

# Investigation of Shear Forces and Deformations in Inclined Cracks in Reinforced Concrete Beams

<sup>1</sup>Matniyazov Bakdurdi Ibragimovich, <sup>1</sup>Sagatov Baxodir Uktamovich, <sup>2</sup>Aproilov Abd Al Islam

<sup>1</sup>Jizzakh Polytechnic Institute, 130100 Jizzakh, Uzbekistan

<sup>2</sup>Moscow Automobile and Road Construction State Technical University (MADI)

**Abstract:** The article presents the results of experimental and theoretical studies of the resistance mechanism of T-beams made of heavy and light concrete, taking into account shear forces and deformations. Their behavior and stress-strain state after crack formation are analyzed. New data on the development of normal and tangential displacements in inclined cracks have been obtained. For the analysis of experimental data, a simplified approach was used, based on the implementation of the truss analogy model, taking into account energy methods and the revealed mechanism of engagement forces in inclined cracks of the beam rib. In the analysis, the conditions for the equilibrium of forces in the inclined section of the beam were used, taking into account the forces in the longitudinal and transverse reinforcement and the engagement forces.

**Keywords:** T-beams, shear forces and deformations, inclined cracks, aggregate interlock, truss model, normal and shear displacements

## 1. Introduction

Despite a large number of recent experimental and theoretical studies [1, 2], the problem of the resistance of reinforced concrete beams to the action of transverse forces is still far from being solved. These studies are being intensively pursued due to the dangerous nature of sudden shear failure and in order to better understand the physical aspects of the problem. Due to the large number and complexity of factors that influence the shear behavior of reinforced concrete beams, the construction of an appropriate comprehensive theory of resistance is associated with great difficulties. This circumstance has long been exacerbated by the predominance of an empirical approach to solving the problem without an in-depth study of the behavior of beams under the action of transverse forces. In this case, the deformation behavior of beams is of particular importance, taking into account stresses in concrete and reinforcement, as well as displacements that occur in inclined cracks.

## 2. Materials and Methods

In connection with the foregoing, the authors carried out special studies of the behavior of reinforced concrete T-beams under the action of transverse forces. The tested beams had a T-section with the dimensions and reinforcement scheme shown in fig. 1. In total, two series of five beams were tested, which were made of heavy (TBT) and expanded clay (KBT) concrete. The general characteristics of the experimental beams are given in Table 1. All beams had a relative shear span equal to  $l/h_0=3,57$ . The required anchoring of the longitudinal reinforcement was ensured by its continuation beyond each support for a length of 250 mm and the installation of 5 clamps  $d = 8$  mm. In each series, the content of transverse reinforcement was changed by changing the pitch of the clamps, with a constant percentage of longitudinal reinforcement. The beams were concreted in two in metal and wooden forms.

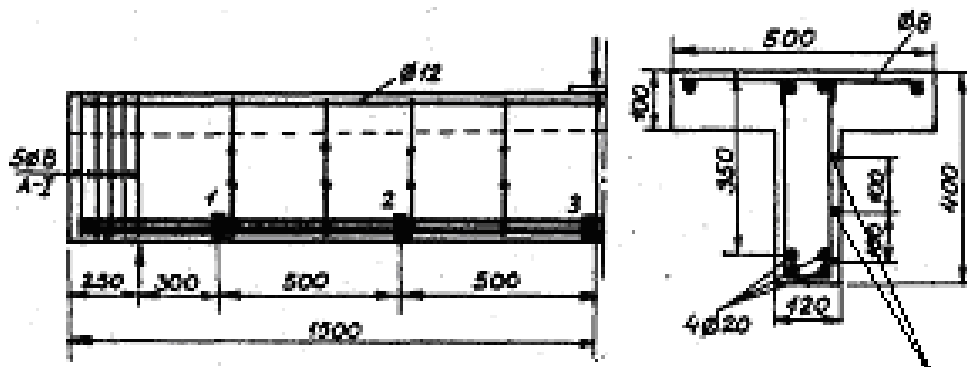


Fig. 1. Scheme of reinforcement of T-beams: 1, 2, 3 - installation locations of strain gauges

Table 1.

