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Special Case of local function in ideal bi topological spaces

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Abstract—*The aim of this paper is to present a new concept of Ideal local function sets by using the concept of ideal and bi topological spaces (X,T,$\boldsymbol{T^{\alpha}}$), in addition to studying some properties and generating theories for these sets.*

Keywords—ideal topological space, α-open set ,α_I-open set, λ-open set, λI-open sets, λI – local function.

1. INTRODUCTION

The local function of ideal topological spaces, which was studied by Kuratowski .K. in 1933[12] and Vaidyanathaswamy.R in 1945[18].

The concept of local function of a subset A of X. After that, this concept was developed by presenting a different of studies related to the concept of ideal topological spaces ,such as Jankovic and Hamlett. T.R.[10], who were among the first to present a studies related to some topological concepts, also, A. Abdel Monsef, Radwan [2], Lashien.E.F. and Nasef.A.A[16], where they presented an importent studied deals with the concept of I-open set, in addition, Al-Swidi.L.A., introduced a different studies with some of researchers in a different types of spaces and sets, and we can see that in [4, 5], also modern studies presenting related to an important type of spaces represented by topological proximity spaces [9,8,7],by using the ideal topological spaces .

A bitopological space is a two topological spaces defined on the space X which was defined by Kelly.J.C.[15] ,and λ −open sets is one of the studies in bitopological space , which was defined by Altalkany .Y.K [1] by using the concept is α-open set which was defined by the scientist Njastad.O [17]as the subset of X satisfy the following condition $A \subset \mathrm{int}(\mathrm{cl}(\mathrm{int}(A)))$.

In this paper we introduce a new definition of λ-open set in ideal bitopological spaces with some of properties and relations.

2. Elementary Materials

2.1 Definition [**10**]
A nonempty collection I of subsets of X is said to be an ideal on X, if it satisfies the following two conditions:

(A) $A \in I$, and, $B \subseteq A \to B \in I$ (heredity).
(B) $A \in I$, and , $B \in I \longrightarrow A \cup B \in I$ (finite additivity).

2.2 Definition [**13**] Let (X, T) be a topological space with an ideal I on X, a set operator $(.)^*: p(X) \to p(X)$, defined as follow $A^*(\mathrm{I}, T) = \{x \in X: A \cap U \notin I$, for every $U_x \in T\}$, Which is called the local function of A with respect to 1 and T.

2.3 Theorem [**10**]
Let (X, T) be a space with I and J ideal on X, and let A and B be two subsets on X. then

1- If $A \subseteq B$, then $\mathrm{A}^* \subseteq \mathrm{B}^*$
2- $\mathrm{A}^* = \mathrm{cl}(\mathrm{A}^*) \subseteq \mathrm{cl}(A)(\mathrm{A}^*$ is a closed subset of $\mathrm{cl}(A))$;
3- $(A^*)^* \subseteq A^*$
4- $(A \cup B)^* = A^* \cup B^*$;
5- $\mathrm{A}^* - \mathrm{B}^* = (\mathrm{A} - \mathrm{B})^* - \mathrm{B}^* \subseteq (\mathrm{A} - \mathrm{B})^*$.
6- for every $I \in I, (A \cup I) *= A *= (A - I) *$.
7 − If $U \in T$, then $U \cap A^* = U \cap (U \cap A)^* \subseteq (U \cap A)^*$
8 − If $I_1 \subseteq I_2$, then $A^*(I_2) \subseteq A^*(I_1)$
9-If $A=A^*$, *then Ais called* $*-perfect$.

2.4 Theorem [2]

For space (X, T, I) and $A \subset X$, we have :

1- If $I = \{\emptyset\}$, then $A^*(I) = Cl(A)$.

2-If $I = p(X)$, then $A^*(I) = \emptyset$.

3- If $A \epsilon I$, $then\ A^*=\{\emptyset\}$.

2.5 Definition [18]

Let (X,T,I) be an ideal topological space ,then $cl^*(.): p(X) \to p(X)$, is defined as follows $cl^*\ (A) = A \sqcup A^*$ for any sub set A of X.

