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# ζ -Dot Cubic AB-subalgebras of AB-algebra

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***Abstract**: The paper introduces the concepts of ζ-dot cubic AB-subalgebras in AB-algebras, and then explores their many features. They are defined, and both the image and inverse image of them in AB-algebras are investigated.*

**Keywords—AB-algebras, ζ -dot cubic AB-algebra, ζ -dot cubic AB-subalgebras, homomorphism of AB-algebra.**

## 1. INTRODUCTION

K. Is'eki and S. Tanaka [22] looked into the ideals and congruences of BCK-algebras. KUS-algebras are a novel type of algebraic structure that were introduced and explored by S. M. Mostafa and coworkers [26]. L.A. Zadeh [30] first proposed the idea of a fuzzy set. Using the fuzzy set notion, O.G. Xi [28] described some of the characteristics of BCK-algebras. The concept of cubic ideals in BCK-algebras was proposed by Y. B. Jun and coauthors [23], who went on to examine some of the features associated with these ideals. Cubic KUS-ideals of KUS-algebra were first described by A.T. Hameed et al. in [21], and their homomorphisms were subsequently investigated. The concept of cubic AT-ideals of AT-algebra was first presented and some of its features were described by A.T. Hameed and coauthors in [1]. Here, we shall define the notion of ζ -dot cubic of AB- subalgebra, and we study some of the relations, theorems, propositions and examples of ζ -dot cubic of AB- subalgebra of AB-algebra. We define and investigate the homomorphic and inverse images of AB-algebraic cubic AB-subalgebra

## 2. Preliminaries

In this section, we introduce the concepts of AB-ideals and fuzzy AB-ideals in AB-algebra and provide some definitions and preliminary properties of these concepts.

**Definition 2.1([2-4])** Let $X$ be a set with a binary operation $*$ and a constant 0. Then $(X;*,ℵ)$ is called **an AB-algebra** if the following axioms satisfied: for all m, y, x ∈X ,

(i) ((m *y) *(x *y)) *(m *x) = ℵ ,
(ii) ℵ *m = ℵ ,
(iii) m *ℵ = m,

**Example 2.2([2-4])** Let $X$ = {ℵ , 1, 2, 3, 4} in which ($*$) is defined by the following table:

| * | ℵ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| ℵ | ℵ | ℵ | ℵ | ℵ | ℵ |
| 1 | 1 | ℵ | 1 | ℵ | ℵ |
| 2 | 2 | 2 | ℵ | ℵ | ℵ |
| 3 | 3 | 3 | 1 | ℵ | ℵ |
| 4 | 4 | 3 | 4 | 3 | ℵ |

Then $(X;*,ℵ)$ is an AB-algebra.

**Remark 2.3([2-4])** Define a binary relation ≤ on AB-algebra $(X;*,ℵ)$ by letting $x \leq y$ if and only if $x * y = 0$.

**Proposition 2.4([2-4])** $In\ any\ AB-algebra$ $(X;*,ℵ)$, $the\ following\ properties\ hold$: for all $x, y, z \in X$,

(1) (m * y) * m = 0.
(2) (m * y) * x = (m * x) * y.
(3) (m * (m * y)) * y = 0.

**Proposition 2.5([2-4])** Let (X; *, ℵ ) be an AB-algebra. X is satisfies for all m, y, x ∈X,

(1) m ≤ y implies m*x ≤ y*x.
(2) m ≤ y implies x*y ≤ x*m.

**Definition 2.6([2-4]).** Let (X; *,ℵ ) be $an\ AB-algebra\ and\ let\ S\ be\ a\ nonempty\ subset\ of\ X.$ S is called an **AB-subalgebra of** X if m *y ∈ S whenever m ∈ S and y ∈ S.

**Definition 2.7([2-4]).** $A\ nonempty\ subset\ I\ of\ an\ AB-algebra$ (X; *,ℵ ) is called **an AB-ideal of X** if it satisfies the following conditions: for any m, y,x ∈ X,

($I_1$) ℵ ∈ I ,
($I_2$) (m * y)*x ∈ I and y ∈ I imply m*x ∈ I.

**Proposition 2.9 ([2-4]).** $Every\ AB-ideal\ of\ AB-algebra\ is\ an\ AB-subalgebra.$

**Proposition 2.8 ([2-4]).** Let {$I_i$ | i∈Λ} be a family of AB-ideals of AB-algebra (X; *, ℵ ). The intersection of $any\ set\ of\ AB-ideals\ of\ X\ is\ also\ an$ AB-ideal.

**Definition 2.9 ([13,14]).** Let (X ; *,ℵ ) and (Y; *`,ℵ `) $be\ nonempty\ sets. The\ mapping$ f:(X; *,ℵ ) → (Y; *`,ℵ `) is called **a homomorphism** if it satisfies: f (m*y) =f (m) *`f (y), for all m , y ∈ X. The set {m∈X | f (m) = ℵ '}is $called$ ***the kernel of f*** $denoted$ by ker f.

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**Theorem 2.10 ([2-4]).** Let f:(X; *,ℵ ) → (Y; *`,ℵ `) *be a homomorphism of an AB − algebra X into an AB* −algebra Y, then :

A. f (ℵ ) = ℵ '.

B. f is *injective* if and only if ker f ={ℵ }.

C. m ≤ y impliesf (m) ≤ f (y).

**Theorem 2.11 ([2-4]).** Let f:(X; *,ℵ ) → (Y; *`,ℵ `) *be a homomorphism of an AB − algebra X into an AB* −algebra Y, then:

($F_1$) If S is an AB-subalgebra of X, *then f (S) is an AB − subalgebra of* Y.

($F_2$) If I is an AB-ideal of X, then f (I) i*s an AB − ideal of Y , where f is onto*.

($F_3$) *If H is an AB − subalgebra of* Y, then$f^{-1}$ (H) is an AB-subalgebra of X .

($F_4$) If J is an AB-ideal of Y, then $f^{-1}$ (J) *s an AB − ideal of X* .

($F_5$) ker f is an AB-ideal of X.

