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On Cartesian Product of Fuzzy q-ideals on KK-algebra

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Abstract *The aim of this paper is introducing the notion of fuzzy q-ideal of KK-algebra, several theorems, properties are stated and proved. The fuzzy relations on KK-algebras are also studied.*

Keywords—KK-algebra, fuzzy ideal, fuzzy q-ideal, the cartesian product of fuzzy q-ideals.

1. Introduction

A BCK-algebra is an important class of logical algebras introduced by K. Is´eki [15] and was extensively investigated by several researchers. The class of all BCK-algebras is a quasivariety. K. Is´eki posed an interesting problem whether the class of BCK-algebras is a variety. In ([18,19]), C. Prabpayak and U. Leerawat the notion of KU-algebras. They gave the concept of homomorphism of KU-algebras and investigated some related properties. The concept of fuzzy subset and various operations on it were first introduced by L.A. Zadeh in [22], then fuzzy subsets have been applied to diverse field. The study of fuzzy subsets and their application to mathematical contexts has reached to what is now commonly called fuzzy mathematics. Fuzzy algebra is an important branch of fuzzy mathematics. They introduced of the concept of fuzzy subgroups in 1971 by A. Rosenfeld [20]. Since these ideas have been applied to other algebraic structures such as semi-groups, rings, ideals, modules and vector spaces. O.G. Xi, ([21]) applied this concept to BCK-algebra, and he introduced the notion of fuzzy sub-algebras (ideals) of the BCK-algebras with respect to minimum, and since then Y. S.M. Mostafa and et al [16] have introduced the notion of KUS-algebras, KUS-ideals and investigates the relations among them. S. Asawasamrit and A. Sudprasert [1-3] have introduced the notion of KK-algebras, ideals, KK-subalgebras and studied the relations among them and gave the concept of homomorphism of KK-algebras and investigated some related properties. In this paper, the study of fuzzy algebraic structures was started with the introduction of the concept of fuzzy q-ideals of KK-algebras and then we investigate several basic properties which are related to fuzzy ideals. We have also proved that the cartesian product of fuzzy q-ideals is a fuzzy q-ideal.

2. Preliminaries

Now, we give some definitions and preliminary results needed in the later sections.

Definition 2.1[1,2]. Let $(X; *, \beth)$ be an algebra with a binary operation $*$ and a nullary operation $\beth$. Then X is called **KK-algebra** if it satisfies the following: for all $x, y, z \in X$,

(KK_1) : $(x*y) * ((y*z) * (x*z)) = \beth$,
(KK_2) : $\beth * x = x$,
(KK_3) : $x * y = \beth$ *and* $y * x = \beth$ if and only if $x = y$.

Definition 2.2[1,2]. Define a binary relation $\leq$ on KK-algebra X by letting $x \leq y$ if and only if $y * x = \beth$.

Examples 2.3[4,19]. Let A be an arbitrary nonempty set and X be the set of all real valued function defined on A for $, g \in X$, we define $f * g$ by $(f * g)(x) = \begin{cases} f(x) - g(x) & f(x) < g(x) \\ \beth & f(x) \geq g(x) \end{cases}$.

Then (X; *, $\beth$) is a KK-algebra.

Examples 2.4[3]. Let X = { $\beth$, 1} and let $*$ be defined by:

*	ב	1
ב	ב	1
1	1	ב

Then $(X; *, \beth)$ is a KK-algebra.

Proposition 2.5 [1,3]. In any KK-algebra $(X; *, \beth)$, the following properties hold: for all x, y, z $\in$X;

(P_1) $x * ((x * y) * y) = \beth$;
(P_2) $x * x = \beth$;
(P_3) $* (y * z) = y * (x * z)$.

Definition 2.6[2]. A nonempty subset I of a KK-algebra $(X; *, \beth)$ is called **an ideal of X** if it satisfies the following conditions: for any $x, y \in X$,

(I_1) $\beth \in I$;
(I_2) $x * y \in I$ *and* $x \in I$ *imply* $y \in I$.