Characteristics of experimental beams

Names	$R_b$ , MPa	Longitudinal reinforcement			Transverse reinforcement				$Q$ kN
		$d$ , mm	$R_s$ N/mm <sup>2</sup>	$E_s \times 10^3$ H/mm <sup>2</sup>	$d$ , mm	step $s$ , mm	$\mu_{sw}$ , %	$R_{sw}$ N/mm <sup>2</sup>	
TBT-1	27,0	4 Ø20	441	200	6	250	0,189	366	118,0
TBT-2	32,0	4 Ø20	441	200	6	125	0,378	366	132,5
TBT-3	25,1	4 Ø20	441	200	8	250	0,419	441	161,0
TBT-4	40,7	4 Ø20	649	150	8	160	0,524	441	219,5
TBT-5	27,5	4 Ø20	649	180	8	125	0,671	441	212,5
KBT-1	23,6	4 Ø20	441	200	6	250	0,189	366	85,0
KBT-2	26,8	4 Ø20	441	200	6	125	0,378	366	118,5
KBT-3	24,4	4 Ø20	441	200	8	200	0,419	441	120,0
KBT-4	28,0	4 Ø20	649	180	8	160	0,524	441	150,0
KBT-5	22,0	4 Ø20	649	180	8	125	0,671	441	150,0

To measure deformations, two strain gauges with a base of 10 mm were glued to the rods of longitudinal reinforcement in each of the selected five sections of the beam. On the diagram of Fig. 1, the installation locations of strain gauges are shown for a half-span of a beam. For observation, rods were selected on one side of the section. Prefabricated frames with glued and insulated strain gauges and metal benchmarks were placed in molds in compliance with the design position of the reinforcing bars. To the transverse rods were welded short-benchmarks, on which the deformation of the clamps was measured by comparators. The load on the beams was applied in steps in the middle of the span. After the appearance of a certain number of inclined cracks, the load (as a rule, not exceeding 50–60% of the destructive design load) was dropped and along the cracks on the surface of the ribs, metal reference points were glued, which were used to measure the mutual displacements of the crack faces during secondary loading of the beam until its destruction. To measure the deformations of the concrete of the rib at an angle of 45° to the axis of the beam, additional reference points were glued onto the surface of the rib of the beams. Simultaneously, the opening of all cracks was duplicated by measurements with an optical microscope. All measurements were carried out at each stage of loading. The scheme of cracks on each beam was transferred to graph paper. Based on the test results, the following information was obtained:

- properties of reinforcement and concrete, the level of breaking load and the nature of destruction;
- results of measurements of relative deformations in longitudinal and transverse reinforcement;
- increments of tangential and normal displacements of crack edges  $\Delta\delta$  and  $\Delta a$  according to the scheme in fig. 2, as well as the results of traditional measurements of the crack opening width with a microscope;
- values of compressive strains in inclined concrete strips of beam ribs.

The general behavior of the beams of both series under load was approximately the same. The normal cracks that appeared at the beginning then bent with an inclination the greater, the closer they were to the support. Simultaneously with the increase in the load, the slope of the cracks decreased. The decrease in the slope was typical either for new cracks that appeared at the second stage of loading (i.e., after the installation of benchmarks on the appeared cracks), or for existing cracks that developed towards the support. Immediately after their appearance, oblique cracks reached the junction of the flange with the rib. The effect of transverse reinforcement on the cracking pattern of the beam in both series was approximately the same and manifested itself in an increase in the number of steeper inclined cracks with increasing  $\mu_{sw}$ . As a rule, this was observed with the same or close to the same pitch of the clamps. Thus, more cracks were found in TBT - 2 and KBT - 2 beams than in TBT - 3 and KBT - 3 beams, than beams TBT - 3 and KBT - 3.