($F_6$) Im(f) *is an AB − subalgebra of Y*.

**Definition 2.12([30]).** Let (X; *,ℵ ) *be a nonempty set, a fuzzy subset* μ of X is a function μ: X → [ℵ ,1].

**Definition 2.13 ([29]).**
*Let X be a nonempty set and μ be a fuzzy subset of* (X; *,ℵ ), for t ∈ [ℵ ,1] , the set $L(\mu, t) = \mu_t$ ={ m ∈ X | μ(m) ≥ t} *is called* ***a level subset of μ*** .

**Definition 2.14([5]).** Let $(X; *, ℵ)$ *be an AB − algebra, a fuzzy subset μ of X is called* ***a fuzzy AB − subalgebra*** **of *X*** if for all m, y ∈ X, μ (m*y) ≥ min {μ (m), μ (y)} .

**Definition 2.15([5]).** Let $(X; *, ℵ)$ be an AB-algebra, *a fuzzy subset μ of X is called* ***a fuzzy AB − ideal of X*** if it *satisfies the following conditions, for all m, y, z ∈ X*,

($FAB_1$) μ (ℵ ) ≥ μ (m) ,

($FAB_2$) μ (m*x) ≥ min {μ ((m∗y)*x), μ (y)} .

**Proposition 2.17([5]).**

1- *The intersection of any set of fuzzy AB − ideals of* AB-algebra is also fuzzy AB-ideal.

2- The union of any set of fuzzy AB-ideals of AB-algebra is also fuzz*y AB − ideal where is chain*.

**Proposition 2.18([5]).** E*very fuzzy AB − ideal of AB − algebra is a fuzz*y AB-subalgebra.

**Proposition 2.19([5]).**

1- Let μ be *a fuzzy subset of AB − algebra* $(X; *, ℵ)$ . If *μ is a fuzzy AB − subalgebra of X i*f and only if for every t ∈ [ℵ ,1], $\mu_t$ is an AB-subalgebra of *X* .

2- Let μ be a fuzzy AB-ideal of AB-algebra $(X;*, ℵ)$, μ is a fuzzy AB − ideal of X if and only if for every t ∈ [ℵ ,1], $\mu_t$ is an AB-ideal of *X* .

**Lemma 2.20([5]).** Let μ be a fuzzy AB-ideal of AB-algebra X and if m ≤ y , then μ(m) ≥ μ(y) , for all m, y∈ X.

**Definition 2.21 ([33]).** Let $f: (X; *, ℵ) \to (Y; *`, ℵ`)$be a mapping nonempty sets X and Y respectively. If μ is a fuzzy subset of *X*, then the fuzzy subset β of Y defined by:

$$f(\mu)(y) = \begin{cases} sup\{\mu(x): x \in f^{-1}(y)\} & if\ f^{-1}(y) = \{x \in X, f(x) = y\} \neq \emptyset \\ ℵ & otherwise \end{cases}$$

is said to be **the image of *μ* under *f*.**

Similarly if β is a fuzzy subset of Y , then the fuzzy subset $\mu = (\beta \circ f)$ of *X* ( i.e the fuzzy subset defined by μ (m) = β (f (m)), for all $x \in X$) is called **the pre-image of *β* under *f*** .

**Definition 2.22 ([29]).**
A fuzzy subset μ of a set X has sup property if for any subset T of X , there exist $t_0$ ∈T such that $\mu(t_ℵ) = sup\,\{\mu(t) | t \in T\}$.

**Proposition 2.23 ([5]).** Let $f: (X; *, ℵ) \to (Y; *`, ℵ`)$ be a homomorphism between AB − algebras *X* and *Y* respectively.

1− For every fuzzy AB − subalgebra *β* of *Y*, $f^{-1}(\beta)$ is a fuzzy AB − subalgebra of *X*.

2- For every fuzzy AB − subalgebra *μ* of *X* , $f(\mu)$ is a fuzzy AB − subalgebra of Y.

3- For every fuzzy AB − ideal β of *Y*, $f^{-1}(\beta)$ is a fuzzy AB − ideal of *X*.

4- For every fuzzy AB − ideal μ of X with sup property, $f(\mu)$ is a fuzzy AB − ideal of Y, where f is onto.

Now, we will recall the concept of interval-valued fuzzy subsets.

**Remark 2.24[1,8].** An interval number is $\tilde{a} = [a^-, a^+]$ , where ℵ $\le a^- \le a^+ \le 1$. Let I be a closed unit interval, (i.e., I = [ℵ , 1]) .
Let D[ℵ , 1] denote the family of all closed subintervals of I = [ℵ , 1] , that is, D[ℵ , 1] = { $\tilde{a} = [a^-, a^+] | a^- \le a^+$, for $a^-, a^+ \in$ I} .

Now, we define what is known as refined minimum (briefly, rmin) of two element in D[ℵ ,1].

**Definition 2.25[1,7].** We also define the symbols (≽) , (≼) , (=) , rmin and rmax in case of two elements in D[ℵ , 1] . Consider two interval numbers (elements numbers ) $\tilde{a} = [a^-, a^+]$ , $\tilde{b} = [b^-, b^+]$in D[ℵ , 1] : Then

(1) $\tilde{a} \succcurlyeq \tilde{b}$ if and only if, $a^- \ge b^-$ and $a^+ \ge b^+$,

(2) $\tilde{a} \preccurlyeq \tilde{b}$ if and only if, $a^- \le b^-$ and $a^+ \le b^+$,

(3) $\tilde{a} = \tilde{b}$ if and only if, $a^- = b^-$ and $a^+ = b^+$,

(4) rmin {$\tilde{a}$, $\tilde{b}$}= [min {$a^-$,$b^-$}, min {$a^+$,$b^+$}],

(5) rmax {$\tilde{a}$, $\tilde{b}$}= [max {$a^-$,$b^-$}, max {$a^+$,$b^+$}],

**Remark2.26[1,7].** It is obvious that (D[ℵ , 1] , ≼ , ∨ , ∧ ) is a complete lattice with $\widetilde{ℵ}$ = [ℵ ,ℵ ] as its least element and $\tilde{1}$ = [1, 1] a sits greatest element. Let $\tilde{a}_i \in$ D[ℵ , 1] where i∈Λ .We define $rinf_{i\in\Lambda}\tilde{a}$= [r $inf_{i\in\Lambda}a^-$ , $rinf_{i\in\Lambda}a^+$], $rsup_{i\in\Lambda}\tilde{a}$= [$rsup_{i\in\Lambda}a^-$ , $rsup_{i\in\Lambda}a^+$].