2.6 Theorem [16]

Let (X,T,I) be an ideal topological space , and let A and B be two subsets on X. then

1- If $A \subseteq B, then\ cl^*\ (A) \subseteq cl^*\ (B)$.

2-$cl^*\ (A\ \sqcup B) = cl^*\ (A) \sqcup cl^*\ (B)$.

3-$cl^*\ (A\ \sqcap B) \subset cl^*\ (A) \sqcap cl^*\ (B)$.

4- $A \subseteq cl^*\ (A) \subseteq Cl(A)$.

5- $cl^*(cl^*\ (A))\ \subseteq cl^*\ (A)$.

2.7 Definition [**2**]

A subset A of an ideal topological space (X, T, I) is said to be (α_I-open set) iff there exists an open set U such that $U - A \in I$ and A - int $(Cl(U)) \in I$.

2.8 Definition :

A subset A of X is called λI-open set iff for each $x \epsilon A$ and for each α_I-open set W_x such that $A \subseteq W$,satisfy that $x \in \{U_x\ \sqcap\ int_T(W) \notin I$, for each $U_x \in T\}$ .The family of all λI-open set denoted by $O_{\lambda I}(X)$.

2.9 Proposition :

If $U \in T, W \in \lambda I - open\ set\ , then\ U \sqcap W \in O_{\lambda I}(X)$.

Proof

Let W is $\lambda I - open\ set\ and\ U\ \in T\ such\ that\ x \in U \sqcap W$, Since W is $\lambda I - open\ set$,then for each $\propto_I - open\ set$ H such that $w \subseteq H$ and $x \in \{x \in X: U_x \sqcap int_T(H) \notin I\}$, for each $U_x \in T$, and since $U \sqcap W \subseteq W, hence\ U \sqcap W \subseteq H\ and\ x \in \{x \in X: \{U_x \sqcap int_T(H) \notin I\}$, so $U \sqcap W\ \lambda I - open\ set$.

2.10 Proposition:

Let (X, T, T^α, I) be an ideal bi topological space then

If $I = \{\Phi\}\ then$ every sub set A of X is $\lambda I - open\ set$.

2.11 Definition :

Let (X, T, T^α, I) be an ideal bitopological space then X is called true space if every open set is * - perfect set .

2.12 Definition :

Let (X,T, T^α, I) be an ideal bitpological space .For any A subset of X we define :

$Cl^{\lambda I}(A)$ (I, T) $= \sqcap \{A \subseteq H : H \text{ is } \lambda I - \text{closed set}\}$

2.13 Theorem :

Let(X,T, T^α,I) be an ideal bitopological space and let A be subset of X then $Cl^{\lambda I}(A) = A \sqcup D^{\lambda I}(A)$.

2.14 Theorem :

Let(X,T, T^α,I) be an ideal bitopological space and let A be subset of X then

1- $If\ I = \{\Phi\}$, then $Cl^{\lambda I}(A) \subseteq Cl(A)$.

2- $X \in Cl^{\lambda I}(A)$, iff $U \sqcap A \neq \Phi$ for each λI − open set U of x .

Proof:

1- Let $x \in Cl^{\lambda I}(A)$, then $U \sqcap A \neq \Phi$, $\forall\ U \in O_{\lambda I}(X)\ by\ proposition$ (2.14)(1) , $then$
$U \sqcap A \neq \Phi\ \forall\ U \in T(x) and\ , then\ x \in Cl(A)$.

2- If $U \sqcap A = \Phi$, then $U \subseteq A^c$,then $x \in A^c, then\ x \notin A, then\ x \in Cl^{\lambda I}(A)$, then $x \in D^{\lambda I}(A)$, then $W \sqcap A/x \neq \Phi$, for each $W \in O_{\lambda I}(X)$

And this is contradiction there for $U \sqcap A \neq \Phi$.

