International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 7 Issue 3, March - 2023, Pages: 8-12

Indigraphic Space and Related Some Properties

Amer Abdullah Amer[1] and Kais Hassan Kulaif[2]

[1]*Directorate* of *Education* in *Babylon,* Iraq , Babylon
rtu85269@gmail.com
[2]*Directorate* of *Education* in *Babylon* , Iraq , Babylon
Kaiskais8282@gmail.com

Abstract: *In this paper, we study the indigraphic topology $\tau_{I Ɗ}$ for a directed edges of a digraph. We give some properties of this topology, in particular we prove that $\tau_{I Ɗ}$ is an Alexandroff topology and when two digraphs are isomorphic, their indigraphic topologies will be homeomorphic. We give some properties matching digraphs and homeomorphic topology spaces. Finally, we investigate the connectedness of this topology and some relations between the connectedness of the digraph and the topology $\tau_{I Ɗ}$.*

Keywords— Digraph, Topology, Alexandroff Topology, Incident Edges System**.**

1.Introduction:

Topological structures are mathematical models that can be used to analyze data without the concept of distance. Topological structures, in our opinion, are a crucial adjustment for the extraction and processing of knowledge [2]. This publication provides a few topological fundamentals that are pertinent to our study. One of the most crucial structures in discrete mathematics is the graph [1]. Two observations explain their pervasiveness. Graphs are mathematically elegant, to start, from a theoretical standpoint. Although a graph merely has a set of vertices and a relationship between pairs of vertices, it is a simple structure, yet graph theory is a vast and diverse field of study. This is partially because graphs can be thought of as topological spaces, combinatorial objects, and many other mathematical structures in addition to being relational structures [1]. This brings us to our second argument about the significance of graphs: many ideas may be abstractly represented by graphs[3], which makes them very helpful in real-world applications. Several earlier studies on the subject of topological graphs we can see in [4-11]. In this paper we discuss a new method to generate topology $\tau_{IƊ}$ on graph by using new method of taking neighborhood is determining a vertex on the digraph and calculate each vertex and its edges indgree of it and we defined $S_{IƊ} = \{ \overrightarrow{Ẹ_{Ṿ}} | Ṿ \in Ṿ\}$, where $Ẹ_{Ṿ}$ is the set of all edges indgree to Ṿ, we have $Ẹ = \cup_{Ṿ\in Ṿ} \overrightarrow{Ẹ_{Ṿ}}$. hence $S_{IƊ}$ forms a subbasis for a topology $\tau_{IƊ}$ on Ẹ, called indigraphic topology,(briefly digtopology) of Ɗ.

Definition 2.1: A digraph Ɗ is a triple consisting of a vertex set (Ṿ(Ɗ), Ẹ(Ɗ)) , an edge set , and a relation that associates with each edge two vertices (not necessarily distinct) called it`s end point and we express a graph to arranged pairs $Ɗ = (Ṿ, Ẹ)$ or $Ɗ = (Ṿ(Ɗ), Ẹ(Ɗ))$.

Definition 2.2: Let $Ɗ = (Ṿ, Ẹ)$ be a digraph, we call H is a subdigraph from D if $Ṿ(H) \subseteq Ṿ(Ɗ)$, $Ẹ(H) \subseteq Ẹ(Ɗ)$, in this case we would write $H \subseteq Ɗ$.

Definition 2.3 : Let $Ɗ = (Ṿ, Ẹ)$ be a digraph, we say that two vertices Ṿ and w of a graph (resp., digraph Ɗ) are adjacent if there is an edge of the form $Ṿw$ (rsep., $\overrightarrow{wṾ}$ or $\overrightarrow{Ṿw}$) joining them, and the vertices v and w are then incident with such an edge.

Definition 2.4 : If Y is non-empty set, a collection $\tau \subseteq P\,(Y)$ is called topology on Y if the following holds:
(1) , $\emptyset \in \tau$.
(2) The intersection of a finite number of elements in , *is in* τ .
(3) The union of a finite or infinite number of elements of sets in τ *belong t* . Then (,) is called a topological space. Any element in (Y, τ) is called open set and it is complement is called closed set .

Definition 2.5 : Let Y is a non-empty set and let τ is the collection of every subsets from Y. Then τ is named the discrete topology on the set Y. The topological space (, τ) is called a discrete space. If τ = { , $\emptyset$}. Then τ is named indiscrete topology and the topological space (, τ) is named an indiscrete topological space .

Definition 2.6 : Let $(Y\ ,\tau)$ be a topological space, $A \subseteq Y$. The closure of A symbolized by (A) is defined as the smallest closed set that includes A.It is thus the intersection of every closed sets that include A .

Definition 2.7 :"Let $(Y\ ,\tau)$ be a topological space, $A \subseteq Y$. The interior of A symbolized by $Int\ (A)$ is defined as the largest open set included in A. It is thus the union of every open sets included in A .