**Definition 2.27[1,7].** An **interval − valued fuzzy subset** $\tilde{\mu}_A$ **on X** is defined as $\tilde{\mu}_A$ ={< m, [$\mu_A^-$(m) ,$\mu_A^+$(m) ]>| m∈ *X*} .Where $\mu_A^-$(m) ≤$\mu_A^+$(m), for all m ∈ X. Then the ordinary fuzzy subsets$\mu_A^-: X \to$ [ℵ , 1] and $\mu_A^+: X \to$ [ℵ , 1] are called a **lower fuzzy subset and an**

**upper fuzzy subset** of $\tilde{\mu}_A$respectively. Let $\widetilde{\mu}_A$ (m) = $[\mu_A^-(m), \mu_A^+(m)]$,
$\widetilde{\mu}_A: X \to D[ℵ, 1]$, then A = {< m, $\widetilde{\mu}_A$ (m) >| m∈ $X$ } .

**Definition 2.28([1,7]).** Let $(X;*,ℵ)$ be a nonempty set. A cubic set Ω in a structure
$\Omega = \{< m, \tilde{\mu}_\Omega(m), \lambda_\Omega(m) >| m\in \}$,
which is briefly denoted by $\Omega = <\tilde{\mu}_\Omega, \lambda_\Omega>$, where $\tilde{\mu}_\Omega : X \to D[ℵ, 1]$, $\tilde{\mu}_\Omega$ is an interval − valued fuzzy subset of $X$ and
$\lambda_\Omega: X \to [ℵ, 1]$, $\lambda_\Omega$ is a fuzzy subset of X.

**Definition 2.29([1,7]).** For a family $\Omega_i = \{\langle m, \tilde{\mu}_{\Omega i}(m)\rangle | m \in X\}$ on fuzzy subsets of X, where i∈Λ and Λ is index set, we define the join (⋁)and meet (⋀) operations as follows:
$\bigvee_{i\in\Lambda} \Omega_i = (\bigvee_{i\in\Lambda} \tilde{\mu}_{\Omega i})(m) = \sup\{\tilde{\mu}_{\Omega i}(m) | i \in \Lambda\}$,
$\bigwedge_{i\in\Lambda} \Omega_i = (\bigwedge_{i\in\Lambda} \tilde{\mu}_{\Omega i})(m) = \inf\{\tilde{\mu}_{\Omega i}(m) | i \in \Lambda\}$,

## 3. ζ -Dot Cubic AB-subalgebras of AB-algebra

In this section, we will introduce a new notion called

cubic AB − subalgebras of AB-
algebra and study several properties of it.

**Definition 3.1[19].** Let $(X;*,ℵ)$ be an AB − algebra. A cubic set $\Omega = < \tilde{\mu}_\Omega(m), \lambda_\Omega(m) >$ of $X$ is called **cubic AB − subalgebra of X** if, for all m, y ∈X:
$\tilde{\mu}_\Omega(m*y) \geqslant \mathrm{rmin}\{\tilde{\mu}_\Omega(m), \tilde{\mu}_\Omega(y)\}$, and $\lambda_\Omega(m*y) \le \max\{\lambda_\Omega(m), \lambda_\Omega(y)\}$.

**Definition 3.2.** Let $(X;*,ℵ)$ be an AB − algebra. A cubic set $\Omega^\zeta = < \tilde{\mu}_\Omega^\zeta(m), \lambda_\Omega^\zeta(m) >$ of $X$ is called **ζ -dot cubic AB-subalgebra of *X*** if ζ ∈ (ℵ, 1] , for all m∈X, $\tilde{\mu}_\Omega^\zeta = \tilde{\mu}_\Omega(x) \cdot \zeta$ and $\lambda_\Omega^\zeta = \lambda_\Omega(x) \cdot \zeta$.

**Example 3.3.** Let $X$ = {ℵ ,1,2,3} in which the operation as in example ∗
be define by the following table:

| ∗ | ℵ | 1 | 2 | 3 |
|---|---|---|---|---|
| ℵ | ℵ | ℵ | ℵ | ℵ |
| 1 | 1 | ℵ | ℵ | ℵ |
| 2 | 2 | 2 | ℵ | ℵ |
| 3 | 3 | 3 | 3 | ℵ |

Then $(X;*,ℵ)$ is an AB − algebra. Define a cubic set $\Omega = < \tilde{\mu}_\Omega(m), \lambda_\Omega(m) >$ of $X$ is fuzzy subset $\mu: X \to [ℵ,1]$ by:

$\tilde{\mu}_\Omega(m) = \begin{cases}[0.3,0.9] & if x = \{ℵ,1\}\\ [0.1,0.6] & otherwise\end{cases}$ and $\lambda_\Omega = \begin{cases}0.1 & if x = \{ℵ,1\}\\ 0.6 & otherwise\end{cases}$.

Define a cubic set $\Omega^\zeta = < \tilde{\mu}_\Omega^\zeta(m), \lambda_\Omega^\zeta(m) >$ of $X$ and $\zeta = 0.5$ as follows:

$\tilde{\mu}_\Omega^\zeta(m) = \begin{cases}[0.15,0.45] & if x = \{ℵ,1\}\\ [0.05,0.3] & otherwise\end{cases}$ and $\lambda_\Omega^\zeta = \begin{cases}0.05 & if x = \{ℵ,1\}\\ 0.3 & otherwise\end{cases}$.