Suppose $U \sqcap A \neq \Phi\ for\ each\ \lambda I - open\ set$ U of x

If $x \notin Cl^{\lambda I}(A)$, $then\ x \notin\ A \sqcup D^{\lambda I}(A)$, $then\ x \notin A, x \notin D^{\lambda I}(A), then\ therexist\ H \in\ \lambda I - open\ set, H \sqcap A/x =\ \Phi$, then $x \notin A$, then $H \sqcap A =\ \Phi$

And this is contradiction then $x \in Cl^{\lambda I}(A)$.

The following example show that $Cl(A) \not\subset\ Cl^{\lambda I}(A)$.

2.15 Example :Let X={a,b,c} , with a topology $T=\{\Phi, X, \{a\}\}$, and I= {Φ} , then $Cl\{a\}=X \not\subset Cl^{\lambda I}\{a\}=\{a\}$.

3.Study on Lambda ideal-local function.

In this section we introduce a new type of local function namely Lambda ideal-local function and study some important characteristics and its relationship with local function .

3.1 Definition :

Let (X,T,I) be an ideal topological space an operator $(.)^{*\lambda I}$:P(X) ⟶ P(X) called λI-local function of A with respect to I and λI-open set is define as follow for any A $\subseteq X$.

$A^{*\lambda I}$ (I, $\lambda I - open$)={x∈ X:U⊓ $A \notin I, for\ every\ subset\ U_X \in O_{\lambda I}$(X)},when there is no chance for confusion $A^{*\lambda I}$ (I, $\lambda I - open$) is denoted by $A^{*\lambda I}$.

3.2 Remark :

A^* and $A^{*\lambda I}$ are independent as in the following example

3.3 Example :

Let X={a,b,c} ,with a topology T={ X,Φ,{a}}, and I={Φ} then $O_{\lambda I}$(X) = {X, Φ,{a},{a,b},{a,c},{ b ,c}} ,clearly that $A^* = X^*$=X while $A^{*\lambda I}$=$X^{*\lambda I}$={a}.

3.4 Remark :

Let(X,T, T^α,I) be an ideal bitopological space and let A , B are sub set of X then

1-If I=P(X), then $A^{*\lambda I}$(I)= {Φ}.

2- If A⊆ B,then $A^{*\lambda I} \subseteq B^{*\lambda I}$.

Proof

1-Suppose if possible x∈ $A^{*\lambda I}$(I) iff for every U∈ $O_{\lambda I}$(X) ,U⊓ $A \notin P(X)$, and this is a contradiction there for $A^{*\lambda I}$(I)={Φ}.

2- Let x∈ $A^{*\lambda I}$(I) then U⊓ $A \notin I$, for every U∈ $O_{\lambda I}$(X) and since A⊆ $B, then\ \ U \sqcap B \notin I$, $for\ every$ U∈ $O_{\lambda I}$(X) and then $A^{*\lambda I}$(I) ⊆ $B^{*\lambda I}$.

3.5 Proposition :

Let(X,T, T^α,I) be an ideal bitopological space and let A , B are sub set of X then

1- $A^{*\lambda I} \subseteq A^*$ for each A⊆ X. If I is condense set.

2 − $A^* \subseteq A^{*\lambda I}$, for each A⊆ X. If X is true space.

Proof

Let x∈ $A^{*\lambda I}$(I) then U⊓ $A \notin I$ for each U∈ $O_{\lambda I}$(X) since x condense space then by

theorem(3.4)(2) we have U⊓ $A \notin I$ there for open set U of X and then x ∈ A^*.

2- Let x∈ A^*, then U⊓ $A \notin I$, for each T –open set U of x ,and by theorem (3.4)(2) we have U⊓ $A \notin I$, for each λI–open set U of x and then x ∈ $A^{*\lambda I}$.

3- $in\ general\ A^* \nsubseteq A^{*\lambda I}$. As the following example

3.6 Example :

Let X={a ,b ,c} ,with a topology T={ X,Φ, $\{a\}$}, and I={Φ, $\{b\}, \{c\}, \{b,c\}$}, ,clearly that

$A^* = \{a\}^* = \{a,b,c\}$, $A^{*\lambda I} = \{a\}^{*\lambda I}$={a} and $\{a,b,c\} \nsubseteq$ {a}.