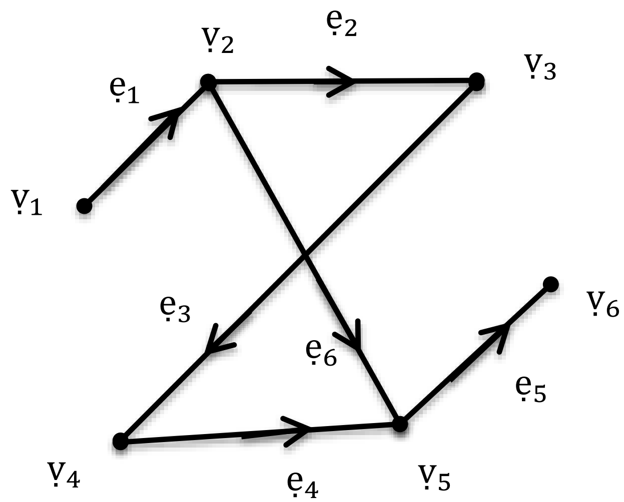
Definition 2.8 : Let $(Y\ ,\tau)$ be a topological space, $A \subseteq Y$ is said dense if $\overline{A} = Y$.

3. Indigraphic Space.

In this section, we offer our novel subbasis family of a digraph Ɗ = (Ṿ, Ẹ) to build a topology on the set of Edges.

Definition 3.1 : Let $Ɗ = (Ṿ,Ẹ)$ be a digraph, we defined $S_{IƊ} = \{ \overrightarrow{Ẹ_{Ṿ}} | Ṿ \in Ṿ\}$, where $Ẹ_{Ṿ}$ is the set of all edges indgree to Ṿ, we have $Ẹ = \cup_{Ṿ\in Ṿ} \overrightarrow{Ẹ_{Ṿ}}$. hence $S_{IƊ}$ forms a subbasis for a topology $\tau_{IƊ}$ on Ẹ, called indigraphic topology,(briefly indigtopology) of Ɗ.

Example 3.2 : Let Ɗ = (Ṿ,Ẹ) be digraph as in Figure (1), such that $Ṿ = \{Ṿ_1, Ṿ_2, Ṿ_3, Ṿ_4, Ṿ_5, Ṿ_6\}$, $Ẹ = \{ẹ_1, ẹ_2, ẹ_3, ẹ_4, ẹ_5, ẹ_6\}$.



Figure(1)

We have, $\overrightarrow{E_{v_1}} = \emptyset$, $\overrightarrow{E_{v_2}} = \{e_1\}$, $\overrightarrow{E_{v_3}} = \{e_2\}$, $\overrightarrow{E_{v_4}} = \{e_3\}$, $\overrightarrow{E_{v_5}} = \{e_4, e_6\}$, $\overrightarrow{E_{v_6}} = \{e_5\}$.and $S_{ID} = \{\emptyset, \{e_1\}, \{e_2\}, \{e_3\}, \{e_5\}, \{e_4, e_6\}\}$.

By taking finitely intersection the basis obtained is : $\{\emptyset, \{e_1\}, \{e_2\}, \{e_3\}, \{e_5\}, \{e_4, e_6\}\}$. Then by taking all unions the topology can be written as:

$\tau_{ID} = \{ E(D), \emptyset, \{e_1\}, \{e_2\}, \{e_3\}, \{e_5\}, \{e_4, e_6\}, \{e_1, e_2\}, \{e_1, e_3\}, \{e_1, e_5\}, \{e_2, e_3\}, \{e_2, e_5\},$
$\{e_3, e_5\}, \{e_1, e_2, e_3\}, \{e_1, e_2, e_5\}, \{e_1, e_3, e_5\}, \{e_2, e_3, e_5\},$
$\{e_1, e_4, e_6\}, \{e_2, e_4, e_6\}, \{e_3, e_4, e_6\}, \{e_5, e_4, e_6\},$
$\{e_1, e_2, e_3, e_5\}, \{e_1, e_2, e_4, e_6\}, \{e_1, e_3, e_4, e_5\},$
$\{e_1, e_5, e_4, e_5\}, \{e_2, e_3, e_4, e_5\}, \{e_2, e_5, e_4, e_5\},$
$\{e_3, e_5, e_4, e_5\}, \{e_1, e_2, e_3, e_4, e_6\}, \{e_1, e_2, e_5, e_4, e_6\},$
$\{e_1, e_3, e_5, e_4, e_6\}, \{e_2, e_3, e_5, e_4, e_6\}\}$.

Then τ_{ID} is topology is called indigtopology τ_{ID} .

Definition 3.3 : Let $D = (V, E)$ be a digraph then $\overrightarrow{E_v}$ is the set of all edges that indgree to the vertice v.

