International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 7 Issue 2, February - 2023, Pages: 7-12

On Homomorphism Fuzzy q-ideals of KK-algebra

Dr. Areej Tawfeeq Hameed[1] , Atwar Ahmed Abboodi Al-Koofee2, Dr. Ahmed Hamzah Abed[3]

1Department of Mathematics, Faculty of Education for Girls, University of Kufa, Najaf, Iraq.
E-mail: areej.tawfeeq@uokufa.edu.iq
2Department of Mathematics, Faculty of Education for Girls, University of Kufa, Najaf, Iraq.
E-mail: areej238@gmail.com
College of Islamic Sciences, AL Iraqia University, Baghdad, Iraq.
E-mail: ahmedh.abed@uokufa.edu.iq

Abstract: *The aim of this paper is introducing the notion of fuzzy q-ideal of KK-algebra, several theorems, properties are stated and proved. The fuzzy relations on KK-algebras are also studied.*

Keywords—KK-algebra, fuzzy ideal, fuzzy q-ideal, homomorphisms and isomorphisms of q-ideal, image and pre-image of fuzzy q-ideals.

1. Introduction

A BCK-algebra is an important class of logical algebras introduced by K. Is´eki [15] and was extensively investigated by several researchers. The class of all BCK-algebras is a quasivariety. K. Is´eki posed an interesting problem whether the class of BCK-algebras is a variety. In ([18,19]), C. Prabpayak and U. Leerawat the notion of KU-algebras. They gave the concept of homomorphism of KU-algebras and investigated some related properties. The concept of fuzzy subset and various operations on it were first introduced by L.A. Zadeh in [22], then fuzzy subsets have been applied to diverse field. The study of fuzzy subsets and their application to mathematical contexts has reached to what is now commonly called fuzzy mathematics. Fuzzy algebra is an important branch of fuzzy mathematics. They introduced of the concept of fuzzy subgroups in 1971 by A. Rosenfeld [20]. Since these ideas have been applied to other algebraic structures such as semi-groups, rings, ideals, modules and vector spaces. O.G. Xi, ([21]) applied this concept to BCK-algebra, and he introduced the notion of fuzzy sub-algebras (ideals) of the BCK-algebras with respect to minimum, and since then Y. S.M. Mostafa and et al [16] have introduced the notion of KUS-algebras, KUS-ideals and investigates the relations among them. S. Asawasamrit and A. Sudprasert [1-3] have introduced the notion of KK-algebras, ideals, KK-subalgebras and studied the relations among them and gave the concept of homomorphism of KK-algebras and investigated some related properties. In this paper, the study of fuzzy algebraic structures was started with the introduction of the concept of fuzzy q-ideals of KK-algebras and then we investigate several basic properties which are related to fuzzy ideals. We describe how to deal with the homomorphism of image and inverse image of fuzzy ideals.

2. Preliminaries

Now, we give some definitions and preliminary results needed in the later sections.

Definition 2.1[1,2]. Let $(X;\ *,\ \beth)$ be an algebra with a binary operation $*$ and a nullary operation $\beth$. Then X is called **KK-algebra** if it satisfies the following: for all $x, y, z \in X$,

(KK_1) : $(x * y) * ((y * z) * (x * z)) = \beth$,

(KK_2) : $\beth * x = x$,

(KK_3) : $x * y = \beth\ and\ y * x = \beth$ if and only if $x = y$.

Definition 2.2[1,2]. Define a binary relation $\leq$ on KK-algebra X by letting $x \leq y$ if and only if $y * x = \beth$.

Examples 2.3[4,19]. Let A be an arbitrary nonempty set and X be the set of all real valued function defined on A for $, g \in X$, we define $f * g$ by $(f * g)(x) =$

$$\begin{cases} f(x) - g(x) & f(x) < g(x) \\ \beth & f(x) \geq g(x) \end{cases}.$$

Then (X; *, $\beth$) is a KK-algebra.

Examples 2.4[3]. Let X = { $\beth$, 1} and let $*$ be defined by:

$*$	$\beth$	1
$\beth$	$\beth$	1
1	1	$\beth$

Then $(X; *,\ \beth)$ is a KK-algebra.

