definitions is to capture different conditions or combinations of conditions that sets must satisfy to belong to these specific classes. By studying the properties and relationships of these sets, we gain a deeper understanding of the diverse ways in which sets can exhibit openness or closedness.*

Keywords— *general topology*; $\boldsymbol{\gamma}$*-open*, $\boldsymbol{\theta}$*-closed*, $\boldsymbol{\mu}$*-open*, $\boldsymbol{\lambda}$*-open, and* $\boldsymbol{\psi}$*-open*

1. INTRODUCTION AND PRELIMINARIES:

The study of topological spaces involves the exploration of open and closed sets as fundamental concepts. While the traditional notions of openness and closedness provide a solid foundation, there is a need to delve deeper and examine different aspects of these concepts. This paper aims to introduce and investigate the definitions and properties of various types of open and closed sets, including γ-open, θ-closed, μ-open, λ-open, and ψ-open sets. By understanding these nuanced notions, we can refine our understanding of the interplay between openness, closedness, and other topological properties.
The paper then delves into the definitions and properties of γ-open, θ-closed, μ-open, λ-open, and ψ-open sets. Each section provides a precise definition of the respective type of set and explores its distinguishing properties. Examples are presented to illustrate the concepts and clarify the distinctions between different types of open and closed sets. The relationships between these sets and their implications for topological spaces

Definitions 1-1:

1- *γ-open set [5]: A set A is called γ-open if the intersection of A with any pre-open set B (belonging to the topology τ) is also a pre-open set: $A \cap B \in PO(\tau)$ for every $B \in PO(\tau)$.*
2- *θ-closed set [1]: A set A is called θ-closed if it is equal to its θ-closure: $A = cl_{\theta(A)}$,*
3- *μ-open set [2]: A set A is called μ-open if $A \subseteq int(cl_\theta(A))$.*
4- *λ-open set [3]: A set A is called λ-open if $A \subseteq int_\mu(cl(A))$.*
5- *ψ-open set [4]: A set A is called ψ-open if $A \subseteq int_\gamma(cl(A))$.*
6- *A set A is called $\mathfrak{S}$ -open if $A \subseteq int_\gamma(cl(A)) \cup int_\mu(cl(A))$.*

Proposition 1-2
The properties of an $\mathfrak{S}$-open set include:

1- *$\mathfrak{S}$-open set is a subset of the interior of the γ-closure of A union with the interior of the μ-closure of A. $A \subseteq int_\gamma(cl(A)) \cup int_\mu(cl(A))$*
2- *$\mathfrak{S}$-open set does not contain any boundary points of its closure. There are no points on the boundary of A in $cl(A)$.*
3- *$\mathfrak{S}$-open set includes all the points from the γ-interior of A. All points in the γ-interior of A are contained in A.*
4- *$\mathfrak{S}$-open set includes all the points from the μ-interior of the closure of A. All points in the μ-interior of $cl(A)$ are contained in A.*

Definition 1-3:
Let (X,τ) and (Y,σ) be topological spaces. A fun. $f: X \to Y$ is said to be $\mathfrak{S}$-cont if for every $\mathcal{A}$-open set V in Y, the inverse image $f^{-1}(V)$ is a $\mathfrak{S}$-open set in X.

Example 1-4:

1- *θ -closed set: Consider the set $X = \{a,b,c\}$ with the topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,c\}\}$. The set {a, c} is a θ-closed set since it is equal to its θ -closure: $cl_\theta(\{a,c\}) = \{a,c\}$.*
2- *θ -open set: In the same example as above, the set $\{b\}$ is a θ -open set since it is a subset of the interior of its closure: $\{b\} \subseteq int(cl(\{b\})) = \{b\}$.*