3.7 Remark :

Let(X,T, T^α*,I) be* an ideal bitopological space and let A , B are sub set of X then

1- If A∈ I, then $A^{*\lambda I}$(I)= {Φ}.

2- If I= {Φ} , then A⊔ $B \subseteq A^{*\lambda I} \sqcup B^{*\lambda I}$.

3- For any ideal , then (A$\sqcup B)^{*\lambda I}$=$A^{*\lambda I} \sqcup B^{*\lambda I}$.

4- For any ideal , then (A$\sqcap B)^{*\lambda I}$=$A^{*\lambda I} \sqcap B^{*\lambda I}$.

5- If I $\subseteq J$, $then A^{*\lambda J} \subseteq A^{*\lambda I}$.

6- If U$\in T$,then U $\sqcap A^{*\lambda I} \subset$ (U$\sqcup A)^{*\lambda I}$

$7-(A^{*\lambda I})^{*\lambda I}$=$A^{*\lambda I}$.

8- $A^{*\lambda I} \subseteq Cl^{\lambda I}(A)$.

9-$A^{*\lambda I} \subseteq Cl^{\lambda I}(A^{*\lambda I})$.

Proof:

1- Suppose that x$\in A^{*\lambda I}$(I) then for every U$\in O_{\lambda I}$(X) ,U$\sqcap A \notin I, since\ A \in I, U \sqcap A \in I\ and\ this\ is\ a\ contradiction\ hence$ $A^{*\lambda I}$(I)={Φ}.

2-Since A$\subset$ A$\sqcup B$ and $B \subset$ A$\sqcup B$ by theorem (3.4)(2) we have A $\subseteq A^{*\lambda I} \sqcup B^{*\lambda I}$ and

B$\subseteq A^{*\lambda I} \sqcup B^{*\lambda I}$, then A$\sqcup B \subseteq A^{*\lambda I} \sqcup B^{*\lambda I}$.

3- Let x$\in$ (A$\sqcup B)^{*\lambda I}$, then U$\sqcap (A \sqcup B) \notin$ I for each U$\in O_{\lambda I}$(X) and U$\sqcap A \sqcup$ U$\sqcap B \notin I\ and\ definition\ (\ ideal)(2), then\ U \sqcap A \notin I\ and$ U$\sqcap B \notin I$ *for any* $U \in O_{\lambda I}(X)$ *and then* $x \in A^{*\lambda I} \sqcup B^{*\lambda I}$.

4-since A$\sqcap B \subseteq A\ andA \sqcap B \subseteq B\ \ by\ (3.4)(2)\ then\ (A \sqcap B)^{*\lambda I} = A^{*\lambda I} \sqcap B^{*\lambda I}$.

5- Let x$\in A^{*\lambda J}$(I) then U$\sqcap A \notin J$, for each $\lambda I - open$ set U $of\ \ x,\ and\ thenU \sqcap A \notin I$,for each $\lambda I - open$ set U of x ,hence x $\in A^{*\lambda I}$.

6- Let x $\in U \sqcap A^{*\lambda I}$, then x$\in U\ and\ x \in A^{*\lambda I}\ hence\ W \sqcap A \notin I\ , for\ each\ \lambda I - open\ set\ W\ of\ x$

If possible that $x \notin (\ U \sqcap A)^{*\lambda I}$ $, then\ there\ exist\ H\ is\ \lambda I - open\ \ set,\ such\ that\ \ H \sqcap\ \ (U \sqcap A) \in I\ , also\ (H \sqcap U) \sqcap A \in I\ and\ by\ proposition\ (\ \ 2.9)\ H \sqcap U\ is\ \lambda I - open\ set, then\ x \notin A^{*\lambda I}\ and\ this\ contradiction.$*Then* $U \sqcap A^{*\lambda I} \subset$(U$\sqcup A)^{*\lambda I}$.

