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Solving N- dimensions Models of Nonlinear Optimization Problems

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Abstract*— In multivariable (large scale) problem of numerical optimization, the Lagrange multiplier method and KKT condition .They an iterative optimization procedure of the N- dimensions order. process for locating a function's local lowest and maximum values. the function can be minimized using this strategy in applied mathematics. this methods is typically taught at we implementation the new approach for Solving constraints (equality or inequality) numerical optimization. A solution's degree of goodness is determined by minimizing or optimizing an objective function (e.g., cost). when searching for a solution, constraints system model are utilized as a guide. instead, optimization aims to improve (or reduce) the value of the objective function while taking into consideration a variety of restrictions.*

Keywords— Lagrange multiplier method, KKT condition method ,python, Numerical Optimization.

1. INTRODUCTION

Optimizing processes is one of the most powerful approaches in process integration. "Best" is a term used in optimization to describe the most advantageous option among a set of feasible alternatives mathematical modeling and numerical simulation. A mathematical model is a representation of physical reality that can be analyzed and calculated. We can compute the using numerical simulation, [1,2,3] calculate a model's solution on a computer in order to make a virtual duplicate of physical reality. PDEs (partial differential equations) or multivariable differential equations will be our major modeling tool in this inquiry (time and space, for example). Applied mathematics has a third fundamental feature: the mathematical study of models. Mathematical analysis is a necessary step It is possible to get some severe shocks from numerical solutions to physical models. A detailed understanding of the underlying mathematical ideas is required to fully appreciate them. and Nonlinear problems and applications are the driving force behind applied mathematics. Difficulties that do not have any random or stochastic aspects. Finally, something must be done in order for this to work[5]. In our efforts to be simple and understandable, we may occasionally use ambiguous language. In our use of mathematics. We may ensure the more discerning reader that an example of modeling that leads to the equation for heat flow. Numerical algorithms must be utilized. This goal is to show and analyze several algorithms that help us to better understand the world around us. To tackle real-world problems, all of the algorithms covered here may be put to work computer-aided to specific optimization issues[10]. All of these algorithms are iterative in nature, beginning with a predetermined initial u_0condition. Each approach creates a sequence (U_n) n ∈N that converges under certain conditions, Optimization consists of an objective function that is a set of variables that reduce (maximize) and constraint set of variables that define the solution area through which the optimal solution is found. The optimization problems are multivariable each problem has special solutions, always numerical methods[12,15].

2. METHODOLOGY

In this section, we talk about how to solve both multivariable (large scale)using the Lagrange multiplier method and KKT condition of numerical optimization

2.1 MULTIVARIABLE (LARGE SCALE) PROBLEM OF NUMERICAL OPTIMIZATION

Multivariate optimization problems, there are many variables in an optimization problem that act as the decision variables

$$Z = f\,(x_1\,, x_2\,, x_3\,, \ldots, x_n)$$

So, like this kind of problem, the general function Z can be a nonlinear function of the decision variables (x_1, x_2, x_3 to x_n). Therefore, there are n variables in which one can improve this z function. We note that one can explain univariate optimization using two-dimensional images. We have the value of the decision variable, because in the x direction and in (y) direction, we have the function value [17]. However the, we have to use images in three dimensions if the optimization is multivariate, and more than 2 the decision variables , it is difficult to visualize. on the constraints depending , multivariate optimization divided into two parts [1, 20].

2.2 SOLVING MULTIVARIABLE OPTIMIZATION WITH INEQUALITY CONSTRAINTS

The general form:

$$\begin{array}{ll} \text{minimize (maximize)} & Z = f(x_i) = f(x_1, \dots, x_n) \\ \text{subject to} & h_j(x_i) \le 0 \quad (j = 1,2, \dots m) \\ & x_i > 0 \end{array}$$

One of the methods that solve multivariable optimization with inequality constraints are Lagrange multiplier method.

2.2.1 LAGRANGE MULTIPLIER METHOD:

The Lagrangian multiples method is a simple and elegant way to find the local minima or local extrema of a function subject to the equality or inequality constraints. Lagrangian multiples are also called indefinite multiples[21,25]. The general Lagrangian multiples method:

Minimize f(x)

Subject to: $g_i(x) \le 0$

The method of Lagrange multipliers first constructs a function called the Lagrange function as given by the following expression.

$$L(x, \lambda) = f(x_i) + \lambda_1 g_1(x) + \lambda_2 g_2(x) + \dots + \lambda_n g_n(x)$$