Proposition 2.5 [1,3]. In any KK-algebra $(X;\ *,\ \beth)$, the following properties hold: for all x, y, z $\in$X;

(P_1) $x * ((x * y) * y) = \beth$;

(P_2) $x * x = \beth$;

(P_3) $* (y * z) = y * (x * z)$.

Definition 2.6[2]. A nonempty subset I of a KK-algebra $(X;\ *,\ \beth)$ is called **an ideal of X** if it satisfies the following conditions: for any $x, y \in X$,

(I_1) $\beth \in I$;

(I_2) $x * y \in I\ and\ x \in I\ imply\ y \in I$.

Definition 2.7[1]. A nonempty subset I of a KK-algebra $(X;\ *,\ \beth)$ is called

International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 7 Issue 2, February - 2023, Pages: 7-12

a q-ideal of X if it satisfies the following conditions: for any $x, y, z \in X$,

(I_1) ב $\in I$;

(I_2) $(x * y) * z \in I$ *and* $y \in I$ *imply* $x * z \in I$.

Examples 2.8[1]. Let X = { ב, a, b, c} and let ∗ be defined by the table:

∗	ב	a	b	c
ב	ב	a	b	c
a	ב	ב	c	c
b	c	c	ב	ב
c	c	b	a	ב

Thus $(X; *,$ ב$)$ is a KK-algebra. And we see that I = { ב, a} and J = { ב, c} are q-ideals of X.

Proposition 2.9[1]. Let $\{I_i \mid i \in \Lambda\}$ be a family of q-ideals of KK-algebra $(X; *,$ ב$)$. The intersection of any set of q-ideals of KK-algebra X is also q-ideal.

Theorem 2.10[1]. A q-ideal of KK-algebra $(X; *,$ ב$)$is an ideal.

Proof. Suppose that I is an q-ideal of X and let $x * y \in I$ *and* $x \in I$. It follows that (ב $* x) * y \in I$ imply ב $* y \in I$. Thus, $y \in I$. ■

Remark 2.11[1]. The converse of Proposition (2.10) needs not be true in general as in the Example.

∗	ב	a	b	c
ב	ב	a	b	c
a	c	ב	a	b
b	b	c	ב	a
c	a	b	c	ב

Example 2.12[1]. Let X ={ ב, a, b, c} and let ∗ be a binary operation defined by the table:

Then $(X; *,$ ב$)$ is a KK-algebra with I = { ב} is an ideal of X, but not a q-ideal of X. Since $(c *$ ב$) * a = a * a =$ ב $\in$ { ב} and ב $\in$ { ב} , but $c * a = b \notin$ { ב}.

Theorem 2.13[1]. If I is an ideal of KK-algebras X, then the following are equivalent:

(1) I is an q-ideal of X;

(2) for any x, $y \in X$, $(x *$ ב$) * y \in I$ implies $x * y \in I$;

(3) for any $x, y, z \in X$, $(x * y) * z \in I$ implies $x * (y * z) \in I$.

Theorem 2.14[1]. Let K and I be ideals of a KK-algebra X with $I \subseteq K$. If I is a q-ideal of X, then so is K.

Corollary 2.15[1]. If zero ideal { ב} of KK-algebra X is a q-ideal, then every ideal of X is a q-ideal.

Theorem 2.16[1]. Let I be an ideal of KK-algebra X. If for any $x \in I$ and $y \in X$ imply $x * y \in I$, then I is q-ideal of X.

Lemma 2.17[1]. If I is a q-ideal of KK-algebra X, then $x * (x *$ ב$) \in I$, for all $x \in X$.

Definition 2.18[2]. Let $(X; *,$ ב$)$ *and* $(Y; *`,$ ב`$)$ be nonempty sets. The mapping $f : (X; *,$ ב$) \rightarrow (Y; *`,$ ב`$)$ is called **a homomorphism** if it satisfies:

$f(x * y) = f(x) *` f(y)$, for all $x, y \in X$. The set $\{x \in X \mid f(x) =$ ב$'\}$is called **the kernel of** f denoted by ker f.