Example 3.4 : According to example 3.2 **,** we get
$\overrightarrow{E_{v_1}} = \emptyset$, $\overrightarrow{E_{v_2}} = \{e_1\}$, $\overrightarrow{E_{v_3}} = \{e_2\}$, $\overrightarrow{E_{v_4}} = \{e_3\}$, $\overrightarrow{E_{v_5}} = \{e_4, e_6\}$, $\overrightarrow{E_{v_6}} = \{e_5\}$.

Proposition 3.5 : Suppose that τ_{ID} is the indigtopology of the adigraph $D = (V, E)$, then $\{e\} \in \tau_{ID}$ if $\overrightarrow{I_e}^v \neq \overrightarrow{I_{é}}^v$ for all $é \in E$.

Prove: Let $e \in E$ then $\overrightarrow{I_e}^v = \{v\}$ for some $v \in V$ and by hypothesis $\overrightarrow{I_e}^v \neq \overrightarrow{I_{é}}^v$ for all $é \in E$ then we get e is only edge is directed to v and hence then $\overrightarrow{E_v} = \{e\}$ and by definition indigtopology τ_{ID} we get $\{e\} \in \tau_{ID}$.

Remark 3.6 : Let $D = (V, E)$ be a digraph, then the indigtopology τ_{ID} is not necessary to be discrete topology in general .

Example 3.7 : Let C_5 be cyclic digraph such that the edges are not all in the same direction, shown in Figure (2).
We have
$\overrightarrow{E_{v_1}} = \{e_1, e_6\}$, $\overrightarrow{E_{v_2}} = \emptyset$, $\overrightarrow{E_{v_3}} = \{e_2\}$, $\overrightarrow{E_{v_4}} = \{e_3, e_4\}$, $\overrightarrow{E_{v_5}} = \{e_5\}$, $\overrightarrow{E_{v_6}} = \emptyset$.
And $S_{ID} = \Big\{\emptyset, \{e_1, e_6\}, \{e_3, e_4\}, \{e_2\}, \{e_5\}\Big\}$.
$\tau_{ID} = \{ E(D), \emptyset, \{e_1, e_6\}, \{e_3, e_4\}, \{e_2\}, \{e_5\}, \{e_2, e_5\}, \{e_1, e_6, e_2\}, \{e_1, e_6, e_5\}, \{e_3, e_4, e_2\},$
$\{e_3, e_4, e_5\}, \{e_1, e_6, e_3, e_4\}, \{e_1, e_6, e_2, e_5\},$
$\{e_1, e_6, e_5, e_2\} \{e_3, e_4, e_2, e_5\}, \{e_3, e_4, e_5, e_2\},$
$\{e_1, e_6, e_3, e_4, e_2\}, \{e_1, e_6, e_3, e_4, e_5\}\}$. Then we get the indigtopology τ_{ID} of C_6 is not discrete topology .

Proposition 3.8 : The indigtopology (E, τ_{ID}) of digraph $D = (V, E)$ is Alexandroff space .

Prove : It is adequate to show that arbitrary intersection of elements of S_{ID} is open , Let $A \subseteq V$: $\bigcap_{v \in A} \overrightarrow{E_v} =$

$$\begin{cases} \overrightarrow{E_v} & \textit{if A contin one vertex } v \\ \emptyset & \textit{owther wise} \end{cases}$$

And by Definition 3.1 of indigtopology τ_{ID} we get $\emptyset, \overrightarrow{E_v} \in \tau_{ID}$, then $\bigcap_{v \in A} \overrightarrow{E_v}$ is open . Hence the indigtopology τ_{ID} is satisfies property of Alexandroff .

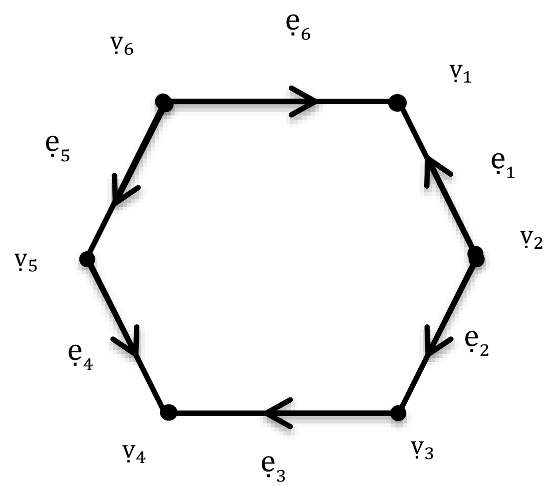


Figure (2)

Definition 3.9 : In any digraph $D = (V, E)$ since (E, τ_{ID}) is Alexandroff space, for each $e \in E$, the intersection of all open set containing e is the smallest open set containing e and denoted by U_e, Also the family $M_D = \{U_e | e \in E\}$ is the minimal basis for the indigtopology τ_{ID} .