The ζ -dot cubic set $\Omega^\zeta = < \tilde{\mu}_\Omega^\zeta(m), \lambda_\Omega^\zeta(m) >$is a ζ -dot cubic AB − subalgebra of $X$.

**Remark 3.4.** Let $(X;*,ℵ)$ be an AB − algebra, then it is clearly that

$\Omega^1 = < \tilde{\mu}_\Omega^1(m), \lambda_\Omega^1(m) > = \Omega = < \tilde{\mu}_\Omega(m), \lambda_\Omega(m) >$

**Proposition 3.5.** Let $(X;*,ℵ)$ be an AB-algebra and $\Omega = < \tilde{\mu}_\Omega(m), \lambda_\Omega(m) >$ is a cubic AB − subalgebra of X such that $\Omega^\zeta = < \tilde{\mu}_\Omega^\zeta(m), \lambda_\Omega^\zeta(m) >$ is ζ -dot cubic AB − subalgebra of X, where ζ ∈ (ℵ, 1], then for all $x, y \in X$,
$\tilde{\mu}_\Omega(m*y)\cdot \zeta \geqslant rmin\{\tilde{\mu}_\Omega(m), \tilde{\mu}_\Omega(y)\}\cdot \zeta$, and
$\lambda_\Omega(m*y)\cdot\zeta \le \max\{\lambda_\Omega(m), \lambda_\Omega(y)\}\cdot\zeta$ .

**Proof.** For all $x, y \in X$ , we have
$\tilde{\mu}_\Omega^\zeta(m*y) = \tilde{\mu}_\Omega(m*y)\cdot\zeta \geqslant \mathrm{rmin}\{\tilde{\mu}_\Omega(m), \tilde{\mu}_\Omega(y)\}\cdot\zeta$,
$= \mathrm{rmin}\{\tilde{\mu}_\Omega(m)\cdot\zeta, \tilde{\mu}_\Omega(y)\cdot\zeta\}$
$= \mathrm{rmin}\{\tilde{\mu}_\Omega^\zeta(m), \tilde{\mu}_\Omega^\zeta(y)\}$
and
$\lambda_\Omega^\zeta(m*y) = \lambda_\Omega(m*y)\cdot\zeta \le \max\{\lambda_\Omega(m), \lambda_\Omega(y)\}\cdot\zeta$
$= \max\{\lambda_\Omega(m)\cdot\zeta, \lambda_\Omega(y)\cdot\zeta\}$
$= \max\{\lambda_\Omega^\zeta(m), \lambda_\Omega^\zeta(y)\}$. ⌂

It is clear that ζ − dot cubic AB − subalgebra of an AB-algebra $(X;*,ℵ)$ is a generalization of a cubic AB-subalgebra of X and a cubic AB-subalgebra of X is special case, when ζ = 1.

**Proposition 3.6.** Let $\Omega^\zeta = < \tilde{\mu}_\Omega^\zeta(m), \lambda_\Omega^\zeta(m) >$be a ζ -dot cubic AB − subalgebra of AB-algebra $(X;*,ℵ)$, then $\tilde{\mu}_\Omega^\zeta(ℵ) \geqslant \tilde{\mu}_\Omega^\zeta(m)$ and $\lambda_\Omega^\zeta(ℵ) \le \lambda_\Omega^\zeta(m)$, for all

$m \in X$ .

**Proof.** For all $m \in X$ , we have
$\tilde{\mu}_\Omega^\zeta(ℵ) = \tilde{\mu}_\Omega(ℵ * m)\cdot\zeta$
$\geqslant rmin\{\tilde{\mu}_\Omega^\zeta((ℵ * m) * ℵ), \tilde{\mu}_\Omega(m)\}\cdot\zeta$
$= rmin\{[\mu_A^-((ℵ * m) * ℵ), \mu_A^-(m)], [\mu_A^+((ℵ * m) * ℵ), \mu_A^+(m)]\}\cdot\zeta$
$= rmin\{[\mu_A^-(ℵ), \mu_A^-(m)], [\mu_A^+(ℵ), \mu_A^+(m)]\}\cdot\zeta$
$= [\mu_A^-(m), \mu_A^+(m)]\cdot\zeta$
$= \tilde{\mu}_\Omega(m)\cdot\zeta$
$= \tilde{\mu}_\Omega^\zeta(m)$ .

Similarly, we can show that
$\lambda_\Omega^\zeta(ℵ) \le max\{[\lambda_\Omega^\zeta(ℵ), \lambda_\Omega^\zeta(m)]\} = \lambda_\Omega^\zeta(m)$ . ⌂

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**Proposition 3.7.** If a ζ -dot cubic set $\Omega^{\zeta} =< \tilde{\mu}^{\zeta}_{\Omega}(m), \lambda^{\zeta}_{\Omega}(m) >$ of AB-algebra $(X;*,\aleph)$ is a ζ -dot cubic AB-subalgebra, then $\Omega^{\zeta}(m*y) = \Omega^{\zeta}(m*((y*\aleph)*\aleph))$, for all $m, y \in X$.

**Proof.**

Let X be an AB − algebra and $, y \in X$, then we know that $y = (y*\aleph)*0$. Hence, $\tilde{\mu}^{\zeta}_{\Omega}(m*y)=\tilde{\mu}^{\zeta}_{\Omega}(m*((y*\aleph)*\aleph))$ and $\lambda^{\zeta}_{\Omega}(m*y)=\lambda^{\zeta}_{\Omega}(m*((y*\aleph)*\aleph))$. Therefore $\Omega^{\zeta}(m*y) = \Omega^{\zeta}(m*((y*\aleph)*\aleph))$. ⌂

**Proposition 3.8.**

Let $(X;*,\aleph)$ be an AB − algebra and $\Omega =< \tilde{\mu}_{\Omega}(m), \lambda_{\Omega}(m) >$ is a cubic subset of X such that $\Omega^{\zeta} =< \tilde{\mu}^{\zeta}_{\Omega}(m), \lambda^{\zeta}_{\Omega}(m) >$ is ζ -dot cubic AB − subalgebra of , for some $\zeta \in [\aleph, 1]$, then $\Omega =< \tilde{\mu}_{\Omega}(m), \lambda_{\Omega}(m) >$ is a cubic AB − subalgebra of $X$.