Definition 2.7[1]. A nonempty subset I of a KK-algebra $(X; *, \beth)$ is called **a q-ideal of X** if it satisfies the following conditions: for any $x, y, z \in X$,

(I_1) $\beth \in I$;
(I_2) $(x * y) * z \in I$ *and* $y \in I$ *imply* $x * z \in I$.

Examples 2.8[1]. Let X = { $\beth$, a, b, c} and let $*$ be defined by the table:

∗	ב	a	b	c
ב	ב	a	b	c
a	ב	ב	c	c
b	c	c	ב	ב
c	c	b	a	ב

Thus $(X;\ \ast,\ ב)$ is a KK-algebra. And we see that I = { ב, a} and J = { ב, c} are q-ideals of X.

Proposition 2.9[1]. Let $\{I_i \mid i\in\Lambda\}$ be a family of q-ideals of KK-algebra $(X;\ \ast,\ ב)$. The intersection of any set of q-ideals of KK-algebra X is also q-ideal.

Theorem 2.10[1]. A q-ideal of KK-algebra $(X;\ \ast,\ ב)$is an ideal.

Proof. Suppose that I is an q-ideal of X and let $x \ast y \in I$ and $x \in I$. It follows that $(ב \ast x) \ast y \in I$ imply $ב \ast y \in I$. Thus, $y \in I$. ■

Remark 2.11[1]. The converse of Proposition (2.10) needs not be true in general as in the Example.

∗	ב	a	b	c
ב	ב	a	b	c
a	c	ב	a	b
b	b	c	ב	a
c	a	b	c	ב

Example 2.12[1]. Let X ={ ב, a, b, c} and let $\ast$ be a binary operation defined by the table:

Then $(X;\ \ast,\ ב)$ is a KK-algebra with I = { ב} is an ideal of X, but not a q-ideal of X. Since $(c \ast ב) \ast a = a \ast a = ב \in$ { ב} and $ב \in$ { ב} , but $c \ast a = b \notin$ { ב}.

Theorem 2.13[1]. If I is an ideal of KK-algebras X, then the following are equivalent:

(1) I is an q-ideal of X;

(2) for any x, $y \in X, (x \ast ב) \ast y \in I$ implies $x \ast y \in I$;

(3) for any $x, y, z \in X, (x \ast y) \ast z \in I$ implies $x \ast (y \ast z) \in I$.

Theorem 2.14[1]. Let K and I be ideals of a KK-algebra X with $I \subseteq K$. If I is a q-ideal of X, then so is K.

Corollary 2.15[1]. If zero ideal { ב} of KK-algebra X is a q-ideal, then every ideal of X is a q-ideal.

Theorem 2.16[1]. Let I be an ideal of KK-algebra X. If for any $x \in I$ and $y \in X$ imply $x \ast y \in I$, then I is q-ideal of X.

Lemma 2.17[1]. If I is a q-ideal of KK-algebra X, then $x \ast (x \ast ב) \in I$, for all $x \in X$.

Definition 2.18[2]. Let $(X;\ \ast,\ ב)$ *and* $(Y;\ \ast`,\ ב`)$ be nonempty sets. The mapping $f:\ (X;\ \ast,\ ב) \rightarrow (Y;\ \ast`,\ ב`)$ is called **a homomorphism** if it satisfies:
$f(x \ast y) = f(x) \ast` f(y)$, for all $x, y \in X$. The set $\{x \in X \mid f(x) = ב'\}$is called **the kernel of** f denoted by ker f.

Theorem 2.19[2]. Let $f:\ (X;\ \ast,\ ב) \rightarrow (Y;\ \ast`,\ ב`)$ be a homomorphism of a KK-algebra X into a KK-algebra Y, then :

A. $f(ב) = ב'$.

B. f is injective if and only if $ker\ f$ = { ב}.

C. $x \leq y$ implies $f(x) \leq f(y)$.

Theorem 2.20[2]. Let $f:\ (X;\ \ast,\ ב) \rightarrow (Y;\ \ast`,\ ב`)$ be a homomorphism of a KK-algebra X into a KK-algebra Y, then :

(1) If I is ideal of X, then $f(I)$ is ideal of .