7-Suppose that x $\in (A^{*\lambda I})^{*\lambda I}$, then $A^{*\lambda I} \sqcap$ U$\notin I$ for every U$\in O_{\lambda I}$(X),then $A^{*\lambda I} \sqcap$ U$\neq$ Φ

For every U$\in O_{\lambda I}$(X) there for ,there exist some y $\in A^{*\lambda I} \sqcap$ U such that U$\in O_{\lambda I}$(y) and

y $\in A^{*\lambda I}$ so A$\sqcap$ U$\notin I$ and x $\in A^{*\lambda I}$, then $(A^{*\lambda I})^{*\lambda I} \subseteq A^{*\lambda I}$.

8- Let x $\in A^{*\lambda I}$ then U$\sqcap A \notin I, for\ each\ U_X\ \in \lambda I - open\ set\ \ and\ U \sqcap A \neq \Phi\ , then$ xϵ $Cl^{\lambda I}(A), then\ A^{*\lambda I} \subseteq Cl^{\lambda I}(A)$.

9 - Let x$\in A^{*\lambda I}$ then U$\sqcap A \notin I, for\ each\ U_X\ \in \lambda I - open\ set\ \ and\ U \sqcap A \neq \Phi$,if x$\notin Cl^{\lambda I}(A^{*\lambda I})$, there exist U$\in O_{\lambda I}$(X) such that U$\sqcap A^{*\lambda I} = \Phi, but\ U \subseteq (A^{*\lambda I})^c$, then x$\in (A^{*\lambda I})^c$ and this is introduction, then x$\in$ $Cl^{\lambda I}(A^{*\lambda I})$.

In the following example we show $l^{\lambda I}(A^{*\lambda I}) \subseteq A^{*\lambda I}$.

3.8 Example :

Let X={a,b,c} , with a topology T={$\Phi, X, \{a\}$}, and I= {Φ} , clearly that

$Cl^{\lambda I}(\{a\}^{*\lambda I}) = \{a\} \subseteq \{a\}^{*\lambda I}$={a} .

3.9 Proposition :

Let(X,T, T^{α}*,I) be an ideal Bi topological space and le*t A , B are sub set of X then

1- If I={ Φ}, then A $\subset A^{*\lambda I}$.

2- If ={ Φ} , then A$\sqcup B \subset A^{*\lambda I} \sqcup B^{*\lambda I}, also\ A \sqcap B \subseteq A^{*\lambda I} \sqcap B^{*\lambda I}$.

Proof

1-Let x$\in A$ $, then\ if\ possible\ \ x\ \notin A^{*\lambda I}, then\ there\ existU \in O_{\lambda I}(\ X), such\ that$ U$\sqcap A \in I$,

And then U$\sqcap A = \{\ \Phi\}, hence\ U \subseteq A^c$,So from that x$\in A^c$ and this is contradiction , then x $\in A^{*\lambda I}$.

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2- Since A $\subset A^{*\lambda I}$,B $\subseteq B^{*\lambda I}$ then A$\sqcup B \subseteq A^{*\lambda I} \sqcup B^{*\lambda I} = (A \sqcup B)^{*\lambda I}$.

3.10 Definition :

Let(X,T, T^{α},I) be an ideal Bi topological space for any A∈X we define:

$Cl^{*\lambda I}(A)(I,T) = A \sqcup A^{*\lambda I}$.

3.11Theorem :

Let(X,T, T^{α},I) be an ideal Bi topological space and let A , B are sub set of X then :-

1- $Cl^{*\lambda I}(X)$=*X.*

2- If A$\subseteq B$*,then* $Cl^{*\lambda I}(A) \subseteq Cl^{*\lambda I}(B)$.

3- A$\subseteq$ $Cl^{*\lambda I}$(A).

4- $Cl^{*\lambda I}(A \sqcup B)$= $Cl^{*\lambda I}(A) \sqcup (B)^{*\lambda I}$

5- $Cl^{*\lambda I}(A \sqcap B) \subseteq Cl^{*\lambda I}(A) \sqcap (B)^{*\lambda I}$

6- $Cl^{*\lambda I}(A) = Cl^{*\lambda I}(Cl^{*\lambda I}(A))$.

7- $Cl^{*\lambda I}(A) \subseteq Cl(A)$. If I is condense.

8- $Cl^{*\lambda I}(A) \subseteq Cl^{*}(A)$ *if X is true space*.