Theorem 2.19[2]. Let $f : (X; *,$ ב$) \rightarrow (Y; *`,$ ב`$)$ be a homomorphism of a KK-algebra X into a KK-algebra Y, then :

A. $f($ ב$) =$ ב$'$.

B. f is injective if and only if $ker f$ = { ב}.

C. $x \leq y$ implies $f(x) \leq f(y)$.

Theorem 2.20[2]. Let $f : (X; *,$ ב$) \rightarrow (Y; *`,$ ב`$)$ be a homomorphism of a KK-algebra X into a KK-algebra Y, then :

(1) If I is ideal of X, then $f(I)$ is ideal of .

(2) If J is ideal of Y, then $f^{-1}(J)$ is ideal of .

(3) $ker f$ is ideal of X.

Definition 2.21[22]. Let (X; ∗, ב) X be a nonempty set, a fuzzy subset μ of X is a function $\mu: X \rightarrow [$ ב$, 1]$.

Definition 2.22[4]. Let X be a nonempty set and μ be a fuzzy subset of X, for t ∈[ב,1],the set μ_t ={x ∈ $X \mid \mu$(x) ≥ t}is called a level subset of μ.

Definition 2.23[4]. Let f : (X;∗, ב) →(Y;∗', ב') be a mapping nonempty sets X and Y respectively. If μ is a fuzzy subset of X, then the fuzzy subset β of Y defined by:

$$f(\mu)(y) = \begin{cases} \sup\{\mu(x) : x \in f^{-1}(y)\} & \text{if } f^{-1}(y) = \{x \in X, f(x) = y\} \neq \phi \\ 0 \quad otherwise & \end{cases}$$

is said to be the image of μ under f.

Similarly if β is a fuzzy subset of Y , then the fuzzy subset $\mu = (\beta \circ f)$ *in X* (i.e. the fuzzy subset defined *by* $\mu(x) = \beta(f(x))$ *for all* $x \in X$) is called the pre-image of β under f .

3. Fuzzy q-ideals and Homomorphism of KK-algebras

In this section, we will discuss a new notion called fuzzy q-ideals of KK-algebras and study several basic properties which are related to fuzzy q-ideals.

Definition 3.1[11]. Let $(X; *,$ ב$)$ be a KK-algebra , a fuzzy subset μ of X is called **a fuzzy ideal of X** if it satisfies the following conditions: , for all x , $y \in X$,

(1) $\mu($ ב$) \geq \mu(x)$,

(2) $\mu(y) \geq min\{\mu(x * y), \mu(x)\}$.

Definition 3.2. Let $(X; *,$ ב$)$ be a KK-algebra , a fuzzy subset μ of X is called **a fuzzy q-ideal of *X*** if it satisfies the following conditions: , for all $x, y, z \in X$,

(1) $\mu($ ב$) \geq \mu(x)$,

(2) $\mu(x * z) \geq min\{\mu((x * y) * z), \mu(y)\}$.

Example 3.3. Consider X = { ב, 1, 2, 3, 4} with $*$ defined by the table

$*$	ב	1	2	3	4
ב	ב	1	2	3	4
1	ב	ב	2	3	4
2	ב	ב	ב	3	4
3	ב	ב	ב	ב	4
4	ב	ב	ב	ב	ב

Then $(X ; * ,$ ב$)$ is a KK-algebra. Define a fuzzy subset μ: $X \rightarrow [$ ב, 1] such that

$\mu($ב$) = 2/3, \mu(1) = \mu(2) = 1/2$ and $\mu(3) = \mu(4) = 1/3$.

Routine calculation gives that μ is a fuzzy q-ideal of KK-algebra .

Example 3.4.

Let X ={ ב, 1, 2, 3} in which $(*)$ is defined by the following table:

$*$	ב	1	2	3
ב	ב	1	2	3
1	ב	ב	3	3
2	3	3	ב	ב
3	3	2	1	ב

Then $(X ; * ,$ ב$)$ is KK-algebra. Define a fuzzy subset $\mu : X \rightarrow [$ ב$, 1]$ by

$\mu(x) = \begin{cases} 0.7 if x \in \{0,1\} \\ 0.3 otherwise \end{cases}$. I_1 = { ב, 1} is ideal of X.
Routine calculation gives that μ is a fuzzy q-ideal of KK-algebras X.