International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 7 Issue 3, March - 2023, Pages: 8-12

Proposition 3.10 : In any digraph $Ɗ = (Ṿ,Ẹ)$, $U_{ẹ} = \overrightarrow{Ẹ_{ṿ}}$ where $\overrightarrow{I_{ẹ}}^{v} = \{ṿ\}$ for every $ẹ \in Ẹ$.

Prove : $\Rightarrow$ Since every $ẹ \in Ẹ$ then $\overrightarrow{I_{ẹ}}^{v} = \{ṿ\}$ for some $ṿ \in Ṿ$ and by Definition3.1.1,of indigtopology τ_{ID}, $Ẹ_{ṿ}$ is open contain $ẹ$ and by of $U_{ẹ}$ then we get $U_{ẹ} \subseteq \overrightarrow{Ẹ_{ṿ}}$.
$\Leftarrow$ since $U_{ẹ}$ is open set and contain $ẹ$ then by definition indigtopology τ_{ID} there exist $\overrightarrow{Ẹ_{ṿ}}$ for some $ṿ \in Ṿ$ such that $ẹ \in \overrightarrow{Ẹ_{ṿ}} \subseteq U_{ẹ}$, and since $é \notin U_{ẹ}$ implies $é \notin \overrightarrow{Ẹ_{ṿ}}$ when $ẹ \in \overrightarrow{Ẹ_{ṿ}}$, then $é \notin \overrightarrow{Ẹ_{ṿ}}$ when $\overrightarrow{I_{ẹ}}^{v} = \{ṿ\}$ thus $\overrightarrow{Ẹ_{ṿ}} \subseteq U_{ẹ}$ when $\overrightarrow{I_{ẹ}}^{v} = \{ṿ\}$, then $U_{ẹ} = \overrightarrow{Ẹ_{ṿ}}$ where $\overrightarrow{I_{ẹ}}^{v} = \{ṿ\}$ for every $ẹ \in Ẹ$.

Remark 3.11 : Let $Ɗ = (Ṿ, Ẹ)$ be a digraph, if $\overrightarrow{I_{ẹ}}^{v} \neq \overrightarrow{I_{é}}^{v}$ for all $é \in Ẹ$ then $U_{ẹ} = \{ẹ\}$.

Proof : clear

Theorem 3.12 : For any $ẹ,é \in Ẹ$ in a digraph $Ɗ = (Ṿ,Ẹ)$ we have $\overrightarrow{I_{ẹ}}^{v} = \overrightarrow{I_{é}}^{v}$ iff $ẹ \in U_{é}$ i.e $U_{ẹ} = \{é \in Ẹ \mid \overrightarrow{I_{ẹ}}^{v} = \overrightarrow{I_{é}}^{v}\}$.

Proof : $\Longrightarrow$ Let $\overrightarrow{I_{ẹ}}^{v} = \overrightarrow{I_{é}}^{v}$ to prove $ẹ \in U_{é}$. Since $\overrightarrow{I_{ẹ}}^{v} = \overrightarrow{I_{é}}^{v} = \{ṿ\}$, implies $ẹ,é \in \overrightarrow{Ẹ_{ṿ}}$ and by Proposition3.11, $U_{ẹ} = \overrightarrow{Ẹ_{ṿ}}$ and $U_{é} = \overrightarrow{Ẹ_{ṿ}}$, and hence $U_{ẹ} = U_{é} = Ẹ_{ṿ}$ then $ẹ \in U_{é}$.
$\Longleftarrow$ let $ẹ \in U_{é}$ and by Proposition 3.111, we get $U_{é} = \overrightarrow{Ẹ_{ṿ}}$ where $\overrightarrow{I_{é}}^{v} = \{ṿ\}$, since $ẹ \in U_{é}$ we get $ẹ \in \overrightarrow{Ẹ_{ṿ}}$ where $\overrightarrow{I_{ẹ}}^{v} = \overrightarrow{I_{é}}^{v} = \{ṿ\}$ hence $\overrightarrow{I_{ẹ}}^{v} = \overrightarrow{I_{é}}^{v}$.

Corollary 3.13 : In any digraph $Ɗ = (Ṿ,Ẹ)$, $U_{ẹ} = \overrightarrow{Ẹ_{ṿ}}$ where $\overrightarrow{I_{ẹ}}^{v} = \{ṿ\}$ for every $ẹ \in Ẹ$.