**Proof.**

Assume that $\Omega^{\zeta} =< \tilde{\mu}^{\zeta}_{\Omega}(m), \lambda^{\zeta}_{\Omega}(m) >$ is a ζ − dot cubic AB − subalgebra of X for some $\zeta \in (\aleph, 1]$. Let $m, y \in X$, then

$$\begin{aligned}
\tilde{\mu}_{\Omega}(m*y)\cdot\zeta &= \tilde{\mu}^{\zeta}_{\Omega}(m*y) \\
&\succcurlyeq min\{\tilde{\mu}^{\zeta}_{\Omega}(m), \tilde{\mu}^{\zeta}_{\Omega}(y)\} \\
&= min\{\tilde{\mu}_{\Omega}(m)\cdot\zeta, \tilde{\mu}_{\Omega}(y)\cdot\zeta\} \\
&= min\{\tilde{\mu}_{\Omega}(m), \tilde{\mu}_{\Omega}(y)\}\cdot\zeta.
\end{aligned}$$

$\tilde{\mu}_{\Omega}(m*y) \geq min\{\tilde{\mu}_{\Omega}(m), \tilde{\mu}_{\Omega}(y)\}$ and so

$$\begin{aligned}
\lambda_{\Omega}(m*y)\cdot\zeta &= \lambda^{\zeta}_{\Omega}(m*y) \\
&\leq max\{\lambda^{\zeta}_{\Omega}(m), \lambda^{\zeta}_{\Omega}(y)\} \\
&= max\{\lambda_{\Omega}(m)\cdot\zeta, \lambda_{\Omega}(y)\cdot\zeta\} \\
&= max\{\lambda_{\Omega}(m), \mu(y)\}\cdot\zeta.
\end{aligned}$$

$\lambda_{\Omega}(x*y) \leq max\{\lambda_{\Omega}(m), \lambda_{\Omega}(y)\}$

Hence $\Omega =< \tilde{\mu}_{\Omega}(m), \lambda_{\Omega}(m) >$ is a cubic AB − subalgebra of $X$. □

**Proposition 3.9.**

Let $(X;*,\aleph)$ be an AB − algebra and $\Omega =< \tilde{\mu}_{\Omega}(m), \lambda_{\Omega}(m) >$ is a cubic subset of X such that $\Omega^{\zeta} =< \tilde{\mu}^{\zeta}_{\Omega}(m), \lambda^{\zeta}_{\Omega}(m) >$ is ζ − dot cubic AB − subalgebra of , for some $\zeta \in [\aleph, 1]$, then then the cubic Ω of $X$ is a fuzzy S − extension of the ζ -dot cubic $\Omega^{\zeta}$ of $X$.

**Proof:**

Since $\tilde{\mu}_{\Omega}(m) \succcurlyeq \mu(m).\zeta = \tilde{\mu}^{\zeta}_{\Omega}(m)$, and $\lambda_{\Omega}(m) \geq \lambda_{\Omega}(m).\zeta = \lambda^{\zeta}_{\Omega}(m)$ then Ω(m) is a fuzzy S-extension of $\Omega^{\zeta}(m)$, for all m∊ X and since Ω is a fuzzy AB − subalgebra of X, then$\Omega^{\zeta}$ of μ is a ζ -dot cubic AB − subalgebra, by Proposition (3.8) . □

**Definition 3.10[19].**

For a fuzzy subset μ of an AB − algebra $(X;*,\aleph)$, $\zeta \in (\aleph, 1]$, $\tilde{t} \in D[\aleph, 1]$ and $s \in [\aleph, 1]$, with $t \leq \zeta$, let $\widetilde{U}(\Omega;\tilde{t},s) = \{ m \in X \mid \tilde{\mu}_{\Omega}(m) \succcurlyeq \tilde{t}, \lambda_{\Omega}(m) \leq s\}$.

**Proposition 3.11.**

Let $(X;*,\aleph)$ be an AB − algebra. A ζ -dot cubic subset $\Omega^{\zeta} =< \tilde{\mu}^{\zeta}_{\Omega}(m), \lambda^{\zeta}_{\Omega}(m) >$ of . If $\Omega^{\zeta}$ is a ζ -dot cubic AB − subalgebra of X, then for all $\zeta \in (\aleph, 1]$, $\tilde{t} \in D[\aleph, 1]$ and $s \in [\aleph, 1]$, with $t \leq \zeta$, then the set $\widetilde{U}(\Omega;\tilde{t},s)$ is an AB-subalgebra of $X$.

**Proof.**

Assume that $\Omega^{\zeta} =< \tilde{\mu}^{\zeta}_{\Omega}(m), \lambda^{\zeta}_{\Omega}(m) >$ is a ζ −dot cubic AB − subalgebra of $X$ and let $\tilde{t} \in D[\aleph, 1]$ and $s \in [\aleph, 1]$, be such that $\widetilde{U}(\Omega;\tilde{t},s) \neq \emptyset$.

Let $x, y \in X$ such that $, y \in \widetilde{U}(\Omega;\tilde{t},s)$, then $\tilde{\mu}^{\zeta}_{\Omega}(m) \succcurlyeq \tilde{t}$, $\tilde{\mu}^{\zeta}_{\Omega}(y) \succcurlyeq \tilde{t}$ and $\lambda^{\zeta}_{\Omega}(m) \leq s$, $\lambda^{\zeta}_{\Omega}(y) \leq s$. Since $\Omega^{\zeta}$ is a ζ -dot cubic AB − subalgebra of X, we get $\tilde{\mu}^{\zeta}_{\Omega}(m*y) \succcurlyeq rmin\{\tilde{\mu}^{\zeta}_{\Omega}(m), \tilde{\mu}^{\zeta}_{\Omega}(y)\} \succcurlyeq \tilde{t}$ and $\lambda^{\zeta}_{\Omega}(m*y) \leq max\{\lambda^{\zeta}_{\Omega}(m), \lambda^{\zeta}_{\Omega}(y)\} \leq s$.