Preposition 3.5. Let A be a nonempty subset of a KK-algebra $(X ; * ,$ ב$)$ and μ be a fuzzy subset of X such that μ is into { ב, 1}, so that μ is the characteristic function of A . Then μ is a fuzzy q-ideal of X if and only if, A is a q-ideal of .

Proof. Assume that μ is a fuzzy q-ideal in X, since $\mu($ ב$) \geq \mu(x)$, for all $x \in X$, clearly we have $\mu($ב$) = 1$, and so ב $\in A$.

Let $(x * y) * z \in$A and $\in A$, since μ is a fuzzy q-ideal of X, it follows that $\mu(x * z) \geq$ min{ $\mu((x * y) * z), \mu(y)\}$, for all x, y, z ∈X, and μ(x* z)=1. This mean that $* z \in A$. i.e. A is a q-ideal of X .

Conversely suppose A is a q-ideal of X, since ב $\in A$, μ(ב)=1≥ μ(x) , for all x∈ X. Let $x, y, z \in X$, if $y \notin$ A, then $\mu(y)$ = ב , and so $\mu(x * z) \geq$ ב=min{ $\mu((x * y) * z), \mu(y)\}$. If $((x * y) * z) \notin A$, and $y \in A$, then $x * z \notin A$ (since A is a q-ideal). Thus $\mu(x * z)$ = ב = min{ $\mu((x * y) * z), \mu(y)\}$. Hence μ is a fuzzy q-ideal of X. ■

Proposition 3.6. Let μ be a fuzzy subset of KK-algebra $(X; *,$ ב$)$. μ is a fuzzy ideal of X if and only if , for every t ∈[ב,1] , μ_t is an ideal of X .

Proof. Assume that μ is a fuzzy ideal of X , by Definition (3.1(1)) , we have $\mu($ ב$) \geq \mu(x)$ for all $x \in X$ therefore $\mu($ ב$) \geq \mu(x) \geq t$ for $x \in \mu_t$ and so ב $\in \mu_t$.

Let $x, y \in X$ be such that $(x * y) \in \mu_t$ and $x \in \mu_t$, then

$\mu(x * y) \geq t$ and $\mu(x) \geq t$, since μ is a fuzzy ideal, it follows that

International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 7 Issue 2, February - 2023, Pages: 7-12

$\mu(y) \geq min\{\mu(x * y), \mu(x)\} \geq t$ and we have that $y \in \mu_t$. Hence μ_t is an ideal of X .

Conversely, we only need to show that (1) and (2) in Definition (3.1) are true . If (1) is false , then there exist $x` \in X$ such that $\mu(ב) < \mu(x`)$. If we take

$t` = (\mu(x`) + \mu(ב))/2$, then $(ב) < t`$ *and* $ב \leq t` < \mu(x`) \leq 1$ , then $x` \in \mu$ and $\mu \neq \emptyset$. As $\mu_{t`}$ is a q-ideal of X, we have $ב \in \mu_{t`}$ and so $\mu(ב) \geq t`$. This is a contradiction.

Now, assume (2) is not true, then there exist $x`, y` \in X$ such that,

$\mu(y`) < min\{\mu(x` * y`), \mu(x`)\}$.

Putting $t` = (\mu(y`) + min\{\mu(x` * y`), \mu(x`)\}/2$, then

$\mu(y`) < t`$ and $ב \leq t` < min\{\mu(x` * y`), \mu(x`)\} \leq 1$, hence

$\mu(x` * y`) > t`$ *and* $\mu(x`) > t`$, which imply that

$(x` * y`) \in \mu_{t`}$, *and* $x` \in \mu_{t`}$. Since $\mu_{t`}$ is an ideal, it follows that

$(y`) \in \mu_{t`}$ and that $\mu(y`) \geq t`$, this is also a contradiction.