Proof : $\Rightarrow$ Since every $ẹ \in Ẹ$ then $\overrightarrow{I_{ẹ}}^{v} = \{ṿ\}$ for some $ṿ \in Ṿ$ and by definition of indigtopology τ_{ID}, $Ẹ_{ṿ}$ is open contain $ẹ$. by Definition 3.12, of $U_{ẹ}$ then we get $U_{ẹ} \subseteq \overrightarrow{Ẹ_{ṿ}}$.
$\Leftarrow$ since $U_{ẹ}$ is open set and contain $ẹ$ then by definition indigtopology τ_{ID} there exist $\overrightarrow{Ẹ_{ṿ}}$ for some $ṿ \in Ṿ$ such that $ẹ \in \overrightarrow{Ẹ_{ṿ}} \subseteq U_{ẹ}$, and since $é \notin U_{ẹ}$ implies $é \notin \overrightarrow{Ẹ_{ṿ}}$ when $ẹ \in \overrightarrow{Ẹ_{ṿ}}$, then $é \notin \overrightarrow{Ẹ_{ṿ}}$ when $\overrightarrow{I_{ẹ}}^{v} = \{ṿ\}$ thus $\overrightarrow{Ẹ_{ṿ}} \subseteq U_{ẹ}$ when $\overrightarrow{I_{ẹ}}^{v} = \{ṿ\}$, then $U_{ẹ} = \overrightarrow{Ẹ_{ṿ}}$ where $\overrightarrow{I_{ẹ}}^{v} = \{ṿ\}$ for every $ẹ \in Ẹ$.

4. Properties of Indigraphic Topology.

In this section, some properties of our new structure we investigated.

Proposition 4.1 : Let τ_{ID} be indigtopology of the a digraph $Ɗ = (Ṿ,Ẹ)$ then we have the following**:**

(i) If $H = \{ẹ \in Ẹ \mid \overrightarrow{I_{ẹ}} \neq \overrightarrow{I_{é}}\ \forall\, é \in Ẹ\}$, then $H \in \tau_{ID}$.

(ii) If $K = \{ẹ \in Ẹ \mid \overrightarrow{I_{ẹ}} = \overrightarrow{I_{é}}\ for\ some\ é \in Ẹ\}$, then K is closed in τ_{ID}.

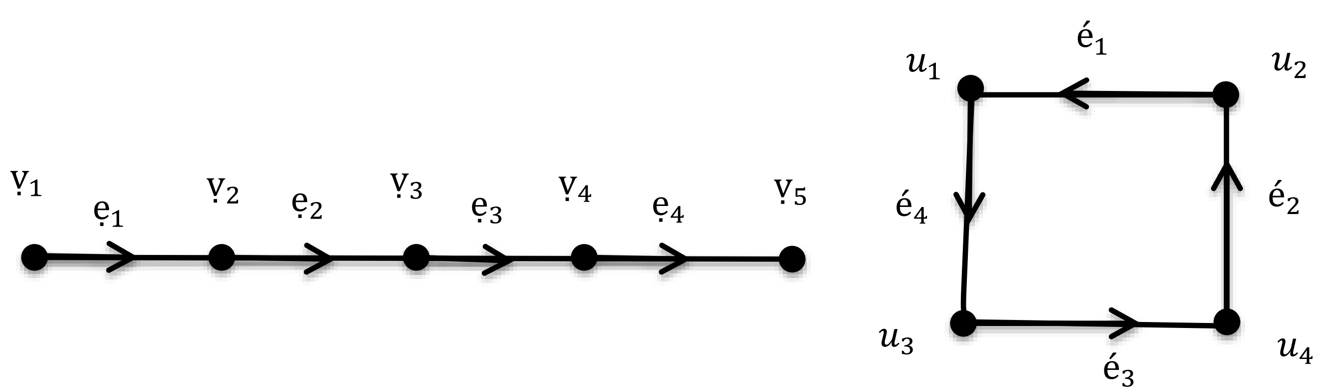
Proof:

(i) Let $ẹ \in H$ then $\overrightarrow{I_{ẹ}} \neq \overrightarrow{I_{é}}$ for all $é \in Ẹ$ and by Remark3.11, $U_{ẹ} = \{ẹ\}$, As result, $ẹ \in U_{ẹ} \subseteq H$ and so $ẹ$ is interior point of H, Hence $H \in \tau_{ID}$

(ii) By assumption $K = \bigcup_{ẹ \in k}\{ẹ\}$ and so, $\overline{K} = \overline{\bigcup_{ẹ \in k}\{ẹ\}} = \bigcup_{ẹ \in k}\overline{\{ẹ\}}$ by proposition of closure. let $é \in \overline{K}$, then $é \in \overline{\{ẹ\}}$ for some $ẹ \in K$. by Theorem 3.13, $\overrightarrow{I_{ẹ}} = \overrightarrow{I_{é}}$ and so $é \in K$ hence $\overline{K} \subseteq K$, and the proof complete.