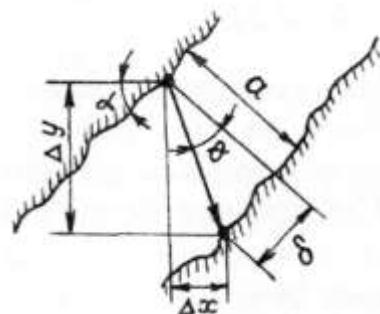


Fig. 2. Scheme for determining the displacements of the crack faces

Beams TBT - 2 and KBT - 1,2,3 collapsed from the cut. Typically, such destruction occurred by crack penetration into the flange and the formation of a fan-shaped fracture plane, which includes the entire width of the compressed zone of the flange (500

mm). However, the destruction of expanded clay beams occurred somewhat differently. In them, the cracks extended to the side face of the shelf to a much greater length. Only when the broken pieces of concrete were removed did it become clear that the fracture had occurred along a fan-shaped plane through the flange. The destruction of the TBT-3 beam occurred in an almost ideal way, since the fluidity of the longitudinal reinforcement was observed in this case. In the beams TBT - 4 and KBT - 4 and KBT - 5, at first, the flow of clamps was observed, and then, immediately before the destruction, the pulling out of the longitudinal rods above and behind the support took place in the absence of fluidity in them, although  $\mu_{SW}$  it was the same.

### 3. Results and Discussions

Analysis of the test results made it possible to draw the following conclusions:

1. All transverse bars that crossed critical inclined cracks reached the yield strength before failure in both series of beams.
2. As a rule, at the corresponding relative load levels, the stresses in the clamps of expanded clay concrete beams were higher than in heavy concrete beams, which indicates their lower ability to transmit engagement forces through a crack.
3. The truss analogy is in good agreement with the values of the measured strains, however, it overestimates the stresses in the clamps by a value corresponding to  $Q_{cre}$ .
4. There was the appearance of residual stresses in the clamps after the initial unloading of the beam to install the benchmarks on the cracks formed, although the applied load did not cause the clamps to flow; a similar phenomenon was also observed in the longitudinal reinforcement. Apparently, in order to ensure equilibrium, it is necessary to assume that the inclined strips of concrete of the rib are somewhat compressed, and the gearing mechanism has come into operation in the cracks. This means that in the near-bearing part of the beam damaged by cracks, the stress-strain state is preserved even if the external load that caused these cracks is completely removed.
5. An analysis of the distribution of deformations along the length of the longitudinal working rods showed that there is a tendency to deviate from the diagram of bending moments from the external load and the distribution of forces in the rods after the formation of cracks. This is due to the addition of the horizontal component of the force in the inclined compressed concrete strip, which is included in the work only after the appearance of inclined cracks.
6. The displacements  $\Delta\delta$  and  $\Delta a$  measured by the reference points and calculated taking into account the angle of inclination of the cracks  $\alpha$  gave a complete picture of the mutual displacements of the blocks of beams. As expected, the largest displacements were found in the KBT-1 beam, which has the lowest percentage of transverse reinforcement and the lowest shear stiffness due to the engagement of crack faces. This is confirmed by Fig. 3 a,b, which shows the graphs of mutual displacements of crack faces in this beam (left half-span, cracks No. 4 and No. 1), inclined at different angles. On fig. 3c, 4d show displacements  $\Delta\delta$  and  $\Delta a$  along cracks with approximately the same slope, measured on two beams TBT-2 (crack No. 4, right half span) and KBT-2 (crack No. 2, left half span), which had the same percentage transverse reinforcement. It can be seen that at the same load levels, the edges of cracks in expanded clay beams experience greater mutual displacements than in similar beams made of heavy concrete. The increase in the summing displacement vector is mainly due to the growth of component  $\Delta\delta$ . This can also be explained by the low shear stiffness of expanded clay concrete with cracks. No significant scatter in the values of the displacement components along the cracks at all loading levels was found. Theoretical analysis was carried out using the average displacement values determined for each crack at each load increment.