(2) If J is ideal of Y, then $f^{-1}(J)$ is ideal of .

(3) $ker\ f$ is ideal of X.

Definition 2.21[22]. Let (X; ∗ , ב) X be a nonempty set, a fuzzy subset μ of X is a function $\mu: X \rightarrow$ [ב, 1].

Definition 2.22[4]. Let X be a nonempty set and μ be a fuzzy subset of X, for t ∈[ב,1],the set μ_t ={x ∈ X | μ(x) ≥ t}is called a level subset of μ.

Definition 2.23[4]. Let f : (X;*, ב) →(Y;*', ב') be a mapping nonempty sets X and Y respectively. If μ is a fuzzy subset of X, then the fuzzy subset β of Y defined by:

$$f(\mu)(y) = \begin{cases} \sup\{\mu(x) : x \in f^{-1}(y)\} & if\ f^{-1}(y) = \{x \in X, f(x) = y\} \neq \phi \\ 0 \quad otherwise \end{cases}$$

is said to be the image of μ under f.

Similarly if β is a fuzzy subset of Y , then the fuzzy subset $\mu = (\beta \circ f)\ in\ X$ (i.e. the fuzzy subset defined $by\ \mu(x) = \beta(f(x))\ for\ all\ x \in X$) is called the pre-image of β under f .

3. Fuzzy q-ideals of KK-algebras

In this section, we will discuss a new notion called fuzzy q-ideals of KK-algebras and study several basic properties which are related to fuzzy q-ideals.

Definition 3.1[11]. Let $(X; \ast, ב)$ be a KK-algebra , a fuzzy subset μ of X is called **a fuzzy ideal of X** if it satisfies the following conditions: , for all $x , y \in X$,

(1) $\mu(ב) \geq \mu(x)$,

(2) $\mu(y) \geq min\{\mu(x \ast y), \mu(x)\}$.

Definition 3.2. Let $(X; *, ב)$ be a KK-algebra , a fuzzy subset μ *of* X is called **a fuzzy q-ideal of *X*** if it satisfies the following conditions: , for all $x, y, z \in X$,

(1) $\mu(ב) \geq \mu(x)$,

(2) $\mu(x * z) \geq min\{\mu((x * y) * z), \mu(y)\}$.

Example 3.3. Consider X = { ב, 1, 2, 3, 4} with * defined by the table

*	ב	1	2	3	4
ב	ב	1	2	3	4
1	ב	ב	2	3	4
2	ב	ב	ב	3	4
3	ב	ב	ב	ב	4
4	ב	ב	ב	ב	ב

Then $(X; *, ב)$ is a KK-algebra. Define a fuzzy subset μ: $X \rightarrow$ [ב, 1] such that $\mu(ב) = 2/3, \mu(1) = \mu(2) = 1/2$ *and* $\mu(3) = \mu(4) = 1/3$.

Routine calculation gives that μ is a fuzzy q-ideal of KK-algebra .

Example 3.4.

Let X ={ ב, 1, 2, 3} in which $(*)$ is defined by the following table:

*	ב	1	2	3
ב	ב	1	2	3
1	ב	ב	3	3
2	3	3	ב	ב
3	3	2	1	ב

Then $(X; *, ב)$ is KK-algebra.

Define a fuzzy subset $\mu : X \rightarrow$ [ב, 1] by $\mu(x) = \begin{cases} 0.7 if x \in \{0,1\} \\ 0.3 otherwise \end{cases}$. I_1 = { ב, 1} is ideal of X.

Routine calculation gives that μ is a fuzzy q-ideal of KK-algebras X.

4. Cartesian Product of Fuzzy q-ideal

In this section, we will discuss, investigate a new notion called cartesian product of fuzzy q-ideals and we study several basic properties which related to fuzzy q-ideals.