Example 4.2: According to Example 3.7, by a cyclic digraph C_6 then we get $H = \{ẹ_2, ẹ_5\}$, $K = \{ẹ_1, ẹ_6\}$.
We note that $\{ẹ_2, ẹ_5\} \in \tau_{ID} \Longrightarrow H \in \tau_{ID}$ and K *is* closed in τ_{ID} since $\{ẹ_1, ẹ_6\}^{c} = \{ẹ_2, ẹ_3, ẹ_4, ẹ_5\} \in \tau_{ID}$.

Proposition 4.3 : Let $Ɗ = (Ṿ,Ẹ)$ be a digraph, then $(Ẹ, \tau_{ID})$ is a compact indigtopology τ_{ID} if and only if $Ẹ$ is finite.



$(Ɗ_1)$ $(Ɗ_2)$

Figure (3)

International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 7 Issue 3, March - 2023, Pages: 8-12

Proof: Let (Ẹ,τ_{ID}) is a compact indigtopologyτ_{ID} suppose that Ẹ is infinite Then $M_{D} = \{U_{ẹ} \mid ẹ \in Ẹ\}$ is an open covering of (Ẹ, τ_{ID}) which has no finite sub cover. Therefore, (Ẹ, τ_{ID}) is incongruous since it is not compact. for the converse, it follows directly that (Ẹ, τ_{ID}) is a compact because the number of open subsets on the finite space is finite.

Definition 4.4 : Two digraph $D_1 = (V_1, E_1)$ and $D_2 = (V_2, E_2)$ are said to be isomorphic to each other, and written $D_1 \cong D_2$ if there is a bijection $\mathcal{F} : V_1 \to V_2$ with $\{x, y\} \in E_1$ if and only if $\{\mathcal{F}(x), \mathcal{F}(y)\} \in E_2$ for all $x, y \in V_1$ the function $\mathcal{F}$ is called an isomorphism .

Example 4.5 : Let $D_1 = (V_1, E_1)$,$D_2 = (V_2, E_2)$ are be digraph in figure (3) such that $V_1 = \{a, b, c, d\}$,$V_2=\{v_1, v_2, v_3, v_4\}$.
Thin the digraph D_1 , D_2 are isomorphic. since $a \to b \to c \to d$ and $v_4 \to v_1 \to v_2 \to v_3$ and put $\mathcal{F} : V_1 \to V_2$ such that $\mathcal{F}(a) = v_4$, $\mathcal{F}(b) = v_1$, $\mathcal{F}(c) = v_2$, $\mathcal{F}(d) = v_3$

Remark 4.6 : It is clear that the indigtopology (E_1, τ_{ID_1}) and (E_2, τ_{ID_2}) are homeomorphic if the digraphs $D_1 = (V_1, E_1)$ and $D_2 = (V_2, E_2)$ are isomorphic but in general the opposite is not true.
The following example is applied to the opposite is not true.

Example 4.7: Let $D_1 = (V_1, E_1)$,$D_2 = (V_2, E_2)$ are be two digraphs in Figure (4), such that $V_1 = \{v_1, v_2, v_3, v_4\}$ and $V_2 = \{u_1, u_2, u_3, u_4\}$.

The indigtopology (E_1, τ_{ID_1}) and (E_2, τ_{ID_2}) are homeomorphic (since both are discrete topology), But they are not isomorphic digraph .

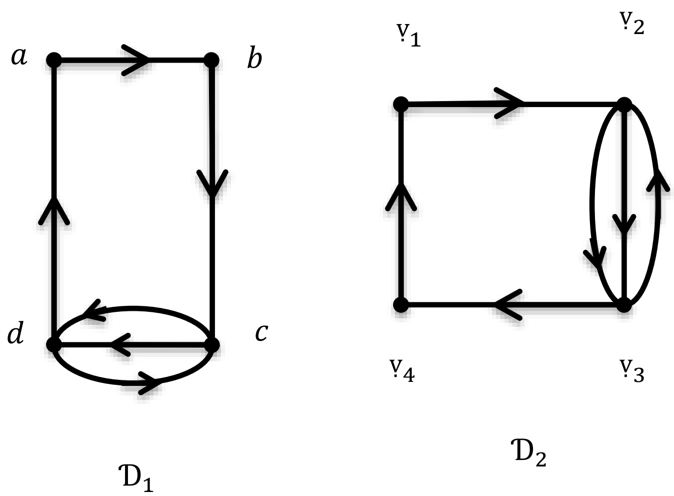


Figure (4)

5. Stipulations on Topological Space to be Digraph Space.

This section illustrates the prerequisite for topology space to be a indigraphic topology space.

Definition 5.1: Any topological space (Ạ,) is called indigtopology if $\mathcal{T} = \tau_{ID}$ for some digraph D with edge set Ạ .