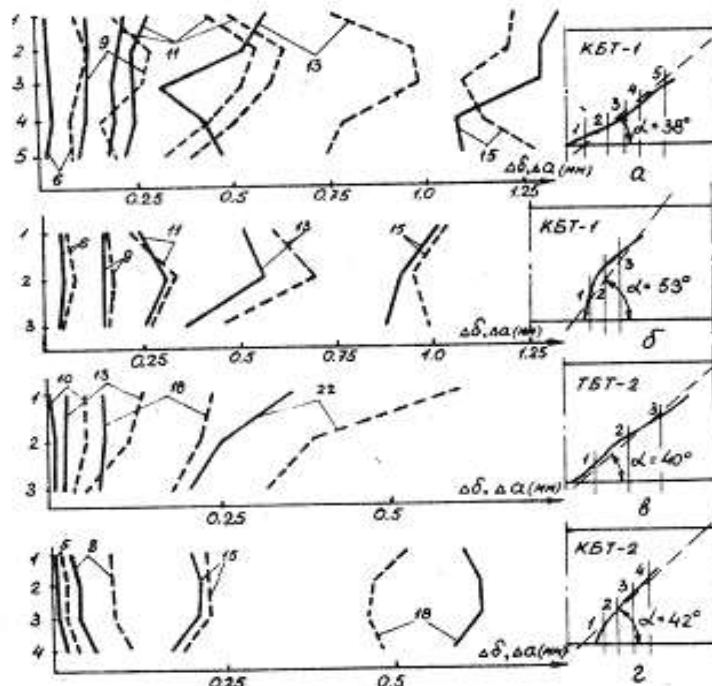


Fig. 3. Graphs of displacements of crack faces in experimental beams: -----  $\Delta\delta$ , ———  $\Delta\alpha$

7. Из рис. 4 видно, что на начальных ступенях приложения нагрузки наклонные трещины сначала раскрываются по всей длине, а затем ее берега сдвигаются. Отсюда следует, что силы зацепления в первых крутых наклонных трещинах в середине пролета влияют на условие совместности в остальной части ребра еще до распространения наклонных трещин на более высоких ступенях нагружения. Подразумевается, что главные растягивающие напряжения или деформации, способствующие появлению наклонных трещин, должны быть равномерно распределены в направлении, перпендикулярном траектории трещины. Следовательно, ясно, что наклонные трещины должны распространяться от плиты до уровня продольной арматуры почти прямолинейно и направление главных растягивающих напряжений (и деформаций) в значительной мере не должно совпадать с траекториями, известными из теории упругости. Деформация бетона наклонных полос ребра замерялась под углом  $45^\circ$  к оси балки. На рис. 4а показано их развитие в балках ТБТ-1 и КБТ-1, взятое как среднее значение по нескольким параллельным базам в каждом полу пролете балки. Видно, что в пределах каждой балки разброс деформаций по отдельным реперам невелик, что позволило использовать их средние значения. Существенная разница в деформациях для тяжелого и легкого бетона связана с различием их модулей упругости.

7. From fig. 4 it can be seen that at the initial stages of load application, inclined cracks first open along the entire length, and then its edges shift. It follows that the engagement forces in the first steep inclined cracks in the middle of the span affect the compatibility condition in the rest of the rib even before the propagation of inclined cracks at higher loading steps. It is understood that the main tensile stresses or strains that contribute to the appearance of oblique cracks must be uniformly distributed in the direction perpendicular to the crack path. Therefore, it is clear that inclined cracks should propagate from the slab to the level of the longitudinal reinforcement almost in a straight line and the direction of the main tensile stresses (and strains) should not largely coincide with the trajectories known from the theory of elasticity. The deformation of the concrete of the inclined strips of the rib was measured at an angle of  $45^\circ$  to the axis of the beam. On fig. 4a shows their development in TBT-1 and KBT-1 beams, taken as an average value over several parallel bases in each half-span of the beam. It can be seen that, within each beam, the spread of deformations over individual reference points is small, which made it possible to use their average values. A significant difference in deformations for heavy and light concrete is associated with the difference in their modulus of elasticity.