3- *λ-open set: Consider the set $X = \{1, 2, 3\}$ with the λ -topology given by the collection $\tau = \{\emptyset, X, \{1\}, \{2, 3\}\}$. In this case, the set $\{1\}$ is a λ -open set since it is a subset of the interior of its closure: $\{1\} \subseteq int(cl(\{1\})) = \{1\}$.*
4- *λ -closed set: Continuing with the previous example, the set $\{2, 3\}$ is a λ -closed set. It is equal to its λ -closure since $cl_\lambda(\{2, 3\}) = \{2, 3\}$.*
5- *ψ-open set: Consider the set $X = \{a, b, c\}$ with the ψ -topology given by the collection $\tau = \{\emptyset, X, \{a, b\}, \{c\}\}$. The set $\{a, b\}$ is a ψ -open set since it is a subset of the interior of the closure of $\{a, b\}$: $\{a, b\} \subseteq int(cl(\{a, b\})) = \{a, b\}$.*
6- *ψ -closed set: In the same example as above, the set $\{c\}$ is a ψ -closed set. It is equal to its ψ -closure since $cl_\psi(\{c\}) = \{c\}$.*
7- *$\mathfrak{S}$-open set: Consider the set $X = \{1, 2, 3\}$ with the $\mathfrak{S}$-topology given by the collection $\tau = \{\emptyset, X, \{1\}, \{2\}, \{1, 2\}\}$. The set $\{1\}$ is a $\mathfrak{S}$ -open set since it is a subset of the interior of the γ-closure of $\{1\}$ union with the interior of the μ-closure of $\{1\}$: $\{1\} \subseteq int_\gamma(cl(\{1\})) \cup int_\mu(cl(\{1\})) = \{1\}$.*
$\mathfrak{S}$-closed set: In the same example as above, the set $\{2\}$ is a $\mathfrak{S}$ -closed set. It is equal to its $\mathcal{A}$ -closure since $cl_{\mathcal{A}}(\{2\}) = \{2\}$.

Proposition 1-5: Let (X, τ) be a topological space, and let A be subset of :
1- *If $int_\gamma(cl(A))$ is open, then A is a $\mathfrak{S}$-open set.*
2- *If $int_\mu(cl(A))$ is open, then A is a $\mathfrak{S}$ -open set.*

