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Some properties of density related to lambda ideal open sets

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Abstract— *The aim of this paper is to introduce a new concept of lambda Ideal dense sets by using the concept of ideal and bitopological spaces (X,T,T^{α}), in addition to studying some properties and generating theories for these sets.*

Keywords— ideal topological space, λI-open sets, λI – local function , λI-dense sets.

1. INTRODUCTION

In this paper we will deals with several concepts that have a wide history in topology, one of These concepts is the ideal topological spaces, which was studied by Kuratowski .K. in 1933[16] and Vaidyanathaswamy.R in 1945[22] . After that, concept was developed by presenting a different of studies related to the concept of dense sets . the other concept is the bitopological space defined by Kelley in 1955 [18]. density concept introduced and studied in a different researcher and spaces ,and one of these spaces is the ideal topological space ,like [4]. J.Dontchev ,M. Ganster in 1999 [15]defended a concept T^*- dense as the subset of X satisfy the following condition CL^*(A) =X and I- dense if A*=X.

this paper deals with a new definition of dense set in ideal bitopological spaces with some of properties and relations.

2. Elementary Materials

2.1 Definition [11]:
A nonempty collection I of subsets of X is said to be an ideal on X, if it satisfies the following two conditions:

(A) $A \in I$, and, $B \subseteq A \rightarrow B \in I$ (heredity).
(B) $A \in I$, and , $B \in I \longrightarrow A \cup B \in I$ (finite additivity).

2.2Definition [17]:
Let (X,T) be a topological space with an ideal I on X, a set operator $(.)^*: p(X) \rightarrow p(X)$, defined as follow $A^*(\mathrm{I},T) = \{x \in X: A \cap U \notin I$, for every $U_x \in T\}$, Which is called the local function of A with respect to 1 and T.
2.3 Definition :
Let (X,T) be a topological space, with an ideal I defined on X, and let A be a subset of X then:-
1- If A*⊂A, then A is called A* - closed.[15]
2-If A=A*, then A is called ∗ - perfect.[12]
2.4 Definition [23]:
An ideal I is called condense, iff T∩I== {Φ}
2.5 Remark [15]:
Every I-dense is T*-dense ,and then T- dense
2.6 Definition [14]:
A subset A of X is called λI-open set iff for each x϶A and for each α_I-open set W_x such that A⊆ W,satisfy that x∈{U_x ⊓ int_T(W)∉ I, for each $U_x \in T\}$ .The family of all λI-open set denoted by $O_{\lambda I}(X)$.
2.7 Definition [14]:
Let (X, T, T^{α}, I) be an ideal bi topological space then X is called true space if every open set is * - perfect set .
2.8 Proposition [14]:
Let (X, T, T^{α}, I) be an ideal bi topological space and let A is $\lambda I - open\ set\ such\ that\ for\ every\ U \epsilon T\ , U\ is\ * - perfect\ set$, then A is $\lambda - open\ set$.
2.9 Definition [14]:

Let (X,T,I) be an ideal topological space an operator $(.)^{*\lambda I}: P(X) \rightarrow p(X)$ called λI-local function of A with respect to I and λI-open set is define as follow for any $A \subseteq X$.

$A^{*\lambda I}(I, \lambda I - open)=\{x \in X: U \sqcap A \notin I$, for every subset $U_X \in O_{\lambda I}(X)\}$, when there is no chance for confusion $A^{*\lambda I}(I, \lambda I - open)$ is denoted by $A^{*\lambda I}$.

2.10 Remark [14]:

Let(X,T, T^{α},I) be an ideal bitopological space and let A , B are sub set of X then

1-If I=P(X), then $A^{*\lambda I}(I)=\{\Phi\}$.

2- If $A \subseteq B$, then $A^{*\lambda I} \subseteq B^{*\lambda I}$.

2.11 Remark [14]:

Let(X,T, T^{α},I) be an ideal bitopological space and let A , B are sub set of X then

1- $A^{*\lambda I} \subseteq A^{*}$ for each $A \subseteq X$. If I is condense set.

2 $- A^{*} \subseteq A^{*\lambda I}$, for each $A \subseteq X$. If x is true space.

2.12 Remark [14]:

Let(X,T, T^{α},I) be an ideal Bi topological space and let A , B are sub set of X then

If $A \in I$, then $A^{*\lambda I}(I)=\{\Phi\}$.

2.13 Proposition [14]:

Let(X,T, T^{α},I) be an ideal Bi topological space and let A , B are sub set of X then

1- If I={ Φ}, then $A \subset A^{*\lambda I}$.

2- If I ={ Φ} , then $A \sqcup B \subset A^{*\lambda I} \sqcup B^{*\lambda I}, also\ A \sqcap B \subseteq A^{*\lambda I} \sqcap B^{*\lambda I}$.

2.14 Definition [14]:

Let(X,T, T^{α},I) be an ideal Bi topological space for any A∈X we define:

$Cl^{*\lambda I}(A)(I,T) = A \sqcup A^{*\lambda I}$.