Hence μ is a fuzzy ideal of . ■

Theorem 3.7. Let μ be a fuzzy subset of KK-algebra $(X; *, ב)$. μ is a fuzzy q-ideal of X if and only if, for every t $\in[ב,1]$, μ_t is a q-ideal of X .

Proof. Assume that μ is a fuzzy q-ideal of X, by Definition (3.2(1)) , we have $\mu(ב) \geq \mu(x)$ for all $x \in X$ therefore $\mu(ב) \geq \mu(x) \geq t$ for $x \in \mu_t$ and so $ב \in \mu_t$.

Let $x, y, z \in X$ be such that $(x * y) * z \in \mu_t$ and $y \in \mu_t$, then

$\mu((x * y) * z) \geq t$ *and* $\mu(y) \geq t$, since μ is a fuzzy q-ideal, it follows that

$\mu(x * z) \geq min\{\mu((x * y) * z), \mu(y)\} \geq t$ and we have that $x * z \in \mu_t$. Hence μ_t is a q-ideal of X .

Conversely, we only need to show that (1) and (2) in Definition (3.2) are true . If (1) is false , then there exist $x` \in X$ such that $\mu(ב) < \mu(x`)$. If we take

$t` = (\mu(x`) + \mu(ב))/2$, then $(ב) < t`$ *and* $ב \leq t` < \mu(x`) \leq 1$ , then $x` \in \mu$ and $\mu \neq \emptyset$. As $\mu_{t`}$ is a q-ideal of X , we have $ב \in \mu_{t`}$ and so $\mu(ב) \geq t`$. This is a contradiction.

Now , assume (2) is not true ,then there exist $x`, y`, z` \in X$ such that ,

$\mu(x` * z`) < min\{\mu((x` * y`) * z`), \mu(y`)\}$.

Putting $t` = (\mu(x` * z`) + min\{\mu((x` * y`) * z`), \mu(y`)\}/2$, then

$\mu(x` * z`) < t`$ and $ב \leq t` < min\{\mu((x` * y`) * z`), \mu(y`)\} \leq 1$, hence

$\mu((x` * y`) * z`) > t`$ *and* $\mu(y`) > t`$, which imply that

$((x` * y`) * z`) \in \mu_{t`}$, *and* $(y`) \in \mu_{t`}$. Since $\mu_{t`}$ is a q-ideal ,it follows that

$(x` * z`) \in \mu_{t`}$,and that $\mu(x` * z`) \geq t`$, this is also a contradiction .

Hence μ is a fuzzy q-ideal of . ■

Proposition 3.8. Every fuzzy q-ideal of KK-algebra $(X; *, ב)$ is a fuzzy ideal of X.

Proof. Since μ is a fuzzy q-ideal of KK-algebra X , then by Theorem (3.7) , for every t $\in[ב,1]$, μ_t is a q-ideal of X . By Theorem (2.10), for every t $\in[ב,1]$, μ_t is ideal of X. Hence μ is a fuzzy ideal of KK-algebra X by Proposition (3.6) . ■

Proposition 3.9. Let A be a q-ideal of KK-algebra $(X; *, ב)$. Then for any fixed number (t) in an open interval (ב,1), there exists a fuzzy q-ideal μ of X such that $\mu_t = A$.

Proof. Define μ: $X \to [ב, 1]$ by $\mu(x) = \begin{cases} t if x \in A \\ 0 otherwise \end{cases}$.

Where (t) is a fixed number in (ב, 1).

Clearly, $\mu(ב) \geq \mu(x)$, for all $x \in X$. Let $x, y, z \in X$. If $((x * y) * z) \notin A$, then $\mu((x * y) * z) = ב$ and so $\mu(x * z) \geq ב = min\{\mu((x * y) * z), \mu(y)\}$.

If $((x * y) * z) \in A$, then clearly $\mu(x * z) \geq min\{\mu((x * y) * z), \mu(y)\}$.

If $(x * z) \notin A, ((x * y) * z) \in A$, then $y \notin$A, since A is a q-ideal. Thus μ$(x * z) = ב = min\{μ((x * y) * z), μ(y)\}$. Hence μis a fuzzy q-ideal of . It is clear that $\mu_t = A$.■

Theorem 3.10.