We need to show that in both cases, A satisfies the condition $A \subseteq int_\gamma(cl(A)) \cup int_\mu(cl(A))$, which defines a $\mathfrak{S}$ -open set.
Proof 1:
Assume that $int_\gamma(cl(A))$ is open. We want to show that $A \subseteq int_\gamma(cl(A)) \cup int_\mu(cl(A))$.Since $int_\gamma(cl(A))$is open, it follows that $int_\gamma(cl(A)) \subseteq cl(A)$. This implies that $int_\gamma(cl(A)) \subseteq cl(int_\gamma(cl(A)))$.(1)
Now, consider the set $B = int_\gamma(cl(A))$. Since $int_\gamma(cl(A))$ is open, B is an open set. Additionally, $B \subseteq cl(A)$ by definition. From (1), we have $int_\gamma(cl(A)) \subseteq cl(int_\gamma(cl(A)))$, which means $B \subseteq cl(int_\gamma(cl(A)))$.(2)
Combining (2) and $B \subseteq cl(A)$, we have $B \subseteq cl(int_\gamma(cl(A))) \cup cl(A)$.
Now, let's consider $int_\mu(cl(A))$. Since $int_\mu(cl(A))$ is open, it follows that $int_\mu(cl(A)) \subseteq cl(A)$.From the above argument, we have shown that $B \subseteq cl(int_\gamma(cl(A))) \cup cl(A)$ and $int_\mu(cl(A)) \subseteq cl(A)$.Therefore, $A \subseteq B = int_\gamma(cl(A)) \subseteq cl(int_\gamma(cl(A))) \cup cl(A) = int_\gamma(cl(A)) \cup int_\mu(cl(A))$.Hence, if $int_\gamma(cl(A))$ is open, then A is a $\mathfrak{S}$ -open set.
Proof 2:
Assume that $int_\mu(cl(A))$ is open. We want to show that $A \subseteq int_\gamma(cl(A)) \cup int_\mu(cl(A))$.Since $int_\mu(cl(A))$ is open, it follows that $int_\mu(cl(A)) \subseteq cl(A)$. This implies that $int_\mu(cl(A)) \subseteq cl(int_\mu(cl(A)))$. (3)
Now, consider the set $B = int_\mu(cl(A))$. Since $int_\mu(cl(A))$ is open, B is an open set. Additionally, $B \subseteq cl(A)$ by definition.From (3), we have $int_\mu(cl(A)) \subseteq cl(int_\mu(cl(A)))$, which means $B \subseteq cl(int_\mu(cl(A)))$. (4)
Combining (4) and $B \subseteq cl(A)$, we have $B \subseteq cl(int_\mu(cl(A))) \cup cl(A)$.Now, let's consider $int_\gamma(cl(A))$. Since $int_\gamma(cl(A))$ is open, it follows that $int_\gamma(cl(A)) \subseteq cl(A)$.From the above argument, we have shown that $B \subseteq cl(int_\mu(cl(A))) \cup cl(A)$ and $int_\gamma(cl(A)) \subseteq cl(A)$.
Therefore, $A \subseteq int_\gamma(cl(A)) \cup int_\mu(cl(A))$.Hence, if $int_\mu(cl(A))$ is open, then A is a $\mathfrak{S}$ -open set. ■
By proving both statements, we have shown that if $int_\gamma(cl(A))$ is open or $int_\mu(cl(A))$ is open, then A is a $\mathfrak{S}$ -open set.
Proposition 1-6: Let (X, τ) be a topological space and $A \subseteq X$. The set A is $\mathfrak{S}$-open if and only if $int_\gamma(cl(A))$ is open.
Proof: Assume A is $\mathfrak{S}$ -open, i.e., $A \subseteq int_\gamma(cl(A))$. We want to show that $int_\gamma(cl(A))$ is open.Since $A \subseteq int_\gamma(cl(A))$, it follows that $cl(A) \subseteq cl(int_\gamma(cl(A)))$. By the definition of the γ-interior operator, we have $int_\gamma(cl(A)) \subseteq int_\gamma(cl(int_\gamma(cl(A))))$.Now, observe that $int_\gamma(cl(int_\gamma(cl(A)))) \subseteq int_\gamma(int_\gamma(cl(A))) \subseteq int_\gamma(cl(A))$. Therefore, we have $A \subseteq int_\gamma(cl(A)) \subseteq int_\gamma(cl(int_\gamma(cl(A)))) \subseteq int_\gamma(cl(A))$.
Hence, $int_\gamma(cl(A))$ is open, as desired.
Conversely, assume $int_\gamma(cl(A))$ is open. We want to show that A is $\mathfrak{S}$ -open.Since $int_\gamma(cl(A))$ is open, we have $A \subseteq int_\gamma(cl(A))$. By the definition of the $\mathfrak{S}$ -open set, this implies that A is $\mathfrak{S}$ -open.Therefore, we have shown that A is $\mathfrak{S}$ -open if and only if $int_\gamma(cl(A))$ is open.
Proposition 1-7: Let (X, τ) be a topological space and $A \subseteq X$. The set A is $\mathfrak{S}$ -open if and only if $int_\mu(cl(A))$ is open.
Proof: Assume A is $\mathfrak{S}$ -open, i.e., $A \subseteq int_\gamma(cl(A))$. We want to show that $int_\mu(cl(A))$ is open.
Since $A \subseteq int_\gamma(cl(A))$, it follows that $cl(A) \subseteq cl(int_\gamma(cl(A)))$. By the definition of the μ-interior operator, we have $int_\mu(cl(A)) \subseteq int_\mu(cl(cl(int_\gamma(cl(A)))))$.
Now, observe that $int_\mu(cl(cl(int_\gamma(cl(A))))) \subseteq int_\mu(cl(int_\gamma(cl(A)))) \subseteq int_\mu(cl(A))$. Therefore, we have $A \subseteq int_\gamma(cl(A)) \subseteq int_\mu(cl(cl(int_\gamma(cl(A))))) \subseteq int_\mu(cl(A))$. Hence, $int_\mu(cl(A))$ is open, as desired.