2.15 Theorem [14]:

Let(X,T, T^{α},I) be an ideal Bi topological space and let A , B are subset of X then :-

1- $Cl^{*\lambda I}(A) \subseteq Cl(A)$. If I is condense.

2- $Cl^{*\lambda I}(A) \subseteq Cl^{*}(A)$ if X is true space.

3-$A^{*\lambda I} \subset Cl^{*\lambda I}(A)$

4- $Cl^{*\lambda I}(X) = X$.

3. dense sets via λI −open set

3.1 Definition :

Let (X,T, T^{α},I) be an ideal bitopological space and let A be subset of X is called $\lambda I - dense\ set$ iff $A^{*\lambda I}$=X

3.2 Example :

Let X={a ,b, c} ,with a topology T={ X,Φ,$\{a\}$}, and I={Φ},

then $\lambda I - dense\ set$={X ,{a ,b},{a ,c}}

3.3Remark :

By remark (2.12) we have that if $A \in I, then\ A\ is\ not\ \lambda I - dense\ set$ by the following example.

3.4 Example :

Let X={a ,b, c} ,with a topology T={ X,Φ,$\{a\}$}, and I={Φ,$\{b\},\{c\},\{b,c\}$}

then $\lambda I - dense\ set$={ Φ }.

3.5 *R*emark :

1- $\Phi\ is\ not\ \lambda I - dense\ set$.

2- X is $not\ ncessary\ \ \lambda I - dense\ set$,we show in the following example.

3.6 Example :

Let X={a ,b, c, d} ,with a topology T={ X,Φ,{a, b},{c, d}}, and I={Φ,{a}} ,

then $\lambda I - dense\ set$ ={ Φ } clearly that X and Φ are not $\lambda I - dense\ set$.

3.7 Remark :

Let (X,T, T^{α},I) be an ideal Bi topological space and let A be subset of X if A is T^*-dense and I-dense ,then A is not necessary $\lambda I -$ dense set by the following example.

3.8 Example :

Let X={a ,b, c} ,with a topology T={ X,Φ,$\{a\}$} and I={Φ}, clearly that {a} is T^*-dense and I-dense but not $\lambda I -$ dense set.

3.9 Remark :

Let (X,T, T^{α},I) be an ideal Bi topological space and let A be subset of X if A is open , then is not necessary A is $\lambda I -$ dense set by the following example.

3.10 Example :

Let X={a ,b, c} ,with a topology T={ X,Φ,$\{a\}$} and I={Φ,$\{b\},\{c\},\{b,c\}$} clearly that {a} is *open* but not $\lambda I -$ dense set.

3.11 Remark :

Let (X,T, T^{α},I) be an ideal Bi topological space ,then the following properties hold.

1- Every $\lambda I -$ dense is I- dense set if (I is condense) .

2- Every $\lambda I -$ dense is $T^* -$ dense set if (I is condense) .

3- Every $\lambda I -$ dense is T- dense set if (I is condense) .

Proof

1- Since A is $\lambda I -$ dense set and A⊆ X ,then $A^{*\lambda I}$=X ,so by theorem (2.11)(1) and I is condense , then

A^*=X , then A is I- dense set .

2- Since A is $\lambda I -$ dense set and A⊆ X ,then $A^{*\lambda I}$=X ,so by theorem (2.15)(3) and I is condense, then $Cl^*(A) = X$ for each A ⊆ X , then A is $T^* -$ dense set.

3- Since A is $\lambda I -$ dense set and A⊆ X ,then $A^{*\lambda I}$=X ,so by theorem (2.15)(2)(3) and I is condense, then $Cl^{*\lambda I}(X) = X$ *and by* theorem(2.15)(1) then $Cl(A) = X$, then A is $T -$ dense set.

3.12 Proposition :

Let (X,T, T^{α},I) be an ideal Bi topological space and let A,B are be subset of X , then the following properties hold

1- If A , B are $\lambda I -$ dense set ,then A⊔ B is $\lambda I -$ dense set .

2- If A , B are $\lambda I -$ dense set ,then A⊓ B is not necessary $\lambda I -$ dense set .

3- If I⊆ J, *then* $\lambda I - dense\ set$ and $\lambda J - dense\ set$ are indpented .

Proof

1-Since A , B are $\lambda I -$ dense set ,then $A^{*\lambda I}$=X , $B^{*\lambda I}$=X if possible that x$\notin (A \sqcup B)^{*\lambda I}$,then there exist $\lambda I -$ open set H , such that H⊓ $(A \sqcup B) \in I$, *and then* $H \sqcap A \in I$ *and this is* contradiction , also H⊓ $B \in I$, and this is contradiction .

2- Let X={a ,b, c} ,with a topology T={ X,Φ,$\{a\}$}, and I={Φ}, clearly that A={a,b} and B={a,c} is $\lambda I -$ dense set but {a} is not $\lambda I - dense\ set$.

3- Let X={a ,b, c} ,with a topology T={ X,Φ,$\{a\}$}, I={Φ} *and* $J = \{\Phi, \{b\}, \{c\}, \{b, c\}\}$, *then*

$\lambda I - dense\ set$= {X, {a,b},{a,c}}, while $\lambda J - dense\ set$ ={Φ}.

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