Hence the set $\widetilde{U}(\Omega;\tilde{t},s)$is an AB − subalgebra of $X$. ⌂

**Proposition 3.12.** Let $(X;*,\aleph)$be an AB − algebra. A ζ − dot cubic subset $\Omega^{\zeta} =< \tilde{\mu}^{\zeta}_{\Omega}(m), \lambda^{\zeta}_{\Omega}(m) >$ of . If the set $\widetilde{U}(\Omega;\tilde{t},s)$ is an AB − subalgebra of $X$, for all $\zeta \in (\aleph, 1]$, $\tilde{t} \in D[\aleph, 1]$ and $s \in [\aleph, 1]$, with $t \leq \zeta$, then $\Omega^{\zeta}$ is a ζ −dot cubic AB − subalgebra of X.

**Proof.**

Suppose that $\widetilde{U}(\Omega;\tilde{t},s)$is an AB − subalgebra of X and let $x, y \in X$ be such that $\tilde{\mu}^{\zeta}_{\Omega}(m*y) \prec rmin\{\tilde{\mu}^{\zeta}_{\Omega}(m), \tilde{\mu}^{\zeta}_{\Omega}(y)\}$ and $\lambda^{\zeta}_{\Omega}(m*y) > max\{\lambda^{\zeta}_{\Omega}(m), \lambda^{\zeta}_{\Omega}(y)\}$.

Consider $\tilde{\zeta} = 1/2\ \{\tilde{\mu}^{\zeta}_{\Omega}(m*y) + rmin\{\tilde{\mu}^{\zeta}_{\Omega}(m), \tilde{\mu}^{\zeta}_{\Omega}(y)\}\}$ and

$$\zeta = 1/2\ \{\lambda^{\zeta}_{\Omega}(m*y) + max\{\lambda^{\zeta}_{\Omega}(m), \lambda^{\zeta}_{\Omega}(y)\}\}.$$

We have $\tilde{\zeta} \in D[\aleph, 1]$ and $\zeta \in (\aleph, 1]$, and $\tilde{\mu}^{\zeta}_{\Omega}(m*y) \prec \tilde{\zeta} \prec rmin\{\tilde{\mu}^{\zeta}_{\Omega}(m), \tilde{\mu}^{\zeta}_{\Omega}(y)\}$, and $\lambda^{\zeta}_{\Omega}(m*y) > \zeta > max\{\lambda^{\zeta}_{\Omega}(m), \lambda^{\zeta}_{\Omega}(y)\}$.

It follows that $x, y \in \widetilde{U}(\Omega;\tilde{t},s)$, and $(m*y) \notin \widetilde{U}(\Omega;\tilde{t},s)$. This is a contradiction and therefore $\Omega^{\zeta}$ is a ζ -dot cubic AB-subalgebra of $X$. ⌂

**Theorem 3.13.** Let $(X;*,\aleph)$ be an AB − algebra. A ζ -dot cubic subset

$\Omega^{\zeta} =< \tilde{\mu}^{\zeta}_{\Omega}(m), \lambda^{\zeta}_{\Omega}(m) >$ of $X$ is a ζ −dot cubic AB − subalgebra of X if and only if, $\mu^{-\zeta}_{\Omega}$, and $\mu^{+\zeta}_{\Omega}$ are fuzzy AB − subalgebras of $X$ and $\lambda^{\zeta}_{\Omega}$ are anti − fuzzy AB − subalgebra of X.

**Proof**. Let $\mu^{-\zeta}_{\Omega}, \mu^{+\zeta}_{\Omega}$ and $\lambda^{\zeta}_{\Omega}$ be fuzzy AB − subalgebras of $X$ and $x, y \in X$, then $\mu^{-\zeta}_{\Omega}(m*y) \geq min\{\mu^{-\zeta}_{\Omega}(m), \mu^{-\zeta}_{\Omega}(y)\}$, $\mu^{+\zeta}_{\Omega}(m*y) \geq min\{\mu^{+\zeta}_{\Omega}(m), \mu^{+\zeta}_{\Omega}(y)\}$and

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$\lambda_\Omega^\zeta(m * y) \le \max\{\lambda_\Omega^\zeta(m), \lambda_\Omega^\zeta(y)\}$. Now,

$\tilde{\mu}_\Omega^\zeta(m * y) = [\mu^{-\zeta}_\Omega(m * y), \mu^{+\zeta}_\Omega(m * y)]$

$\succcurlyeq [\min\{\mu^{-\zeta}_\Omega(m), \mu^{-\zeta}_\Omega(y)\}, \min\{\mu^{+\zeta}_\Omega(m), \mu^{+\zeta}_\Omega(y)\}]$

$= \mathrm{rmin}\{[\mu^{-\zeta}_\Omega m), \mu^{+\zeta}_\Omega(m)], [\mu^{-\zeta}_\Omega(y), \mu^{+\zeta}_\Omega(y)]\}$

$= \mathrm{rmin}\{\tilde{\mu}_\Omega^\zeta(m), \tilde{\mu}_\Omega^\zeta(y)\}$,

therefore $\Omega$ is a $\zeta$-dot cubic AB $-$subalgebra of $X$.

Conversely, assume that $\Omega^\zeta$ is a $\zeta$ $-$dot cubic AB $-$ subalgebra of $X$, for any $x, y \in X$,

$[\mu^{-\zeta}_\Omega(m * y), \mu^{+\zeta}_\Omega(m * y)] = \tilde{\mu}_\Omega^\zeta(m * y) \succcurlyeq \mathrm{rmin}\{\tilde{\mu}_\Omega^\zeta(m), \tilde{\mu}_\Omega^\zeta(y)\}$

$= \mathrm{rmin}\{[\mu^{-\zeta}_\Omega(m), \mu^{+\zeta}_\Omega(m)], [\mu^{-\zeta}_\Omega(y), \mu^{+\zeta}_\Omega(y)]\}$

$= [\min\{\mu^{-\zeta}_\Omega(m), \mu^{-}_\Omega(m), \min\{\mu^{+\zeta}_\Omega(y), \mu^{+\zeta}_\Omega(y)\}]$.