A homomorphic pre-image of a fuzzy q-ideal is also a fuzzy q-ideal.

Proof:

Let f : (X; ∗, ב) →(Y; ∗', ב') be an into homomorphism of KK-algebras, β a fuzzy q-ideal of Y and μ the pre-image of β under f, then β (f (x)) = μ (x) , for all x ∈ X . Since f (x) ∈ Y and β is a fuzzy q-ideal of Y, it follows that β(ב') ≥ β(f (x)) = μ(x) , for every x ∈ X, where ב' is the zero element of Y.

But β(ב') = β(f (ב)) = μ(ב) and so μ(ב) ≥ μ(x) , for x ∈ X.

Now, let x, y, z ∈ X , then we get

μ (x*z) = β (f (x*z))

≥ min {β ((f (x) ∗` $f$ (y)) ∗` f (z)), β(f (y))}

= min {β (f ((x ∗ y)*z),β(f (y)) }

= min{μ((x∗y)*z), μ (y)}

i.e., μ (x*z) ≥ min{μ((x∗y)*z), μ (y)}, for all x, y, z ∈ X. This completed the Proof. ⌂

Definition 3.11 ([11]).

A fuzzy subset μ of a set X has sup property if for any subset T of X , there exist t_0 ∈T such that μ(t_0) = sup {μ(t)| t∈T}.

Theorem 3.12.

Let f : (X; ∗, ב) →(Y; ∗', ב') be a homomorphism between KK-algebras X and Y respectively. For every fuzzy q-ideal μ of X with sup property, f (μ) is a fuzzy q-ideal of Y.

Proof:

By definition β(y') = f (μ)(y') := sup{μ(x)| $x \in f^{-1}(y')$ },for all y'∈ Y (sup ϕ = ב).

We have to prove that β(x'∗'z') ≥ min {β((x'∗'y') ∗' z'),β(y')}, for all x', y' , z'∈ Y.

Let f : (X; ∗, ב) →(Y; ∗', ב') be a onto homomorphism of KK-algebras, μ is a fuzzy q-ideal of X with sup property and β the image of μ under f . Since μ is a fuzzy q-ideal of X , we have μ(ב) ≥ μ(x) for all x ∈ X. Note that ב ∈ f^{-1} (ב'), where ב and ב' are the zero elements of X and Y respectively. Thus $\beta(0') = \sup_{t \in f^{-1}(x')} \mu(t) = \mu(0) \geq \mu(x)$, for all x ∈ X, which implies that $\beta(0') \geq \sup_{t \in f^{-1}(x')} \mu(t) = \beta(x')$ for any $x' \in Y$

For any x', y', z'∈ Y , let $x_ב \in f^{-1}$(x') , $y_ב \in f^{-1}$(y') and $z_ב \in f^{-1}$(z')be such that: $\mu(x_ב * z_ב) = \beta(f(x') * f(z')) = sup\, \mu_{t\in f^{-1}(x' * z')}(t)$,

$\mu(y_ב) = \beta(f(y')) = sup\, \mu_{t\in f^{-1}(y')}(t)$ and

$\mu((x_ב * y_ב) = \beta(((f(x') * f(y')) * f(z')) = sup\, \mu_{t\in f^{-1}((x' * y') * z')}(t)$, then

$\beta(f(x') * f(z')) = sup\, \mu_{t\in f^{-1}(x' * z')}(t) = \mu(x_ב * z_ב)$

$\geq min\{\mu((x_ב * y_ב) * z_ב), \mu(y_ב)\}$

$= min\{sup\, \mu_{t\in f^{-1}((x' * y') * z')}(t), sup\, \mu_{t\in f^{-1}(y')}(t)\}$

$= min\{\beta(((f(x') * f(y')) * f(z')), \beta(y')\}$

Hence β is a fuzzy q-ideal of Y. ⌂

References

[1] S. Asawasamrit and A. Sudprasert, (2013), **On The Special Ideals in KK-algebras,** International Journal of Pure and Applied Mathematics, vol.82, no. 4, pp:605-613.
[2] S. Asawasamrit, (2012), **KK-isomorphism and Its Properties**, International Journal of Pure and Applied Mathematics, vol. 78, no. 1, pp:65-73.
[3] S. Asawasamrit, A. Sudprasert, (2015), **On P-semisimple in KK-algebras**, International Journal of Pure and Applied Mathematics, vol. 98, no. 1, pp:23-32.