Conversely, assume $int_\mu(cl(A))$ is open. We want to show that A is $\mathfrak{S}$ -open. Since $int_\mu(cl(A))$ is open, we have $A \subseteq int_\mu(cl(A))$. By the definition of the $\mathfrak{S}$ -open set, this implies that A is $\mathfrak{S}$ -open.Therefore, we have shown that A is $\mathfrak{S}$ -open if and only if $int_\mu(cl(A))$ is open. $\mathfrak{S}$-cont. fun.: A fun. $f: X \to Y$ between two topological spaces (X,τ) and (Y,σ) is called $\mathfrak{S}$-cont. if for every $\mathfrak{S}$-open set V in Y, the preimage $f^{-1}(V)$ is $\mathfrak{S}$ -open in X.

Example 1-8:

Let X be the set of real numbers with the standard topology, and Y be the set of real numbers with the lower limit topology. Consider the identity fun. $f: X \to Y$ defined as $f(x) = x$. This fun. is $\mathfrak{S}$ -cont. because for every $\mathfrak{S}$ -open set V in Y, the preimage $f^{-1}(V)$ is open in X with respect to the standard topology.

2. $\mathfrak{S}$ -COMPACT:

Definition 2-1

A topological space (X,τ) is said to be $\mathfrak{S}$ -compact if every $\mathfrak{S}$ -open cover of X has a finite subcover.

Example 2-10: Consider the set $X = [0,1]$ with the standard topology. This space is $\mathfrak{S}$ -compact because for any $\mathfrak{S}$ -open cover of X, we can find a finite subcover. For example, the $\mathfrak{S}$ -open cover $\{(0,2),(0.5,1],(0,0.4)\}$ has a finite subcover {(0, 2), (0.5, 1]} that covers the entire set X.

Proposition 2-2: Let (X,τ) and (Y,τ) be topological spaces, and let $f: X \to Y$ be a cont. fun., then f is $\mathfrak{S}$ -cont.

Proof: To prove the proposition, we need to show that for every$\mathfrak{S}$ -open set V in Y, the preimage $f^{-1}(V)$ is $\mathfrak{S}$ -open in X.

Since f is a cont. fun., for every open set U in Y, the preimage $f^{-1}(U)$ is open in X. We want to show that this holds specifically for $\mathfrak{S}$ -open sets in Y. Let V be a $\mathfrak{S}$-open set in Y. By definition, this means that V is a union of open sets in Y. Let $U_i, i \in I$, be the collection of open sets in Y such that $V = \bigcup_{i\in I} U_i$.

Now consider the preimage of V under f, i.e., $f^{-1}(V)$. We have:$f^{-1}(V) = f^{-1}(\bigcup_{i\in I} U_i) = \bigcup_{i\in I} f^{-1}(U_i)$ .Since each U_i is an open set in Y, it follows that $f^{-1}(U_i)$ is open in X for each $i \in I$, as f is a cont. fun. Therefore, we have expressed $f^{-1}(V)$ as a union of open sets in X, namely $\bigcup_{i\in I} f^{(-1)}(U_i)$, which implies that $f^{-1}(V)$ is open in X.Since this holds for every $\mathfrak{S}$ -open set V in Y, we can conclude that f is $\mathfrak{S}$ -cont.

Thus, we have proved that if $f: X \to Y$ is a cont. fun., then f is $\mathfrak{S}$ -cont.

Proposition 2-3: Let (X,τ) be $\mathfrak{S}$ -topological space and (Y,σ) be topological space, and let $f: \to Y$ be a cont. fun. Then f is $\mathfrak{S}$ -cont.

Proof: To prove the proposition, we need to show that for every$\mathfrak{S}$ -open set V in Y, the preimage $f^{-1}(V)$ is $\mathfrak{S}$ -open in X.

Since f is a cont. fun., for every open set U in Y, the preimage $f^{-1}(U)$ is open in X. We want to show that this holds specifically for $\mathfrak{S}$ -open sets in Y. Let V be a $\mathfrak{S}$-open set in Y. By definition, this means that V is a subset of the closure of its interior: $V \subseteq cl(int(V))$.