Definition 4.1[18]. A fuzzy relation R on any set S is a fuzzy subset

R: S×S →[ב,1].

Definition 4.2[18]. If R is a fuzzy relation on sets S and β is a fuzzy subset of S , then R is a fuzzy relation on β if R(x,y) ≤ min {β(x), β(y)}, for all x, y∈S .

Definition 4.3[18]. Let μ and β be fuzzy subsets of a set S . The cartesian product of μ and β is defined by $(\mu \times \beta)$(x,y) = min {μ(x), β(y)}, for all x, y∈S .

Lemma 4.4[18]. Let μ and β be fuzzy subsets of a set S . Then,

(1) $(\mu \times \beta)$ is a fuzzy relation on S ,

(2) $(\mu \times \beta)_t = \mu_t \times \beta_t$, for all t ∈[ב,1].

Definition 4.5[18]. If β is a fuzzy subset of a set S , the strongest fuzzy relation on S ,that is, a fuzzy relation on β is R_β given by R_β (x,y) = min {β(x) , β(y)}, for all x, y∈S .

Lemma 4.6[18]. For a given fuzzy subset β of a set S, let R_β be the strongest fuzzy relation on S. Then for t ∈[ב,1], we have $(R_\beta)_t = \beta_t \times \beta_t$.

Proposition 4.7. For a given fuzzy subset β of a KK-algebra $(X; *, ב)$, let R_β be the strongest fuzzy relation on X . If β is a fuzzy q-ideal of $X \times X$, then R_β (x,x) ≤ R_β (ב, ב) , for all x∈ X .

Proof. Since R_β is a strongest fuzzy relation of $X \times X$, it follows from that,

$R_\beta(x, x)$ = min {β(x) , β(x)}≤ min{β(ב), β(ב)}= $R_\beta(0,0)$, which implies that R_β (x,x) ≤ R_β(ב, ב).■

Proposition 4.8. For a given fuzzy subset β of a KK-algebra $(X; *, ב)$, let R_β be the strongest fuzzy relation on X . If R_β is a fuzzy q-ideal of $X \times X$, then β(x) ≤ β(ב), for all x∈X .

Proof. Since R_β is a fuzzy q-ideal of $X \times X$, it follows by proposition (4.7), R_β (x,x) ≤ R_β (ב, ב) . Where (ב, ב) is the zero element of $X \times X$. But this means that, min{β(x), β(x)} ≤ min{β(ב), β(ב)}which implies that β(x) ≤ β(ב) . ■

Remark 4.9. Let $(X; *, ב)$ *and* $(Y; *`, ב`)$ be KK-algebras, we define (·) on $X \times Y$ by: for all (x,y),(u,v) ∈ $X \times Y$, (x,y) · (u,v) = (x∗u, y∗′v). Then clearly $(X \times Y, \cdot, ($ ב, ב'$))$ is a KK-algebra.

Theorem 4.10.

Let μ and β be fuzzy q-ideals of KK-algebra $(X; *, ב)$. Then $\mu \times \beta$ is a fuzzy q-ideal of $X \times X$.

Proof. Note first that for every (x,y)∈ $X \times X$,

$(\mu \times \beta)$(ב, ב) = min { μ (ב), β (ב)}≥ min{μ (x), β(y)}= $(\mu \times \beta)$(x,y) .

Now let (x_1, x_2) , (y_1, y_2) , $(z_1, z_2) \in X \times X$. Then

$(\mu \times \beta)(x_1 \cdot z_1, x_2 \cdot z_2)$ = min { $\mu(x_1 \cdot z_1)$, $\beta(x_2 \cdot z_2)$}

≥ min { min{ $\mu((x_1 \cdot y_1) \cdot z_1)$, $\mu(y_1)$}, min{ $\beta((x_2 \cdot y_2) \cdot z_2)$, $\beta(y_2)$}}

= min { min{ $\mu(((x_1 \cdot y_1) \cdot z_1))$, $\beta((x_2 \cdot y_2) \cdot z_2))$ }, min{ $\mu(y_1)$, $\beta(y_2)$}}

= min { $(\mu \times \beta)(((x_1 \cdot y_1) \cdot z_1), ((x_2 \cdot y_2) \cdot z_2))$, $(\mu \times \beta)(y_1, y_2)$}

Hence $(\mu \times \beta)$ is a fuzzy q-ideal of $X \times X$. ■

Theorem 4.11.