[4] P. Bhattacharye and N.P. Mukheriee, (1985), **Fuzzy relations and fuzzy group**, Inform . Sci ,vol .36, pp:267-282

[5] A.T. Hameed and E.K. Kadhim, 2020, **Interval-valued IFAT-ideals of AT-algebra**, Journal of Physics: Conference Series (IOP Publishing), pp:1-5.

[6] A.T. Hameed and N.H. Malik, (2021), **(β, α)-Fuzzy Magnified Translations of AT-algebra**, Journal of Physics: Conference Series (IOP Publishing), 2021, pp:1-13.

[7] A.T. Hameed and N.H. Malik, (2021), **Magnified translation of intuitionistic fuzzy AT-ideals on AT-algebra,** Journal of Discrete Mathematical Sciences and Cryptography, (2021), pp:1-7.

[8] A.T. Hameed and N.J. Raheem, (2020), **Hyper SA-algebra**, International Journal of Engineering and Information Systems (IJEAIS), vol.4, Issue 8, pp.127-136.

[9] A.T. Hameed and N.J. Raheem, (2021), **Interval-valued Fuzzy SA-ideals with Degree (λ,κ) of SA-algebra**, Journal of Physics: Conference Series (IOP Publishing), 2021, pp:1-13.

[10] A.T. Hameed, F. F. Kareem and S.H. Ali, **Hyper Fuzzy AT-ideals of AT-algebra**, Journal of Physics: Conference Series (IOP Publishing), 2021, pp:1-15.

[11] A.T. Hameed, H.A. Faleh and A.H. Abed, (2021), **Fuzzy Ideals of KK-algebra**, Journal of Physics: Conference Series (IOP Publishing), 2021, pp:1-7.

[12] A.T. Hameed, I.H. Ghazi and A.H. Abed, (2020), **Fuzzy α-translation AB-ideal of AB-algebras**, Journal of Physics: Conference Series (IOP Publishing), 2020, pp:1-19.

[13] A.T. Hameed, N.J. Raheem and A.H. Abed, (2021), **Anti-fuzzy SA-ideals with Degree (λ,κ) of SA-algebra**, Journal of Physics: Conference Series (IOP Publishing), 2021, pp:1-16.

[14] A.T. Hameed, S.H. Ali and , R.A. Flayyih, 2021,**The Bipolar-valued of Fuzzy Ideals on AT-algebra**, Journal of Physics: Conference Series (IOP Publishing), pp:1-9.

[15] K. Is´eki and S. Tanaka, (2013), **Ideal theory of BCK-algebras**, Math. Japon. , vol.21 (1976), pp:351-366.

[16] S. M. Mostafa, M. A. Abd-Elnaby, F. Abdel-Halim and H A.T.ameed, **On KUS-algebras**, International Journal of Algebra, vol.7, no.3, pp: 131-144.

[17] T.K. Mukherjee and M.K. Sen, (1987), **On Fuzzy Ideals of a Ring I**, Fuzzy Sets and Systems, vol.21, pp: 99-104.

[18] Prabpayak and U. Leerawat, (2009) , **On ideals and congurences in KU-algebras** , Scientia Magna Journal , vol.5, no .1, pp:54-57 .

[19] C. Prabpayak and U. Leerawat, (2009), **On isomorphisms of KU-algebras** , Scientia Magna Journal , vol.5, no .3 , pp:25-31 .

[20] A. Rosenfeld, (1971), **Fuzzy group**, J. Math. Anal. Appl., vol.35, pp:512-517.

[21] O. G. Xi, (1991), **Fuzzy BCK-algebras**, Math. Japon. , vol.36, pp:935-942.

[22] L.A. Zadeh, (1965), **Fuzzy sets** , inform . and control ,vol.8, pp:338-353 .