Remark 5.2 :

(i) if $\mathcal{T}$ is discrete topology on Ạ, then by Corollary 3.8, $\mathcal{T} = \tau_{ID}$ for some digraph D with edges set Ạ, such that $\overrightarrow{I_{ẹ}}^{v} \neq \overrightarrow{I_{é}}^{v}$ for all every distinct pairs of edge ẹ, é ∈ Ẹ. hence $\mathcal{T}$ is an indigtopology.

(ii) If $\mathcal{T}$ is not discrete topology on Ạ, if all the elements of basis of τ_{ID} are disjoint then $\mathcal{T} = \tau_{ID}$ for some digraph D with edges set Ạ, such that all open set u of basis of τ_{ID} then u is set of edges are directed to the same vertex.

Example 5.3 : Let Ạ = {a ,b ,c ,d } we note that the topology $\mathcal{T}_1 = \{\emptyset$, Ạ,{ a},{b},{c},{d},{a, b},{a, c},{a ,d},{b, c},{b, d},{c, d}, {a, b , c} , {a, b ,d}, {b ,c ,d},{a, c, d}} is indigtopology τ_{ID} , since $\mathcal{T}_1$ is discrete topology. shown in Figure (5)(A).

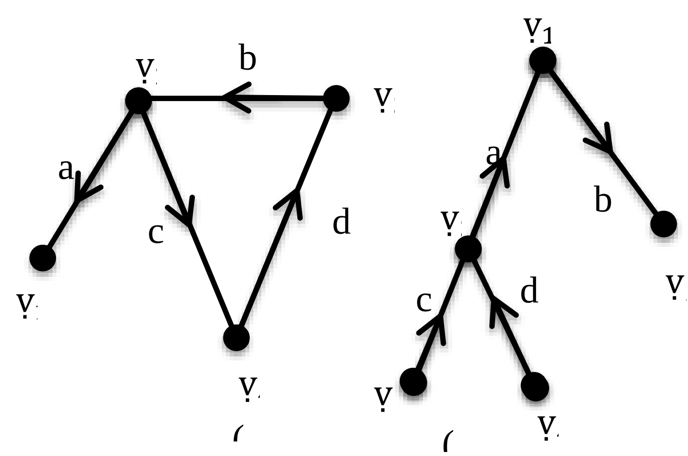


Figure (5)

Also, we note that the topology $\mathcal{T}_2 = \{\emptyset$,Ạ,{$a$},{b},{a, b},{c, d},{a, c , d}, {b, c, d}} is indigtopology τ_{ID} since every element in basis β={{a},{b},{c,d}} are disjont, shown in Figure(5)(B)

But the topology $\mathcal{T}_3 = \{\emptyset$, Ạ,{ a},{b},{a, b},{a,c},{a, b , c}}is not indigtopology τ_{ID} since {a}∩{a,c}={a} in basis β ={{a},{b},{a,c}} hence $\mathcal{T}_3$ is not form indigtopology τ_{ID} .

proposition 5.4: Let τ_{ID} be the a indigtopology of the a digraph D = (V,Ẹ) the topological space (E^*,$\mathcal{T}$) is indigtopology τ_{ID} if it is homeomorphic to (Ẹ,τ_{ID}).

International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 7 Issue 3, March - 2023, Pages: 8-12

Proof : Suppose that $\mathcal{F}$: (Ẹ, $\tau_{\mathrm{I}Ɗ}$) $\rightarrow$ (Ẹ*,$\mathcal{T}$) is a homeomorphism. since (Ẹ,$\tau_{\mathrm{I}Ɗ}$) is an Alexandroff space and (Ẹ,$\tau_{\mathrm{I}Ɗ}$) $\cong$ (Ẹ*,$\mathcal{T}$), (Ẹ*,$\mathcal{T}$) is an Alexandroff space. to construct on Ẹ* adigraph Ɗ* = (Ṿ*, Ẹ*) we put { $\mathcal{F}$(é), $\mathcal{F}$(ẹ)} is indegree edges in Ẹ* if anf only if {é,ẹ} is indegree in Ẹ for every é,ẹ $\in$ Ẹ. Then we have $\mathcal{F}$({é,ẹ}) ={ $\mathcal{F}$(é), $\mathcal{F}$(ẹ)} and so $\mathcal{T} = \tau_{\mathrm{I}Ɗ^*}$. As a result, U$^*_{ẹ}$ = $M_{é}$ such that U$^*_{é}$, $M_{é}$ are the smallest open set containing é in (Ẹ*, $\tau_{\mathrm{I}Ɗ^*}$) and (Ẹ*,$\mathcal{T}$) respectively. Since $\mathcal{F}$ is ahomeomorphism, $\mathcal{F}$(U$^*_{é}$) = $M_{\mathcal{F}(ẹ)}$ such that U$^*_{é}$ is the smallest open set containing é in (Ẹ*, $\tau_{\mathrm{I}Ɗ^*}$). also $\mathcal{F}$ is an isomorphism between Ɗ and Ɗ*, then $\mathcal{F}$(U$_{é}$) = U$^*_{\mathcal{F}(é)}$.