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Let μ and β be fuzzy subsets of KK-algebra X such that $\mu \times \beta$ is a fuzzy q-ideal of $X \times X$. Then for all $x \in X$

(i) either $\mu($ב$) \geq \mu(x)$ or $\beta($ב$) \geq \beta(x)$.

(ii) If $\mu($ב$) \geq \mu(x)$, for all $x \in X$, then either $\beta($ב$) \geq \mu(x)$ or $\beta($ב$) \geq \beta(x)$.

(iii) If $\beta($ב$) \geq \beta(x)$, for all $x \in X$, then either $\mu($ב$) \geq \mu(x)$ or $\mu($ב$) \geq \beta(x)$.

(iv) Either μ or β is a fuzzy q-ideal of X.

Proof.

(i) suppose that $\mu(x) > \mu($ב$)$ and $\beta(y) > \beta($ב$)$, for some $x, y \in X$. Then
$(\mu \times \beta)(x,y) = \min\{\mu(x), \beta(y)\} > \min\{\mu($ב$), \beta($ב$)\} = (\mu \times \beta)($ב, ב$)$. This is a contradiction and we obtain (i).

(ii) Assume that there exist $x, y \in X$ such that $\mu(x) > \beta($ב$)$ and $\beta(y) > \beta($ב$)$. Then $(\mu \times \beta)($ב, ב$) = \min\{\mu($ב$), \beta($ב$)\} = \beta($ב$)$ it follows that
$(\mu \times \beta)(x,y) = \min\{\mu(x), \beta(y)\} > \beta($ב$) = (\mu \times \beta)($ב, ב$)$. which is a contradiction. Hence (ii) holds.

(iii) is by similar method to part (ii).

(iv) Since by (i) either $\mu($ב$) \geq \mu(x)$ or $\beta($ב$) \geq \beta(x)$, for all $x \in X$.

Without loss of generality we may assume that $\beta($ב$) \geq \beta(x)$, for all $x \in X$, from (iii) it follows that either $\mu($ב$) \geq \mu(x)$ or $\mu($ב$) \geq \beta(x)$. If $\mu($ב$) \geq \beta(x)$, for any $x \in X$, then $(\mu \times \beta)($ב$,x) = \min\{\mu($ב$), \beta(x)\} = \beta(x)$.

Let $(x_1,x_2), (y_1,y_2), (z_1,z_2) \in X \times X$, since $\mu \times \beta$ is a fuzzy q-ideal of $X \times X$, we have

$(\mu \times \beta)(x_1 \cdot z_1, x_2 \cdot z_2) \geq \min\{(\mu \times \beta)(((x_1 \cdot y_1) \cdot z_1), ((x_2 \cdot y_2) \cdot z_2)), (\mu \times \beta)(y_1, y_2)\}$ ------- (F).

If we take $x_1 = y_1 =$ ב, then

$\beta(x_2 \cdot z_2) = (\mu \times \beta)($ב$, (x_2 \cdot z_2))$

$\geq \min\{(\mu \times \beta)($ב$, ((x_2 \cdot y_2) \cdot z_2)), (\mu \times \beta)($ב$, y_2)\}$

$= \min\{\min\{\mu($ב$), \beta((x_2 \cdot y_2) \cdot z_2)\}, \min\{\mu($ב$), \beta(y_2)\}\}$

$= \min\{\beta((x_2 \cdot y_2) \cdot z_2), \beta(x_2)\}$

This prove that β is a fuzzy q-ideal of .