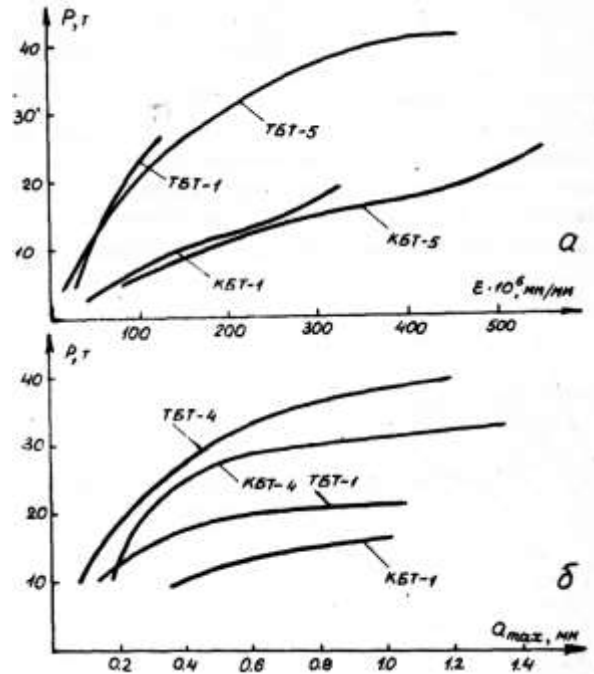


Fig. 4. Deformation of concrete (a) and opening of cracks (b) in the rib of T-beams.

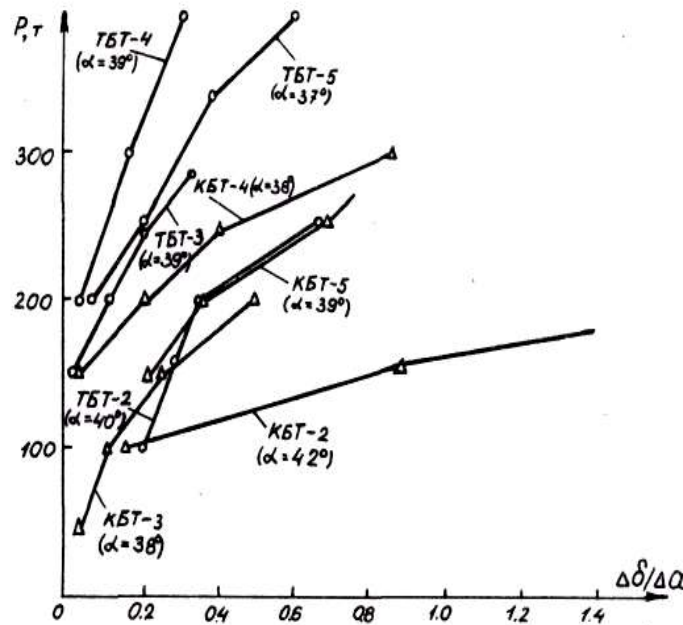


Fig. 6. Dependence of the ratio  $\Delta\delta/\Delta\alpha$  on the level of loading for individual cracks.

8. Measurements of crack opening width in beams with a microscope were carried out on all tested beams. On fig. 5b comparisons are given for both series of beams. As can be seen, the reduction in the pitch of the clamps allows better control of crack opening. The crack width for TBT beams is less than for KBT beams. This confirms the measurement data obtained from the displacement components of the crack faces.

9. The dependence of the strength of the beams on the magnitude of the transverse reinforcement shows a significant difference between the tested series. Beams made of heavy concrete have a large bearing capacity.

For the analysis of experimental data, a simplified approach was used based on the implementation of the truss analogy, taking into account energy methods, as well as the role of engagement forces in inclined cracks of the beam rib. In this case, we proceeded from the consideration of the equilibrium condition relative to the level of the longitudinal working reinforcement, including the forces in the rib and longitudinal reinforcement. It is also necessary to take into account the balance of forces in the

inclined section, taking into account the forces in the longitudinal and transverse reinforcement, as well as the engagement forces. It was shown in [1] that the latter determine the shear stiffness of a section with a crack, which can be reduced to shear stresses by the expression:

$$\tau = A \Delta\delta \quad \text{or} \quad \tau = K (\Delta\delta / \Delta a),$$

where:  $A$  – shear stiffness of the mechanism of engagement in a crack, N/mm<sup>2</sup>;  $K$  – coefficient of proportionality, N/mm<sup>2</sup>;  $\Delta a$  – width of normal crack opening, mm.