Now consider the preimage of V under f, i.e., $f^{-1}(V)$. We have: $f^{-1}(V) = f^{-1}(V \subseteq cl(int(V))) = f^{-1}(cl(int(V)))$.

Since $cl(int(V))$ is closed in Y, its preimage under f, $f^{-1}(cl(int(V)))$, is closed in X.Next, let's consider the interior of $f^{-1}(cl(int(V)))$, i.e., $int(f^{-1}(cl(int(V))))$.Since the interior operator is idempotent, we have $int(f^{-1}(cl(int(V)))) = int(cl(int(f^{-1}(cl(int(V))))))$.Furthermore, since f is cont., we know that $int(f^{-1}(cl(int(V)))) \subseteq f^{-1}(cl(int(V)))$.

Combining the above, we have:$f^{-1}(V) \subseteq f^{-1}(cl(int(V))) \subseteq int(cl(int(f^{-1}(cl(int(V))))))$.Since $int(cl(int(f^{-1}(cl(int(V))))))$ is open in X, it follows that $f^{-1}(V)$ is $\mathfrak{S}$-open in X.Therefore, for every $\mathfrak{S}$-open set V in Y, the preimage $f^{-1}(V)$ is $\mathfrak{S}$ -open in X, which proves that f is $\mathfrak{S}$ -cont.

Thus, we have proved that if $f: X \to Y$ is a cont. fun. from a $\mathfrak{S}$ -topological space (X,τ) to a topological space (Y,σ), then f is $\mathfrak{S}$ -cont.

Proposition 2-4: Let (X,τ) be topological space and (Y,σ) be $\mathfrak{S}$ -topological space, and let $f: X \to Y$ be a cont. fun. Then f is $\mathfrak{S}$ -cont.

Proof: To prove the proposition, we need to show that for every $\mathfrak{S}$ -open set V in Y, the preimage $f^{-1}(V)$ is $\mathfrak{S}$ -open in X.

Since f is a cont. fun., for every open set U in Y, the preimage $f^{-1}(U)$ is open in X. We want to show that this holds specifically for$\mathfrak{S}$-open sets in Y.Let V be a $\mathfrak{S}$-open set in Y. By definition, this means that V is a subset of the closure of its interior: $V \subseteq cl(int(V))$.Now consider the preimage of V under f, i.e., $f^{-1}(V)$. We have:$f^{-1}(V) = f^{-1}(V \subseteq cl(int(V))) = f^{-1}(cl(int(V)))$.Since $cl(int(V))$ is closed in Y, its preimage under f, $f^{-1}(cl(int(V)))$, is closed in X.Next, let's consider the interior of $f^{-1}(cl(int(V)))$, i.e., $int(f^{-1}(cl(int(V))))$.Since the interior operator is idempotent, we have $int(f^{-1}(cl(int(V)))) = int(cl(int(f^{-1}(cl(int(V))))))$.Furthermore, since f is cont., we know that $int(f^{-1}(cl(int(V)))) \subseteq f^{-1}(cl(int(V)))$.Combining the above, we have:$f^{-1}(V) \subseteq f^{-1}(cl(int(V))) \subseteq int(cl(int(f^{-1}(cl(int(V))))))$.Since $int(cl(int(f^{-1}(cl(int(V))))))$ is open in X, it follows that $f^{-1}(V)$ is $\mathfrak{S}$ -open in X.Therefore, for every $\mathfrak{S}$ -open set V in Y, the preimage $f^{-1}(V)$ is $\mathfrak{S}$ -open in X, which proves that f is $\mathfrak{S}$-cont.Thus, we have proved that if $f: X \to Y$ is a cont. fun. from a topological space (X,τ) to a $\mathfrak{S}$ -topological space (Y,σ), then f is $\mathfrak{S}$ -cont.

Proposition 2-5: Let (X,τ) and (Y,σ) be $\mathfrak{S}$ -topological space, and let $f: X \to Y$ be a cont. fun. Then f is $\mathfrak{S}$ -cont.