Thus

$\mu^{-\zeta}_\Omega(m * y) \ge \min\{\mu^{-\zeta}_\Omega(m), \mu^{-\zeta}_\Omega(m)\}, \mu^{+\zeta}_\Omega(m * y)$

$\ge \min\{\mu^{+\zeta}_\Omega(m), \mu^{+\zeta}_\Omega(m)\}$ and

$\lambda_\Omega^\zeta(m * y) \le \max\{\lambda_\Omega^\zeta(m), \lambda_\Omega^\zeta(y)\}$,

Therefore, $\mu^{-\zeta}_\Omega$ and $\mu^{+\zeta}_\Omega$ are fuzzy AB $-$ subalgebras of X and $\lambda_\Omega^\zeta$ are anti $-$ fuzzy AB $-$ subalgebra of $X$. □

**Proposition 3.14.**

Let $(X; *, ℵ)$ be an AB $-$ algebra and $\Omega = <\tilde{\mu}_\Omega(m), \lambda_\Omega(m)>$ is a cubic AB $-$ subalgebra of X and $\zeta_1, \zeta_2 \in (ℵ, 1]$. If $\zeta_2 \ge \zeta_1$, then the $\zeta$ $-$dot cubic AB $-$ subalgebra $\Omega^{\zeta_2}$ is a fuzzy S $-$ extension of the $\Omega^{\zeta_1}$ $\zeta$-dot cubic AB $-$ subalgebra of $X$.

**Proof:** For every $m \in X$ and $\zeta_1, \zeta_2 \in (ℵ, 1]$ and $\zeta_2 \ge \zeta_1$, we have

$\tilde{\mu}_\Omega^{\zeta_2}(m) = \tilde{\mu}_\Omega(m).\zeta_2 \succcurlyeq \tilde{\mu}_\Omega(m).\zeta_1 = \tilde{\mu}_\Omega^{\zeta_1}(m)$, and

$\lambda_\Omega^{\zeta_2}(m) = \lambda_\Omega(m).\zeta_2 \ge \lambda_\Omega(m).\zeta_1 = \lambda_\Omega^{\zeta_1}(m)$, then

$\tilde{\mu}_\Omega^{\zeta_2}(m) \succcurlyeq \tilde{\mu}_\Omega^{\zeta_1}(m)$, and $\lambda_\Omega^{\zeta_2}(m) \ge \lambda_\Omega^{\zeta_1}(m)$, therefore $\Omega^{\zeta_2}$ is a fuzzy S-extension of $\Omega^{\zeta_1}$.

Since $\Omega$ is a cubic AB $-$ subalgebra of $X$, then $\Omega^\zeta$ is a $\zeta$-dot cubic AB $-$ subalgebra of $\mu$, by Proposition (3.8). Hence $\Omega^{\zeta_2}$ of $X$ is a fuzzy S$-$extension of the $\zeta$ $-$ dot cubic AB $-$subalgebra $\Omega^{\zeta_1}$ of $X$. □

## 4. Homomorphism of ζ -Dot Cubic AB-ideals (AB-subalgebras) of AB-algebra

*In this section*, we will present *some results on images* and preimages of $\zeta$-*dot cubic AB $-$ ideals of AB $-$ algebras*.

**Definition 4.1[3].**

Let $: (X; *, ℵ) \to (Y; *', ℵ')$ *be a mapping from the set* $X$ to a set Y. If $\Omega^\zeta = <\tilde{\mu}_\Omega^\zeta(m), \lambda_\Omega^\zeta(m)>$ is a $\zeta$-*dot cubic subset of* X, *then the cubic subset* $\beta = <\tilde{\mu}_\beta, \lambda_\beta>$ of $Y$ defined by:

$f(\tilde{\mu}_\Omega^\zeta)(y)$

$$= \begin{cases} \underset{m \in f^{-1}(y)}{rsup} \tilde{\mu}_\Omega^\zeta(x) \, if f^{-1}(y) = \{m \in X, f(m) = y\} \ne \varphi \\ ℵ \quad otherwise \end{cases}$$

$f(\lambda_\Omega^\zeta)(y)$

$$= \begin{cases} \underset{m \in f^{-1}(y)}{inf} \lambda_\Omega^\zeta(m) \, if f^{-1}(y) = \{m \in X, f(m) = y\} \ne \varphi \\ 1 \quad otherwise \end{cases}$$

is said to be **the image of Ω under f.**

Similarly if $\beta^\zeta = <\tilde{\mu}_\beta^\zeta(m), \lambda_\beta^\zeta(m)>$ is a $\zeta$-dot cubic subset of Y, then the cubic subset $\Omega^\zeta = (\beta^\zeta \circ f)$ in X ( i.e the $\zeta$-dot cubic subset defined by $\tilde{\mu}_\Omega^\zeta(m) = \tilde{\mu}_\beta^\zeta(f(m))$, $\lambda_\Omega^\zeta(m) = \lambda_\beta^\zeta(f(m))$ for all $m \in X$) is called **the preimage of β under *f***).

**Theorem4.2.** A homomorphic preimage of $\zeta$-dot cubic AB $-$ subalgebra is also

$\zeta$-dot cubic AB-subalgebra.

**Proof.** Let $f: (X; *, ℵ) \to (Y; *', ℵ')$ *be homomorphism from an* AB-algebra $X$ into an AB-algebra $Y$.

If $\beta^\zeta = <\tilde{\mu}_\beta^\zeta(m), \lambda_\beta^\zeta(m)>$ is a cubic AB-subalgebra of Y and $\Omega^\zeta = <\tilde{\mu}_\Omega^\zeta(m), \lambda_\Omega^\zeta(m)>$ the preimage of $\beta^\zeta$ under f, then $\tilde{\mu}_\Omega^\zeta(m) = \tilde{\mu}_\beta^\zeta(f(m))$, $\lambda_\Omega^\zeta(m) = \lambda_\beta^\zeta(f(m))$, for all $m \in X$.