Example 1 : Solving the following minimization problem

$$\text{minimize } f(x_1, x_2, x_3) = x_1 (x_1 - 10) + x_2 (x_2 - 50) - 2x_3$$

$$\begin{array}{ll} \text{subject to} & x_1 + x_2 \le 10 \\ & x_3 \le 10 \end{array}$$

To convert them to equality constraint introduce new variables(s) corresponding equality constraints

$$L(x, \lambda, s) = f(x) + \lambda^T (h(x) + s$$

$$h(x) = \begin{pmatrix} x_1 + x_2 - 10 \\ x_3 - 10 \end{pmatrix} \qquad s = \begin{pmatrix} s_1^2 \\ s_2^2 \end{pmatrix}$$

$$\frac{\partial L}{\partial x_1} = 2x_1 - 10 + \lambda_1 = 0$$

$$\frac{\partial L}{\partial x_2} = 2x_2 - 50 + \lambda_1 = 0$$

$$\frac{\partial L}{\partial x_3} = -2 + \lambda_2 = 0$$

The Kuhn-Tucker conditions are $\lambda_1, \lambda_2 \ge 0$ and

$$\lambda_1 (x_1 + x_2 - 10 = 0$$

$$\lambda_2 (x_3 - 10) = 0$$

$$\lambda_2 = 2 \quad , \quad x_3 = 10$$

After substituting in the above equations

$$\lambda_1 = 20, \quad x_2 = 15 \quad , \quad x_1 \; - 5$$

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2.3 SOLVING MULTIVARIABLE OPTIMIZATION WITH EQUALITY CONSTRAINTS

The general form:

$$\text{minimize (maximize)} \quad Z=f(x_i)=f(x_1,\ldots,x_x)$$
$$\text{subject to} \quad h_j(x_i) = 0 \quad (j=1,2,\ldots m)$$

One of the methods that solve multivariable optimization with equality constraints are Karush-Kuhn-Tucker Conditions (KKT) method[27]. In order for the solution to be the optimal solution, we apply the KKT terms are first-order derived tests (necessary conditions). These conditions generalize the idea of Lagrangian multiples, as they allow not only equality constraints to be included, [3 , 7]. Given general problem

$$\text{minimize} \quad f(x_i)$$
$$\text{subject to} \quad g_j(x_i) \quad j = 1, \ldots, R \;\; , \;\; i = 1, \ldots M$$
$$x \in R^n$$

Now we will use Python code in KKT condition to find the optimal solution to constrained optimization problem by using KKT condition

$$\text{minimize} \quad f(x) = x_1 + x_2$$
$$\text{subject to} \;\; g(x) = x_1 + 2x_2 - 4$$
$$h(x) = 2x_1 + x_2 - 6$$
$$x \in R^n$$

```
from sympy import *
x1,x2,k,n = symbols('x1,x2,k,n')
f = x1+x2
g = x1+2*x2-4
h = 2*x1+x2-6
# I will construct the Lagrange equation
L = f+k*g+n*h
# Finding Derivatives, Building a KKT Case
dx1 = diff(L, x1) #Find the partial derivative of x1
print("dx1=",dx1)
dx2 = diff(L,x2)
#Find the partial derivative of x2
print("dx2=",dx2)
dk = diff(L,k) # partial derivative of k
print("dk=",dk)
dn = diff(L,n) # partial derivative of n
print("dn=",dn)
# Looking for a different solution
m= solve([dx1,dx2,dk,dn],[x1,x2,k,n])
print(m)
# Reset variables
x1=m[x1]
x2=m[x2]
k=m[k]
n=m[n]
# Calculate the value of the equation
f = x1+x2
print("The maximum value of the equation is:",f)
```

2.4 RESULTS AND DISCUSSION

In this article, we check how multivariable (large scale) problem of numerical optimization, and it is used in solving problems containing constrained optimization by using Lagrange multiplier method and KKT condition when solving problems that contain a objective function and constraint The results were (x_1 = -5 , x_2 = 15 , x_3 = 10 , λ_1 = 20 , $\lambda_2 = 2$) and {k = -1/3 , n = - 1/3 , $x_1 = \frac{8}{3}$, $x_2 = 2/3$ and the maximum value of the equation is 10/3.

3. CONCLUSION

In this study, we used two of the most important numerical optimization methods in applied mathematics, which is called the Lagrange multiplier method , using to solve multivariable (large scale) problem of numerical optimization that contain a objective function and inequality constraint .And KKT condition method using to solve multivariable (large scale) problem of numerical optimization that contain a objective function and equality constraint. In this study, we used a new approach to solve equality constrained and inequality constrained of numerical optimization problems in a more accurate way and in less time to find the optimal solution.

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