The use of the experimental values of the shear stiffness of the section with a crack  $K$  obtained in [1] shows satisfactory agreement between these methods and with the energy method. The application of the energy method for the stages of elastic operation of transverse reinforcement is somewhat limited. Since the flow of stirrups is not associated with the general destruction of the beam, it is advisable to consider an additional equilibrium condition based on overcoming the engagement forces of the crack edges. At the same time, the issues of bonding and anchoring of reinforcement in the tested beams should not be overlooked.

The total vector of mutual displacements of the crack faces can be divided into two components  $\Delta\delta$  and  $\Delta a$  in the direction of any coordinates. Considering that the crack inclination angle  $\alpha$  to the beam axis is not equal to 90°, the displacements  $\delta$  and  $a$  were expressed in terms of  $\Delta x$  and  $\Delta y$  as follows (see Fig. 2):

$$a = \sqrt{\Delta x^2 + \Delta y^2} \times \cos(\alpha - \theta); \quad \delta = \sqrt{\Delta x^2 + \Delta y^2} \times \sin(\alpha - \theta),$$

where:  $\theta = \text{tg}^{-1}(\Delta x / \Delta y)$ .

It was shown in [3] that the stress transmitted through the crack is  $K (\Delta\delta / \Delta a)$ , i.e. at a constant  $K$  it depends on the ratio  $\Delta\delta / \Delta a$ . Then the graphs in Fig. 3 can be analyzed as follows. For example, the dependence of the growth of the ratio  $\Delta\delta / \Delta a$  with an increase in the load for each of the marked benchmarks along the cracks in the beams can be represented in the form shown in Fig. 5, where, for a clearer presentation, the averaged values of  $\Delta\delta / \Delta a$  are given for the entire fracture. The graphs show the boundaries of the load that causes the yield of clamps, determined by the measured deformations in the clamps. In this case, the physical yield point in any clamp is taken as the lower limit, and the estimate for the average relative deformation in the clamps is taken as the upper limit. As can be seen, the ratio  $\Delta\delta / \Delta a$  increases with increasing load, which indicates an increase in engagement forces. It should be noted that the considered cracks have the same slope to the beam axis.

#### 4. Conclusions

In connection with the foregoing, the following conclusions can be drawn about the behavior of the tested beams:

1. The change in stresses in the longitudinal bars of the beam in accordance with the diagram of bending moments is associated with adhesion, and the formation of oblique cracks partially worsens this adhesion of the reinforcement to the concrete of the rib. This contributes to a certain increase in the efforts of the longitudinal reinforcement. Consequently, these efforts are still quite large (Fig. 4) and the destruction of the beam from pulling out the reinforcement may follow. Thus, the formation of inclined cracks increases the probability of this type of failure. The destruction of beams with more saturated transverse reinforcement in the form of spallation of the protective layer along the longitudinal reinforcement near the supports is undoubtedly associated with a high level of the above-mentioned adhesion forces.

2. A splitting crack, as a rule, begins at the end of the beam cut span in front of the support and then quickly propagates along the longitudinal reinforcement to the end of the beam. Such a destruction mechanism indicates the inexpediency of increasing the length of the embedment of the rods by more than 250 mm per support, since this will not reduce the risk of such destruction. Studies show that the adhesion strength in this case increases insignificantly even in the presence of clamps. Therefore, for logical reasons, it would be more appropriate to increase the thickness of the protective layer over the supports.

3. The noted mechanism of destruction of beams requires an accurate assessment of the deformed state of the considered zone of the beam rib, which is associated with significant difficulties. They are exacerbated by the presence of inclined cracks near the supports (which, in turn, is associated with the assessment of initial deformations), as well as the influence of clamps and support reaction.

#### References

1. Ashrabov A.A. Lightweight concrete and reinforced concrete for industrial construction. Tashkent. Publishing house “Mekhnat”, 1988, 252 p.
2. Zalesov A.S., Klimov Yu.A. Strength of reinforced concrete structures under the action of transverse forces. - Kyiv.: Budivelnik, 1989. - 5c.
3. Ashrabov A.A., Samarin A.V. To the method for determining the forces of engagement in the cracks of reinforced concrete beams under the action of transverse forces. // Issues of reliability of bridge structures: Mezhdvuz. Sat. / LISI. - L., 1984, p. 62-68.