Let $m \in X$, then

$(\tilde{\mu}_\Omega^\zeta)(ℵ) = \tilde{\mu}_\beta^\zeta(f(ℵ)) \succcurlyeq \tilde{\mu}_\beta^\zeta(f(m)) = \tilde{\mu}_\Omega^\zeta(m)$, and

$(\lambda_\Omega^\zeta)(ℵ) = \lambda_\beta^\zeta(f(ℵ)) \le \lambda_\beta^\zeta(f(m)) = \lambda_\Omega^\zeta(m)$.

Now, let $x, y \in X$, then

$\tilde{\mu}_\Omega^\zeta(m*y) = \tilde{\mu}_\beta^\zeta(f(m*y)) = \tilde{\mu}_\beta^\zeta(f(m) *' f(y))$

$\succcurlyeq \mathrm{rmin}\{\tilde{\mu}_\beta^\zeta(f(m), \tilde{\mu}_\beta^\zeta(f(y))\}$

$= \mathrm{rmin}\{\tilde{\mu}_\Omega^\zeta(m), \tilde{\mu}_\Omega^\zeta(y)\}$, and

$\lambda_\Omega^\zeta(m*y) = \lambda_\beta^\zeta(f(m*y)) = \lambda_\beta^\zeta(f(m) *' f(y))$

$\le \max\{\lambda_\beta^\zeta(f(m), \lambda_\beta^\zeta(f(y))\}$

$= \max\{\lambda_\Omega^\zeta(m), \lambda_\Omega^\zeta(y)\}$. □

**Definition 4.3.** Let $f: (X; *, ℵ) \to (Y; *', ℵ')$ be a mapping from a set X into a set Y. $\Omega^\zeta = <\tilde{\mu}_\Omega^\zeta(m), \lambda_\Omega^\zeta(m)>$ is a $\zeta$-dot cubic subset of $X$ **has sup and inf properties** if for any subset T of $X$, there exist t, s ∈ T such that

$$\tilde{\mu}_\Omega^\zeta(t) = \underset{t0 \in T}{rsup} \tilde{\mu}_\Omega^\zeta(t0) \text{ and } \lambda_\Omega^\zeta(s) = \underset{s0 \in T}{inf} \lambda_\Omega^\zeta(s0).$$

**Theorem 4.4.** Let $f: (X; *, ℵ) \to (Y; *', ℵ')$ be an epimorphism from an AB $-$ algebra X into an AB-algebra Y. For every $\zeta$-dot cubic AB $-$ subalgebra

$\Omega^{\zeta} =< \tilde{\mu}^{\zeta}_{\Omega}(m), \lambda^{\zeta}_{\Omega}(m) >$ of with **sup and inf properties**, then $f(\Omega^{\zeta})$ is a

$\zeta$-dot cubic AB − subalgebra of $Y$.

**Proof.** By definition $\tilde{\mu}^{\zeta}_{\beta}(y') = f(\tilde{\mu}^{\zeta}_{\Omega})(y') = \underset{m \in f^{-1}(y')}{rsup} \tilde{\mu}^{\zeta}_{\Omega}(m)$ and

$\lambda^{\zeta}_{\beta}(y') = f(\lambda^{\zeta}_{\Omega})(y') = \underset{x \in f^{-1}(y')}{inf} \lambda^{\zeta}_{\Omega}(m)$ for all y' ∈ Y and

rsup(∅) = [ℵ , ℵ ] and inf (∅) = ℵ .

We have prove that

$\tilde{\mu}^{\zeta}_{\beta}$ (m'∗y') ≽ rmin {$\tilde{\mu}^{\zeta}_{\beta}$ (m'), $\tilde{\mu}^{\zeta}_{\beta}$ (y' )}, and

$\lambda^{\zeta}_{\beta}$ (m'∗y') ≤ max{$\lambda^{\zeta}_{\beta}$ (m'), $\lambda^{\zeta}_{\beta}$ (y' )}, for all m', y' ∈ Y .

$$\tilde{\mu}^{\zeta}_{\beta}(m' * y') = \underset{t \in f^{-1}(x' * y')}{rsup} \tilde{\mu}^{\zeta}_{\Omega}(t) = \tilde{\mu}^{\zeta}_{\Omega}(x_{\aleph} * y_{\aleph})$$
$$\succcurlyeq rmin\{\tilde{\mu}^{\zeta}_{\Omega}(x_{\aleph}), \tilde{\mu}^{\zeta}_{\Omega}(y_{\aleph})\},$$
$$\succcurlyeq rmin\{\underset{t \in f^{-1}(x')}{rsup} \tilde{\mu}^{\zeta}_{\Omega}(t), \underset{t \in f^{-1}(y')}{rsup} \tilde{\mu}^{\zeta}_{\Omega}(t)\},$$
$$= \text{rmin}\{\tilde{\mu}^{\zeta}_{\beta}(m'), \tilde{\mu}^{\zeta}_{\beta}(y')\} \text{ and}$$

$$\lambda^{\zeta}_{\Omega}(m' * y') = \underset{t \in f^{-1}(x' * y')}{inf} \lambda^{\zeta t}_{\Omega}(t)$$
$$\leq \max\{\lambda^{\zeta}_{\Omega}(m_{\aleph}), \lambda^{\zeta}_{\Omega}(y_{\aleph})\}$$
$$= \max\{\underset{t \in f^{-1}(x')}{inf} \lambda^{\zeta}_{\Omega}(t), \underset{t \in f^{-1}(y')}{inf} \lambda^{\zeta}_{\Omega}(t)\}$$

Hence, $\Omega^{\zeta} =< \tilde{\mu}^{\zeta}_{\Omega}(m), \lambda^{\zeta}_{\Omega}(m) >$ is a $\zeta$-dot cubic AB − subalgebra of . □

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