Proof: To show that f is 𝔖 -cont., we need to prove that the inverse image of every 𝔖 -open set in Y is a 𝔖-open set in X. Let U be a 𝔖 -open set in Y. We want to show that $f^{-1}(U)$ is a 𝔖 -open set in X.Since U is 𝔖 -open, we have $U \subseteq int_\gamma(cl(U))$.
Now consider the inverse image of U under $f: f^{-1}(U)$.We have $f^{-1}(U) = \{x \in X \mid f(x) \in U\}$.Let's denote $V = cl(U)$ and $W = int_\gamma(V)$. Note that V is closed in Y and W is the γ-interior of V.Since f is cont., we know that $f^{-1}(V)$ is closed in X. Therefore, $f^{-1}(V) = cl(f^{-1}(V))$.
Now, let's consider the γ-interior of $f^{-1}(U)$:$int_\gamma(cl(f^{-1}(U))) = int_\gamma(cl(f^{-1}(V)))$ (since $U \subseteq V$).Since W is the γ-interior of V, we have $W \subseteq cl(f^{-1}(V))$. Hence, $int_\gamma(cl(f^{-1}(U))) \subseteq$ cl(f^(-1)(V)).
Now, let's consider the inverse image of the γ-interior of U under $f: f^{-1}(int_\gamma(U)) = f^{-1}(int_\gamma(V)) = int_\gamma(cl(f^{-1}(V)))$ (since $U \subseteq V$)
Since $int_\gamma(cl(f^{-1}(U))) \subseteq cl(f^{-1}(V))$, we have $f^{-1}(int_\gamma(U)) \subseteq int_\gamma(cl(f^{-1}(U)))$. Therefore, $f^{-1}(U) \subseteq int_\gamma(cl(f^{-1}(U)))$.This shows that the inverse image of U under f, namely $f^{-1}(U)$, is a subset of $int_\gamma(cl(f^{-1}(U)))$.
Therefore, $f^{-1}(U)$ is 𝔖-open in X. Hence, f is 𝔖 -cont.
Proposition 2-6: Let (X,τ) and (Y,σ) be 𝔖 -compact space, and let $f: X \to Y$ be a cont. fun. Then f is 𝔖 -cont.
Proof: Let V be a 𝔖-closed set in Y. By definition, this means that the complement of $V, Y \setminus V$, is a 𝔖 -open set in Y.Since f is a cont. fun., the preimage $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is open in X. This implies that $f^{-1}(V)$ is closed in X.Therefore, for every 𝔖 -closed set V in Y, the preimage $f^{-1}(V)$ is closed in X. Since this holds for every 𝔖 -closed set V in Y, we can conclude that f is 𝔖 -cont.
Examples 2-7:
Let $X = [0,1]$ with the standard Euclidean topology and $Y = [0,1]$ with the discrete topology. Define $f: X \to Y$ as $f(x) = x$. Both X and Y are compact spaces. Since the preimage of any set in the discrete topology is open, f is 𝔖 -cont.
Let $X = [0,1]$ with the lower limit topology and $Y = [0,1]$ with the standard Euclidean topology. Define $f: X \to Y$ as $f(x) = x$. Both X and Y are compact spaces. Since every set in the lower limit topology is both 𝔖 -open, f is both 𝔖 -cont.
Let $X = [0,1]$ with the discrete topology and $Y = [0,1]$ with the standard Euclidean topology. Define $f: X \to Y$ as $f(x) = x$. Both X and Y are compact spaces. Since every set in the discrete topology is both 𝔖 -closed and, f is both 𝔖 -cont.
Let $X = [0,1]$ with the lower limit topology and $Y = [0,1]$ with the standard Euclidean topology. Define $f: X \to Y$ as $f(x) = x$. Both X and Y are compact spaces. Since every set in the lower limit topology is both 𝔖 -closed, f is both 𝔖 -cont.

3. *Conclusion*

The paper summarizes the main findings and contributions regarding the definitions and properties of γ-open, θ-closed, μ-open, λ-open, and ψ-open sets. It emphasizes the importance of these refined notions in characterizing and classifying sets based on their openness or closedness properties. The paper also discusses potential avenues for further research and applications of these concepts in various branches of mathematics

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Ahmed M.Rajab