Now, we consider the case $\mu($ב$) \geq \mu(x)$, for all $x \in X$. Suppose that $\mu($ב$) < \mu(y)$, for some $y \in X$. then $\beta($ב$) \geq \beta(y) > \mu($ב$)$. Since $\mu($ב$) \geq \mu(x)$, for all $x \in X$, it follows that $\beta($ב$) \geq \mu(x)$ for any $x \in X$. Hence $(\mu \times \beta)(x,$ ב$) = \min\{\mu(x), \beta($ב$)\} = \mu(x)$, taking $x_2 = y_2 =$ ב in (F), then

$\mu(x_1 \cdot z_1) = (\mu \times \beta)((x_1 \cdot z_1),$ ב$)$

$\geq \min\{(\mu \times \beta)((x_1 \cdot y_1) \cdot z_1),$ ב$), (\mu \times \beta)(y_1,$ ב$)\}$

$= \min\{\min\{\mu((x_1 \cdot y_1) \cdot z_1)), \beta($ב$)\}, \min\{\mu(y_1), \beta($ב$)\}\}$

$= \min\{\mu((x_1 \cdot y_1) \cdot z_1)), \mu(y_1)\}$

Which proves that μ is a fuzzy q-ideal of X. Hence either μ or β is a fuzzy q-ideal of X.

Theorem 4.12. Let β be a fuzzy subset of a KK-algebra X and let R_β be the strongest fuzzy relation on , then β is a fuzzy q-ideal of X if and only if R_β is a fuzzy q-ideal of $X \times X$.

Proof: Assume that β is a fuzzy q-ideal of X. By . proposition (4.7), we get,

$R_\beta($ב, ב$) \geq R_\beta(x,y)$, for any $(x,y) \in X \times X$.

Let $(x_1,x_2), (y_1,y_2), (z_1,z_2) \in X \times X$, we have:

$R_\beta((x_1 \cdot z_1), (x_2 \cdot z_2)) = \min\{\beta(x_1 \cdot z_1), \beta(x_2 \cdot z_2)\}$

$\geq \min\{\min\{\beta((x_1 \cdot y_1) \cdot z_1), \beta(y_1)\}, \min\{\beta((x_2 \cdot y_2) \cdot z_2), \beta(y_2)\}\}$

$= \min\{\min\{\beta((x_1 \cdot y_1) \cdot z_1), \beta((x_2 \cdot y_2) \cdot z_2)\}, \min\{\beta(y_1), \beta(y_2)\}\}$

$= \min\{R_\beta(((x_1 \cdot y_1) \cdot z_1), ((x_2 \cdot y_2) \cdot z_2)), R_\beta(y_1, y_2)\}$

Hence R_β is a fuzzy q-ideal of $X \times X$.

Conversely, suppose that R_β is a fuzzy q-ideal of $X \times X$, by Proposition (4.8)

$\beta($ב$) \geq \beta(x)$, for all $x \in X$, which prove (1).

Now, let $(x_1,x_2), (y_1,y_2), (z_1,z_2) \in X \times X$, then

$\min\{\beta(x_1 \cdot z_1), \beta(x_2 \cdot z_2)\} = R_\beta((x_1 \cdot z_1), (x_2 \cdot z_2))$

$\geq \min\{R_\beta(((x_1,x_2) \cdot (y_1, y_2)) \cdot (z_1,z_2)), R_\beta(y_1, y_2)\}$

$= \min\{R_\beta(((x_1 \cdot y_1) \cdot z_1), ((x_2 \cdot y_2) \cdot z_2)), R_\beta(y_1,y_2)\}$

$= \min\{\min\{\beta((x_1 \cdot y_1) \cdot z_1), \beta((x_2 \cdot y_2) \cdot z_2)\}, \min\{\beta(y_1), \beta(y_2)\}\}$

In particular if we take $x_2 = y_2 =$ ב, then $\beta(x_1 \cdot z_1) \geq \min\{\beta((x_1 \cdot y_1) \cdot z_1), \beta(y_1)\}$.

This proves (2) and β is a fuzzy q-ideal of .■

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