6. Density Indigraphic Topology space .

This section examines several prerequisites for digraph-related dense subsets of the indigraphic space.

Remark 6.1: It is known that in (Ẹ, $\tau_{\mathrm{I}Ɗ}$)) the subset K $\subseteq$ Ẹ is dense in Ẹ if and only if the complement of K has empty interior .

Proposition 6.2 : Let (Ẹ,$\tau_{\mathrm{I}Ɗ}$) be indigtopology $\tau_{\mathrm{I}Ɗ}$ of a digraph Ɗ = (Ṿ, Ẹ) then the set K={ẹ $\in$ Ẹ| $\overrightarrow{\mathrm{I}_{ẹ}}^{v} \neq \overrightarrow{\mathrm{I}_{é}}^{v}$, $\forall$é $\in$ Ẹ}is dance in (Ẹ, $\tau_{\mathrm{I}Ɗ}$).

Proof : By Remark6.1, it is enough to prove that the complement of K has empty interior. For every ẹ $\in K^c$, ẹ is a edge such that $\overrightarrow{\mathrm{I}_{ẹ}}^{v} = \overrightarrow{\mathrm{I}_{é}}^{v}$ for some é $\in$ Ẹ. And hence by Proposition3.1.14, we get {ẹ}$\notin \tau_{\mathrm{I}Ɗ}$ for every ẹ $\in K^c$. As a result, B $\subseteq K^c$, for every B cannot be written as a union of finitely intersection of elements of $S_{\mathrm{I}Ɗ}$, i.e B $\notin \tau_{\mathrm{I}Ɗ}$ hence int(K^c) = $\emptyset$. and this means K is dense subset in (Ẹ, $\tau_{\mathrm{I}Ɗ}$) .

Example 6.3: According to Example 3.2, we note that H = {ẹ$_1$, ẹ$_2$, ẹ$_3$, ẹ$_5$} implies H^c ={ẹ$_4$,ẹ$_6$}$\in \tau_{\mathrm{I}Ɗ}$ then $(H^c)^o$ = {ẹ$_4$, ẹ$_6$} so H is not dense in indigtopology $\tau_{\mathrm{I}Ɗ}$.

Also, in Example 3.2, we note that if K={ẹ$_1$, ẹ$_6$} then K^c = {ẹ$_2$, ẹ$_3$, ẹ$_4$, ẹ$_5$} $\in \tau_{\mathrm{I}Ɗ}$ implies $(K^c)^o$ = {ẹ$_2$, ẹ$_3$, ẹ$_4$, ẹ$_5$} so K is not dense in indigtopology $\tau_{\mathrm{I}Ɗ}$.

References

[1] J. Bondy, D. S. Murty, Graph theory with applications, North- Holland, 1992.

[2] J. R. Munkres, Topology, Prentice- Hall, Inc., Englewood Cliffs, New Jersey, 1975.

[3] R. J. Wilson, Introduction to Graph Theory, Longman Malaysia, 1996.

[4] S.P. subbaih, A study of Graph Theory: Topology, Steiner Domination and Semigraph Concepts, Ph.D. thesis, Madurai Kamaraj University, India, 2007.

[5] K.Karunakaran, Topics in Graph Theory-Topological Approach, Ph.D. thesis, University of Kerata, India 2007

[6] U.Thomas, A study on Topological set-indexers of Graphs ph.D. thesis, Mahatma Gandhi university, India, 2013.

[7] M. Shorky, Generating Topology on Graphs by Operations on Graphs, Applied Mathematical Science, 9(54),PP 2843-2857, 2015.

[8] Kh. Sh Al,Dzhabri, A.M, Hamza and Y.S. Eissa, On DG-Topological spaces Associated with directed graphs, Journal of Discrete Mathematical Sciences and Cryptograph, 12(1): 60-71 DoI: 10.1080109720529.2020.1714886

[9] Kh. Sh Al,Dzhabri and M.F.Hani, On Certain Types of Topological spaces Associated with Digraphs, Journal of Physics: Conference Series 1591(2020)012055 doi:10.1088/1742-6596/1/012055

[10] Kh.Sh. Al'Dzhabri and et al, DG-domination topology in Digraph. Journal of Prime Research in Mathematics 2021, 17(2), 93–100. http://jprm.sms.edu.pk/

[11] Kh.Sh. Al Ɗzhabri, Enumeration of connected components of acyclic digraph. Journal of Discrete Mathematical Sciences and Cryptography, 2021, 24(7), 2047–2058. DOI: 10.1080/09720529.2021.1965299