9- $Cl(A^{*\lambda I}) = A^{*\lambda I}$.

10- $Cl^{*}(A) \subset Cl^{*\lambda I}(A)$ if I is condense

11-$A^{*\lambda I} \subset Cl^{*\lambda I}(A)$.

Proof:

1- $Cl^{*\lambda I}(X)$= $X \sqcup X^{*\lambda I}$=*X.*

2- Since A$\subseteq B$ *, then by theorem (If A*$\subseteq B$*,then* $A^{*\lambda I} \subseteq B^{*\lambda I}$)

$A^{*\lambda I} \subseteq B^{*\lambda I}$*, and* $(A \sqcup A^{*\lambda I}) \subseteq (B \subseteq B^{*\lambda I})$ *implise that* $Cl^{*\lambda I}(A) \subseteq Cl^{*\lambda I}(B)$.

3- A$\subseteq A \sqcup A^{*\lambda I} = Cl^{*\lambda I}(A)$.

4- $Cl^{*\lambda I}(A \sqcup B) = (A \sqcup B) \sqcup (A \sqcup B)^{*\lambda I}$

$=(A \sqcup B) \sqcup (A^{*\lambda I} \sqcup B^{*\lambda I})$

$=(A \sqcup A^{*\lambda I}) \sqcup (B \sqcup B^{*\lambda I})$

$= Cl^{*\lambda I}(A) \sqcup Cl^{*\lambda I}(B)$.

5- $Cl^{*\lambda I}(A \sqcap B) = (A \sqcap B) \sqcup (A \sqcup B)^{*\lambda I}$ *, then by theorem(3-8)*

$(A \sqcap B) \sqcup (A \sqcup B)^{*\lambda I} \subseteq (A \sqcap B) \sqcup ((A^{*\lambda I} \sqcup B^{*\lambda I}) \subseteq (A \sqcup A^{*\lambda I}) \sqcap (B \sqcup B^{*\lambda I})$

$= Cl^{*\lambda I}(A) \sqcap (B)^{*\lambda I}$.

6- $Cl^{*\lambda I}(Cl^{*\lambda I}(A))$= $Cl^{*\lambda I}(A \sqcup A^{*\lambda I})$

$=(A \sqcup A^{*\lambda I}) \sqcup (A \sqcup A^{*\lambda I})^{*\lambda I}$

$= Cl^{*\lambda I}(A). \sqcup (A^{*\lambda I} \sqcup A^{*\lambda I})$

$= Cl^{*\lambda I}(A)$.

7- Let x $\in Cl^{*\lambda I}(A)$= $A \sqcup A^{*\lambda I} \subseteq Cl$(A)

8-11-Let x∈ $Cl^{*\lambda I}$(A), then x $\in A \sqcup A^{*\lambda I}$ by x is true space ,then $A^{*\lambda I} \subseteq A^{*}$and then

$A \sqcup A^{*\lambda I} \subset A \sqcup A^{*}$, then $Cl^{*\lambda I}(A) \subset Cl^{*}(A)$.

9- Let x$\in A^{*\lambda I}$, then U$\sqcap A \notin I, for\ each\ U is\ \lambda I - open\ set$, $then\ U \sqcap A \neq \Phi$, $for\ each\ U\ is\ \lambda I - open\ set$, then x $\in Cl(A^{*\lambda I})$.

10-Since I is condense set then $A^{*} \subset A^{*\lambda I}$,then A$\sqcup A^{*} \subset A \sqcup A^{*\lambda I}$,then $Cl^{*}(A) \subset Cl^{*\lambda I}(A)$.

11- Let x$\in A^{*\lambda I}$ then U$\sqcap A \notin I, for\ each\ U is\ \lambda I - open\ set$ if possible A$\notin Cl^{*\lambda I}(A)$, then x $\notin A \sqcup A^{*\lambda I}$,then x$\notin A\ and\ x \notin A^{*\lambda I}$ and this is contradiction x$\in A^{*\lambda I}$,then A$\subset Cl^{*\lambda I}(A)$.

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